Wireless Communications

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***Abstract*—Wireless communication is classically performed by transmitting an encoded message signal using electromagnetic waves as a physical channel which is received and decoded after estimation to get the message. We implemented a very simplified version of this model, which encoded the text message into bits using codes (like Huffman codes, repetition codes etc) to reduce error probability. The codes are then converted into a time signal, which is scaled and transmitted into an EM wave channel, the received signal contains added Gaussian noise, so the signal is rounded off, and parity bits are estimated and then decoded to give the original text message. Error could be further reduced by using a minimum distance decoder or other plausible decoders at the receiver end as the channel is probabilistic, and extensive use of probability concepts is done for accurate estimation in more complex systems. This way we could simulate a very basic wireless communication system.**

***Index Terms*—wireless communication, probability, random process**

I. Introduction

Wireless communication is one of the most vibrant areas in the communication field today.It focuses on the transmission of information in the form of signals from transmitter/s to receiver/s without the use of any physical objects such as Electrical wires, optical fibres etc. Electro-Magnetic (EM) Waves are used in this kind of communication. The distance of communication ranges from few meters which is used in Bluetooth to billions of kilometers used in deep-space radio communication.

There is a heavy dependence of Probability, Random Processes and Statistics in the study of wireless communication. This is because the transmitted EM signal reaches the receiver through wireless channel which may add noise to the signal and corrupt the transmitted information. Hence there is quite a lot of uncertainty in decoding the correct information.In order to minimize the chances of such errors the study of Probability Theory in wireless communication becomes important.

II. Noise in Wireless Communication

In wireless communication noise is any unwanted signals which interferes with the actual information sent by the transmitter and this corrupts the message Signal.

One way to measure the Noise in a Signal is by the Signal to Noise Ratio (SNR) which is defined as

$$SNR = \frac{Power\ of\ Signal}{Power\ of\ Noise}$$

Hence from the above equation we can understand that higher the SNR ratio of a channel better is the closeness of transmitted signal to the Original Signal.

There are many kinds of noise which are induced into a Signal of these one of the most common ones is Additive White Gaussian Noise (AGWN)

A. Additive White Gaussian Noise - AGWN

AGWN is the kind of probability distribution which is used to mimic the noises which are naturally added to a Signal which is transmitted. It has uniform distribution across all frequencies and hence it is known is as uniform.

If $x(t)$ is the input Signal and let $w(t)$ be the noise signal induced into the input signal which has a Gaussian distribution with mean = 0 and variance = σ^2, Then the signal received by the receiver $y(t)$ is

$$y(t) = x(t) + w(t)$$

$$where\, pdf\ of\ w\ f_w(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}} \quad (the\, gaussian\, noise)$$

B. Auto-correlation function of Gaussian Noise

Let the mean of the noise X(t) at some time instance be μ_x then by the definition of auto-correlation

$$R_{xx}(\tau) = E(X(t).X(t+\tau))$$

$$now\ if\ \tau = 0 \;\; R_{xx}(0) = E(X(t)^2)$$

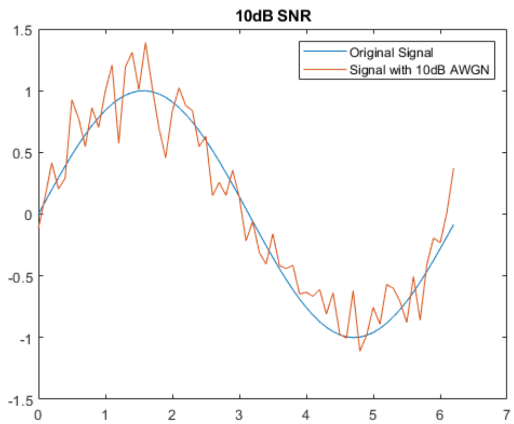


Fig. 1. Transmitted Signal which contains 10dB SNR Noise

This implies $R_{xx}(0) =$ Second moment of X

$$Now\ Var(X) = E(X^2(t)) - E(X(t))^2$$

$$Var(X) = R_{xx}(0) - \mu^2$$

Hence pdf of X at time instant t is

$$f_X(x) = \frac{1}{\sqrt{2\pi(R_{xx}(0) - \mu_x^2)}} e^{-\frac{(x-\mu_x)^2}{2.(R_{xx}(0)-\mu_x^2)}}$$

