

Problem 1

a.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Total : 36

 $P[A_k]$ for $k =$

$2 = \frac{1}{36}$

$7 = \frac{6}{36}$

$3 = \frac{2}{36}$

$8 = \frac{5}{36}$

$4 = \frac{3}{36}$

$9 = \frac{4}{36}$

$5 = \frac{4}{36}$

$10 = \frac{3}{36}$

$6 = \frac{5}{36}$

$11 = \frac{2}{36}$

$12 = \frac{1}{36}$

 $P[B_k]$ for $k =$

$2 = 1$

$8 = \frac{15}{36}$

$3 = \frac{35}{36}$

$9 = \frac{16}{36}$

$4 = \frac{33}{36}$

$10 = \frac{6}{36}$

$5 = \frac{30}{36}$

$11 = \frac{3}{36}$

$6 = \frac{26}{36}$

$12 = \frac{1}{36}$

$7 = \frac{21}{36}$

$$b. \quad P[E|B_7] = \frac{P[E \cap B_7]}{P[B_7]} = \frac{\frac{9}{36}}{\frac{21}{36}} = \frac{9}{21} = 42.9\%$$

$$c. \quad P[A_{10} \cup A_{11} | B_7] = \frac{5}{21}$$

$$d. P[B_7 | E] = \frac{9}{18} = \frac{1}{2}$$

$$e. P[B_9 | B_6] = \frac{10}{26} = \frac{5}{13}$$

$$f. P[O \cap B_8 | B_7] = \frac{6}{21}$$

$$g. \frac{6 \times 5}{6 \times 6} = \frac{5}{6} = 83.3\%$$

Problem 2

$$S = \{a, b, c, d\}$$

$$P[\{a, b\}] = \frac{2}{3}$$

$$P[\{a, c\} | \{c, d\}] = \frac{1}{4}$$

$$P[\{a, d\} | \{a, b\}] = \frac{1}{2}$$

$$P[\{a, b\}] = P[a] + P[b]$$

$$P[\{a, c\} | \{c, d\}] = \frac{P[\{a, c\} \cap \{c, d\}]}{P[\{c, d\}]} = P[c] = \frac{1}{4} [P[c, d]]$$

$$P[\{a, d\} | \{a, b\}] = \frac{P[\{a, d\} \cap \{a, b\}]}{P[\{a, b\}]} = \frac{P[a]}{\frac{2}{3}} = \frac{1}{2}$$

$$P[a] = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$$P[\{a, b\}] = P[a] + P[b] = \frac{1}{3} + P[b] = \frac{2}{3} \rightarrow P[b] = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$P[c] = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

↳ remaining

$$P[d] = \frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$$

$$P[a] = \frac{1}{3}$$

$$P[b] = \frac{1}{3}$$

$$P[c] = \frac{1}{12}$$

$$P[d] = \frac{1}{4}$$

Problem 3

$$P[C] = 0.2$$

$$P[A] = 0.1$$

$$P[C \cap A | C \cup A] = 0.5$$

$$a. P[A \cap C] = 0.5 [P(A) + P(C)] = 0.15$$

$$b. P[A|C] = \frac{P[A \cap C]}{P(C)} = \frac{0.15}{0.2} = \frac{3}{4} = 0.75$$

$$c. P(C|A) = \frac{P(A \cap C) \cdot P(C)}{P(A)} = \frac{0.15 \times 0.2}{0.1} = 0.3$$

Problem 4.

$$P(A) + P(B) - 2P(A \cap B)$$

only one occurring =

$$\begin{aligned} & P(\bar{A} \cap B) \cup (A \cap \bar{B}) \\ &= P[(\bar{A} \cap B)] + P(A \cap \bar{B}) \\ &= P[B - (A \cap B)] + P[A - (A \cap B)] \\ &= P(B) - P(A \cap B) + P[A - (A \cap B)] \\ &= P(B) - P(A \cap B) + P(A) - P(A \cap B) \\ &= P(A) + P(B) - 2P(A \cap B) \end{aligned}$$

Problem 5

$$\begin{aligned} P(A \cap B \cap C) &= P[A | B \cap C] P[B \cap C] P(C) \\ &= \frac{P(A \cap B \cap C)}{P(B \cap C)} \times \frac{P(B \cap C)}{P(C)} \times P(C) \\ &= P[A \cap B \cap C] \end{aligned}$$

Problem 6

$$P(A) = 10^{-3} \quad P(B) = 3 \times 10^{-3} \quad P(B|A) = 6 \times 10^{-3}$$

a. $P(A \cap B) =$

$$\begin{aligned} P(B|A) &= \frac{P(B \cap A)}{P(A)} = \frac{6 \times 10^{-3} \times 10^{-3}}{10^{-3}} \\ &= 6 \times 10^{-6} \end{aligned}$$

b. $P(A|B) = \frac{P(B|A) \times P(A)}{P(B)} = \frac{6 \times 10^{-6}}{3 \times 10^{-3}} = 2 \times 10^{-3}$