Now since X is a white random Process as stated above hence it satisfies the below property by definition of white random process

$$R_{xx}(t) = \frac{\eta}{2}\delta(t) \quad \forall\ t \quad here\ \eta\ is\ a\ constant$$

This implies that X and X+τ are correlated at t = 0 and uncorrelated for t $\neq$ 0

C. Power Spectral Densiy of AWGN

By definition Power Spectral density it is the Fourier Transform of the Auto-correlation function. Therefore, the PSD of White Noise is

$$S_{xx}(F) = \frac{\eta}{2} \quad (Since\ fourier\ transform\ of\ \delta(t)\ is\ 1)$$

Here F is the variable in frequency domain.

III. Fading in Wireless Channels and Relation to Random Processes

Fading refers to the variation in the power of a signal due to multiple variables. Typically, this can be referred to as the characteristic of a wireless channel, that varies with time and frequency of the signal.

Fading occurs due to multiple reasons that can be characterized primarily into two types

- Large-scale fading
- Small-scale fading

Large-scale fading can be thought of as fading occurring to due obstruction by obstacles, like trees, buildings, mountains etc. In other words, large-scale fading is the signal power attenuation over large areas.

Small-scale fading typically results from the signal taking multiple paths through the channel from the source to the receiver, which causes constructive and destructive interferences. Small-scale fading can further be classified into

- Slow fading
- Fast fading
- Flat fading
- Frequency-selective fading

Slow and fast fading is mainly characterised by Doppler spreads, while flat fading and frequency-selective fading is characterized by delay spreads.

Fading can be modelled as a random process, and typically the fading process is characterized by Rician distributions for a line of sight path between the source and the receiver where there is one dominant signal; and by Rayleigh distributions for non line of sight paths where there could be multiple dominant signals.

On occasions when strong destructive interferences occur, there is a significant decline in the signal-to-noise ration (SNR) and typically leads to outages in communication and this is commonly referred to as deep fade. We define the outage probability as the probability that the SNR falls below a certain threshold value.

Since fading depends on signal attenuation and propagation delays (primarily due to multiple paths), for a fading multipath channel, for a transmitted signal $x(t)$, the received signal $y(t)$ can be written as,

$$y(t) = \sum_i \alpha_i(t)x(t - \tau_i(t))$$

where $\alpha_i(t)$ are the signal attenuations that vary with time, and $\tau_i(t)$ are the signal propagation delays that vary with time. Moreover, certain noise is bound to come into play during transmission and this is primarily assumed to be at the receiver end. This noise can be modelled as an Additive White Gaussian Noise (AWGN) $w(t)$ with a power spectral density of $N_0/2$ and zero mean. So, $y(t)$ can be written as,

$$y(t) = \sum_i \alpha_i(t)x(t - \tau_i(t)) + w(t)$$

Here, if we were to consider uniformly spaced samples of $w(t)$ as $w[m]$, then they could be considered as independent identically distributed Gaussian random variable with mean 0 and variance $N_0/2$.

The input and output signals at either end of a channel can be modelled through discrete-time signals, and with the help of 'filter taps' denoted by h_l. Then the input and output signals can be related as,

$$y[m] = \sum_l h_l[m]x[m - l] + w[m]$$

where w[m] is the noise. It is easier to model the input-output relations using filter taps.

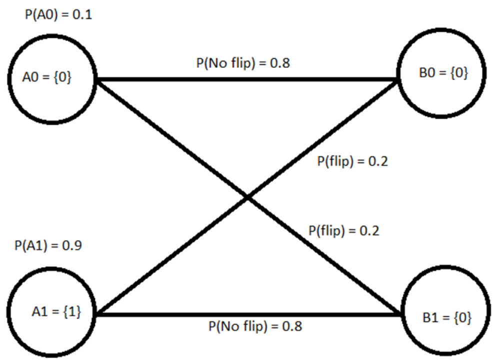


Fig. 2. The Binary symmetric Channel which is denoted by BSC(0.2)

IV. Applications of Baye's Theorem in Wireless Communications

A. MAP receiver (or) maximum a posteriori probability receiver

Consider the sample space $S = \{0, 1\}$ which is nothing but the binary information symbols.

Now consider the events $A_0 = \{0\}$ and $A_1 = \{1\}$

Here $A_0\ and A_1$ are mutually exhaustive and exclusive as the $A_0 \cap A_1 = \phi$

Now in the context of wireless communication one of the uses of probability theorems can be modelled as follows through a simple example of binary symmetric channels.
• Consider a BSC with p = 0.2 and that $P(A_0)$ = 0.1 and $P(A_1)$ = 0.9
So, the BSC or the binary symmetric channel can be visualised as seen in Fig.2.
• Here, the prior probabilities are $P(A_0)$ = 0.1 and $P(A_1) = 0.9$

• Likelihoods are given by $P(B_0|A_1)$ = 0.2 and $P(B_1|A_0)$ = 0.2 which tells us the probability that the symbols are getting flipped and also we have $P(B_0|A_0)$ = 0.8 and $P(B_1|A_1)$ = 0.8 which is telling us the probability of the symbols retaining their previous state.

• Now to calculate the aposterioric probability that is $B_0 = \{0\}$ is given to be received then what is probability that $A_0 = \{0\}$ is received.For this we use the Bayes theorem from the concepts of probability and find the required aposterioric probability in terms of the prior and the likelihood probabilities.

$$P(A_0|B_0) = \frac{P(B_0|A_0).P(A_0)}{P(B_0|A_0).P(A_0) + P(B_0|A_1).P(A_1)}$$
$$= \frac{0.8 * 0.1}{0.8 * 0.1 + 0.2 * 0.9}$$
$$= \frac{0.08}{0.26} = \frac{8}{26}$$

Similarly, we can calculate the other probabilities which is nothing but $P(A_1|B_0)$ as following

$$P(A_0|B_0) = \frac{P(B_0|A_1).P(A_1)}{P(B_0|A_1).P(A_1) + P(B_0|A_0).P(A_0)}$$
$$= \frac{0.9 * 0.2}{0.8 * 0.1 + 0.2 * 0.9}$$
$$= \frac{0.18}{0.26} = \frac{18}{26}$$

• So here the posterior probabilities means that when we observe the output 0 then we can conclude that the input was 0 with probability $P_0 = \frac{8}{26}$ and input 1 with probability $P_1 = \frac{18}{26}$

• If any of the probability $P_0\ or\ P_1$ were very low, then we could require 0 bits to transmit data by having some low probability of error.

• Hence the theory of probability has helped us to decrease the length of the input symbols if the posterior probabilities were very less by the cost of some less error.

V. AWGN Channel and MAP Receiver

A simple AWGN wireless channel can be modelled as

$$y[m] = x[m] + w[m]$$

where $w[m]$ is the additive white Gaussian noise. Typically, AWGN channels are considered as the bedrock for other more complex wireless channels.

Using maximum a posteriori probability (MAP) estimation, we can find out the originally transmitted signal with very low probability of error. We denoted estimation error as the event when the transmitted message m is not equal to the estimated message of the receiver y.

$$P_{\text{error}} = P(m \neq y)$$

Consider the input-output relation $r = s + n$ where n is the Gaussian noise. Assume that the message signal m is binary (0 or 1) and is encoded into s. The probability density function of the Gaussian noise with mean 0 and variance σ^2 can be written as,

$$P(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-n^2}{2\sigma^2}}$$

We take the assumption that s can take two values a and $-a$ (Assume that $0 -> -a$ and $1 -> a$ and a is a positive real number). Then, the estimate of the MAP receiver is,

$$y = \arg\max_{s_i \in s} \frac{P(s_i)P(r|s_i)}{P(r)}$$

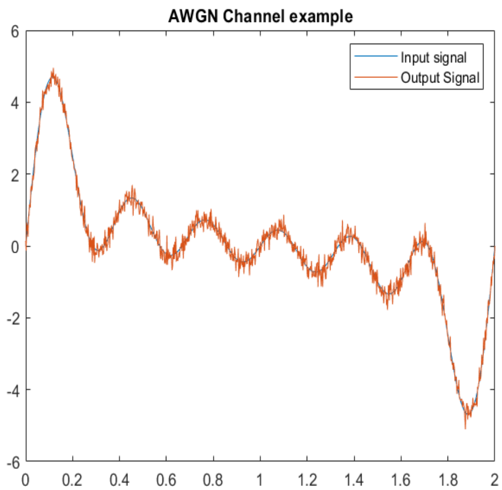


Fig. 3. Simple AWGN channel input and output

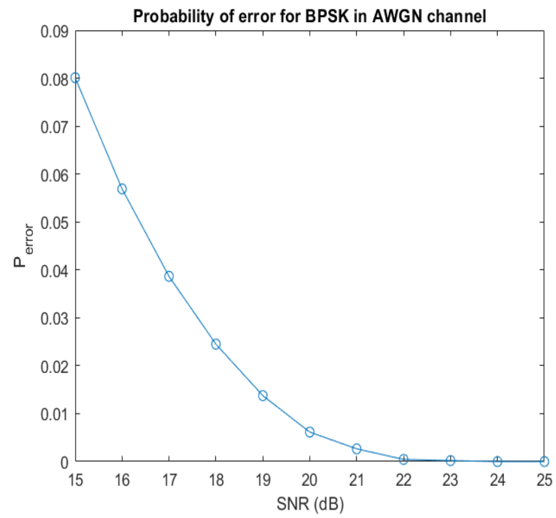


Fig. 4. Error probability in AWGN channel with BPSK

Since $P(r|s_i)$ depends on the Gaussian noise, it can be written as $(n = r - s_i)$,

$$P(r|s_i) = P(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(r-s_i)^2}{2\sigma^2}}$$

Considering the inequality assuming that both message bits are equally likely,

$$P(r|a)P \geq P(r|-a)$$

If the above inequality is satisfied, then the estimate is a (i.e. 1) and if not, the estimate is $-a$ (i.e. 0). On expanding the inequality,

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(r-a)^2}{2\sigma^2}} \geq \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(r+a)^2}{2\sigma^2}}$$

On cancelling the common terms and taking the natural logarithm on both sides we get

$$-(r-a)^2 \geq -(r+a)^2$$

$$\implies -(r-a)^2 + (r+a)^2 \geq 0$$

$$\implies r \geq 0$$

So, if $r \geq 0$, the estimate is 1, and if not, the estimate is 0 and this is yet again another simple application of probability in wireless communications. If a is made sufficiently large enough, the probability of error can be significantly reduced.

The condition $r \geq 0$ is equivalent to the condition that the noise content is greater than a. So,

$$P_{error} = P(n \geq a)$$

$$= \int_a^\infty f_N(n)dn = \int_a^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}} dn$$

Let $a = \sqrt{\rho}$. Then, on simplification, the integral reduces to,

$$\int_{\frac{\sqrt{\rho}}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{n^2}{2}} dn$$

The above integral corresponds to a Gaussian distribution with zero mean and variance 1, and hence, we can write the probability of error as,

$$P_{error} = P(\mathcal{N}(0,1) \geq \sqrt{\rho}/\sigma)$$

The RHS of the above equation is typically written in terms of the Q-function given by,

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{n^2}{2}} dn$$

So,

$$P_{error} = Q(\sqrt{P/\sigma^2}) = Q(\sqrt{SNR})$$

So, the error probability reduces exponentially on increasing the SNR of the received signal. The above error probability is often called the bit error rate (BER).

VI. Rayleigh Fading Channel and BER

Modulating the bits as $+\sqrt{P}$ and $-\sqrt{P}$ (to get the signal power as P) is often called BPSK (Binary Phase-Shift Keying). In a Rayleigh fading channel, the taps are circularly symmetric complex Gaussian random variables. A random variable X is said to be circularly symmetric if $e^{j\phi}X$ has the same probability distribution as X.

The power of the signal in a Rayleigh fading channel can be thought to be of the form of an exponential random variable X with probability density function of the from $\lambda e^{-\lambda x}$ (i.e. an exponential distribution). Then, $Y = \sqrt{X}$ would denote the amplitude of the signal in fading wireless channel. So,

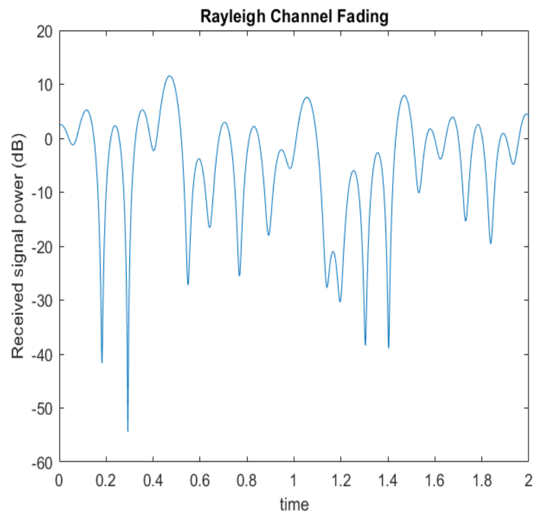


Fig. 5. Rayleigh channel

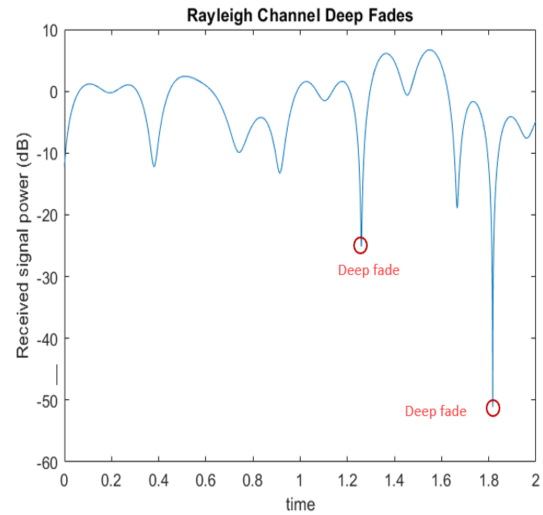


Fig. 6. Rayleigh channel deep fades

$$F_Y(y) = P(Y \leq y)$$
$$= P(\sqrt{X} \leq y) = P(X \leq y^2) = F_X(y^2)$$
$$\implies \frac{dF_Y(y)}{dy} = \frac{dF_X(y^2)}{dy}$$
$$\implies f_Y(y) = 2yf_X(y^2) = 2y\lambda e^{-\lambda y^2},\ y \geq 0$$

Hence, we can observe that Y represents a Rayleigh distribution. For sufficiently large bandwidth, the filter tap coefficients of the Rayleigh channel depict circularly symmetric Gaussian random variables from the Central Limit Theorem and so we can assume that the magnitude of any given filter tap has a Rayleigh distribution.

Consider the input-output relation given by,

$$y = hx + n$$

If x can take values $+a$ and $-a$, the power received at the receiver is $|h|^2$. However, due to the fading nature of the Rayleigh fading channel, the fading SNR is $|h|^2a^2/\sigma^2$, which is equivalent to $|h|^2SNR$, where SNR is that of the non-fading AWGN channel. The, the BER would be,

$$P_{error} = Q(\sqrt{|h|^2SNR})$$

$|h|^2$ itself behaves like a random variable, and so, the BER would be the expectation of $Q(\sqrt{|h|^2SNR})$. Since h is circularly symmetric with zero mean and on assuming the variance to be 1, we get,

$$P_{error} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

For large SNR values, the above equation can be approximated to,

$$P_{error} \approx \frac{1}{2}\left(1 - \left(1 + \frac{2}{SNR}\right)^{-1/2}\right)$$

$$\approx \frac{1}{2}(1 - (1 - SNR))$$
$$\approx \frac{1}{2SNR} \propto \frac{1}{SNR}$$

So, unlike the AWGN channel case, in the Rayleigh fading channel, the error probability is inversely proportional to the SNR.

When the SNR drops below a certain threshold, it becomes difficult for the receiver to differentiate between the actual received signal and the noise and can cause momentary lapses in communication. We call such an event as 'deep fade'. Deep fades typically occur when the received power is less than the noise threshold and so, the probability of deep fade can be approximated to,

$$P_{\text{deep fade}} = P\left(|h|^2 < \frac{1}{SNR}\right) \approx \frac{1}{SNR}$$

Hence, as the probability of deep fade increases, as the probability of bit errors increase .

$$P_{error} \propto P_{\text{deep fade}}$$

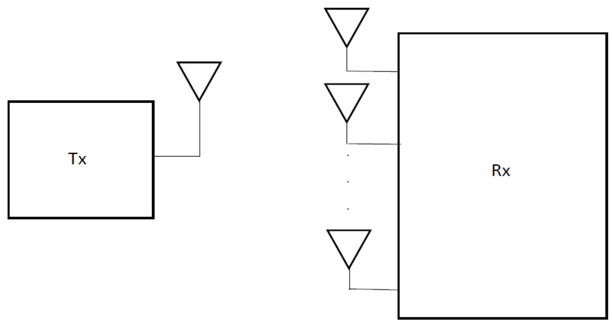
VII. Diversity and Probability

Deep fades are unwanted situations in any wireless communication system, and to overcome this we often use principles of diversity. Diversity increases the reliability of communication by using multiple channels for communication which would greatly decrease the probability of errors. There are various diversity schemes in practice today including but not limited to,

- Time diversity (For example, repetition coding)
- Frequency diversity (For example, frequency hopping)
- Antenna diversity (Transmit diversity, receive diversity etc)

A. Receive diversity

Consider a system where there is a single transmission antenna and L receiver antennas.



In such a system, each antenna receives a different signal as,

$$y_1 = h_1 x_1 + n_1$$

$$y_2 = h_2 x_2 + n_2$$

$$.$$

$$.$$

$$y_L = h_L x_L + n_L$$

Since $h_1, h_2, ...h_L$ can be considered as independent identically distributed random variables, the overall channel filter coefficient h can be written as,

$$|h|^2 = \sum_{i=1}^{L} |h_i|^2$$

$|h|^2$ can be approximated to a Chi-squared random variable with $2L$ degrees of freedom. Then, the probability of error for Rayleigh fading and BPSK can be written as,

$$P_{error} = Q(E(|h|^2 SNR))$$

On integration, we get,

$$P_{error} = \left(\frac{1-\lambda}{2}\right) \sum_{l=1}^{L} \binom{L-l+1}{l} \left(\frac{1+\lambda}{2}\right)^{l}$$

where

$$\lambda = \sqrt{\frac{SNR}{2+SNR}}$$

For large values of SNR, the error probability can be approximated to,

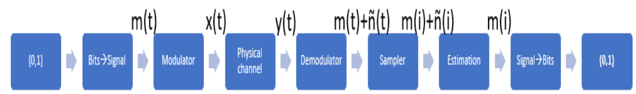
$$P_{error} \approx \binom{2L-1}{L} \left(\frac{1}{2SNR}\right)^{L}$$

So, on increasing the value of L, there is more than an exponential decrease in bit error probability. Typically, such models can be seen in Rake receivers that employ the principle of receive diversity.

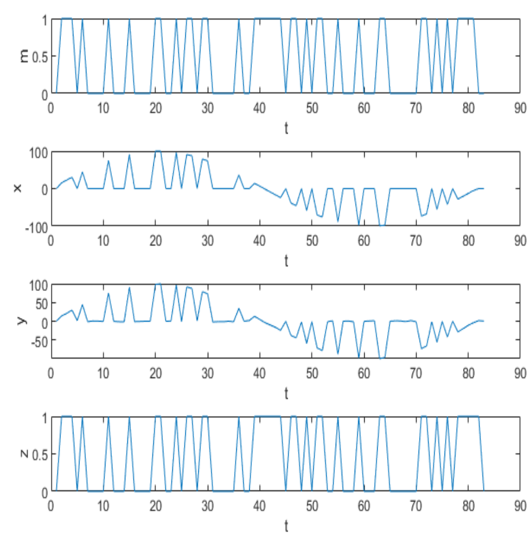
VIII. Simulations

We can model a very basic MATLAB model to simulate the wireless communication system. A simple wireless communication can be modelled as shown below.

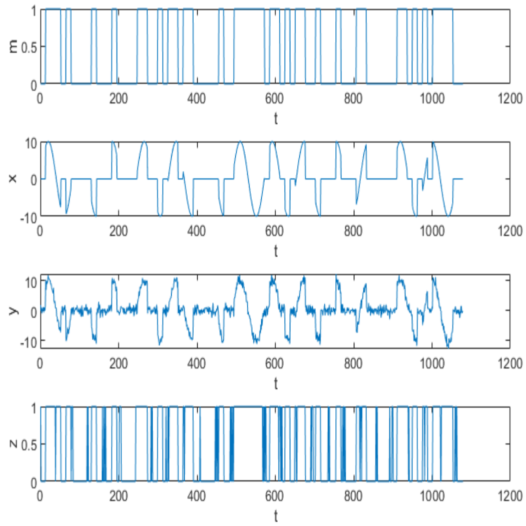
However, the transmitter and receiver usually contains other subparts as shown below.



Simple Huffman encoding can be perform to transmit message strings which are then modulated and then sent from the transmitter. The receiver receives the signal, demodulates the signal and then decodes the signal to get the transmitted message. However, noise will creep in throughout the transmission and so, proper decoding is necessary to estimate the signal. We transmit a simple encoded message m(for example "wireless communication") the modulated signal x of which can be seen in the second plot. The receiver receives y and demodulates it to give the message z. We say that an error has occurred when m does not match z. Based on our simulations, when the amplitude of the signal is set to 100 (increasing the amplitude boosts the SNR), we could observe an average bit error rate of around 2%.



However, repetition coding can further decrease the error probability and can even achieve better performances even with a much smaller amplitude as given below,



The above simulations show how reliable and advanced modern wireless communication systems are and how much we've come over the years, and at the same time achieving much lower probabilities of error.

IX. Conclusion

We have seen how the theory and practices of probability and random processes are used in the field of wireless communications. Usually, signal transmission involves noise and it is modelled as additive white Gaussian noise (AWGN), and a simple channel using this principle is shown. The applications of random processes and random variables can be seen in fading channels, which provide a more realistic approximation to real-life channels that we often face in wireless communication. We have seen how fading channels can be modelled through Rayleigh distributions. Estimation and decoding of the transmitted signal further employs approaches based on probability. A simple example is that of the MAP receiver which employs Baye's theorem. We have derived and stated expressions for the probability of error and deep fades on changing the SNR for AWGN and Rayleigh fading channels using BPSK. Diversity principles are also used to reduce the probability of error and deep fades.

The communication system model we implemented gives quite low percentage of error (mode = 0), for signal amplitude boosted to 10, and 12 extra parity bits for each bit to reduce error. The rate and be improved by boosting the signal power and reducing the parity bits for similar probability of error. The probability of error can be further reduced by using a minimum distance decoder instead of just round of estimation for higher accuracy. There is lots of scope for improvement as this is just a very basic model just to simulate the concept.

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Contributions

Snehit Gupta - Introduction, Abstract, conclusion, communication model simulation codes, repetition code model and other related concepts and code.

Prashant Gupta - Introduction, AWGN, AWGN channel, fading, AWGN channel simulation codes and other related concepts and code.

Jewel Benny - Fading, Rayleigh channel, error probabilities in AWGN channel (and code simulation) and Rayleigh channel, diversity and other related concepts and code.

Nitin Shrinivas - Baye's theorem and related codes, MAP receiver and applications to BSC and other related concepts and codes.