HW#2

Due: October 16, 2023

- Boldface letters indicate points ($\mathbf{x}$), lines ($\mathbf{l}$), vectors ($\mathbf{p}$) or matrices ($\mathbf{H}$).
- All the quantities related to the second camera are identified by the superscript *prime* (').
- The notation $\mathbf{H} : A \mapsto A'$ means a matrix $\mathbf{H}$ which maps A to A'.
- The notation $\mathbf{x} \leftrightarrow \mathbf{x}'$ means that $\mathbf{x}$ and $\mathbf{x}'$ are corresponding elements (points, lines, etc.) in the two image planes.
- An affine transformation can be described as follows:

$$\mathbf{x}' = \begin{bmatrix} a_1 & a_3 & t_x \\ a_2 & a_4 & t_y \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}$$

- A generic projective (homography) transformation can be described as follows:

$$\mathbf{x}' = \begin{bmatrix} h_1 & h_4 & h_7 \\ h_2 & h_5 & h_8 \\ h_3 & h_6 & h_9 \end{bmatrix} \mathbf{x}$$

- *The following you may find useful in your calculations.*
  Equation of a line connecting two points (x_1, y_1) and (x_2, y_2) in 2D:
  $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1}$
  Equation of a line connecting two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in 3D:
  $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$

Question 1: Coordinate Systems

Two lines l_1 and l_2 are represented in the 2-D Cartesian coordinate system as follows:

$$l_1 : \ 2x + 5y = 4, \quad l_2 : \ 3x + 10y = 5$$

(a) How are the lines l_1 and l_2 represented in the 2-D projective space?

(b) Evaluate P, the point of intersection of l_1 and l_2, in both projective and Cartesian space.

(c) Given the projective transformation $\mathbf{H} = \begin{bmatrix} 3 & -20 & 7 \\ -7 & 10 & 6 \\ 4 & 5 & -3 \end{bmatrix}$, where does the point P map to? Write the answer in projective and in cartesian coordinates.

Question 2:

(a) Consider two lines on a plane in the homogeneous coordinate system $l_1 = (8, 3, 4)^T$ and $l_2 = (5, -7, 1)^T$. Where do they intersect?

(b) Where is the ideal point corresponding to l_1?

(c) Where would the ideal point from part (b) be located under the following projective transformation given by matrix $\mathbf{H}$?

$$\mathbf{H} = \begin{pmatrix} 3 & 7 & 5 \\ -4 & 8 & 2 \\ 6 & 1 & 9 \end{pmatrix}$$

Question 3: Line-Intersections and Transformations

(a) The following two lines are represented in the homogeneous coordinate system $l_1 = [5,\ 2,\ 1]^T$ and $l_2 = [2,\ 1,\ 4]^T$. Where do the these lines intersect?

(b) Compute the the ideal point of the line l_2?

(c) Where is the ideal point from part (b) located under the projective transformation given by

$$\mathbf{H} = \begin{bmatrix} -50 & 125 & 200 \\ 125 & -300 & 175 \\ -25 & 75 & 150 \end{bmatrix} ?$$

Write the transformed point in Cartesian form. Is the point computed in (b) still an ideal point? Explain.

Question 4: Projections

In the following figure a 3D line connects point a with *Cartesian coordinates* (0,0,10) to point b with coordinates (2, 2, 12).

(a) What is the 3D equation of the line? What is the 2D equation of the line when projected on the 2D image plane 3 units away from the center of projection (COP)?

(b) Where is the vanishing point of the line located on the image plane?

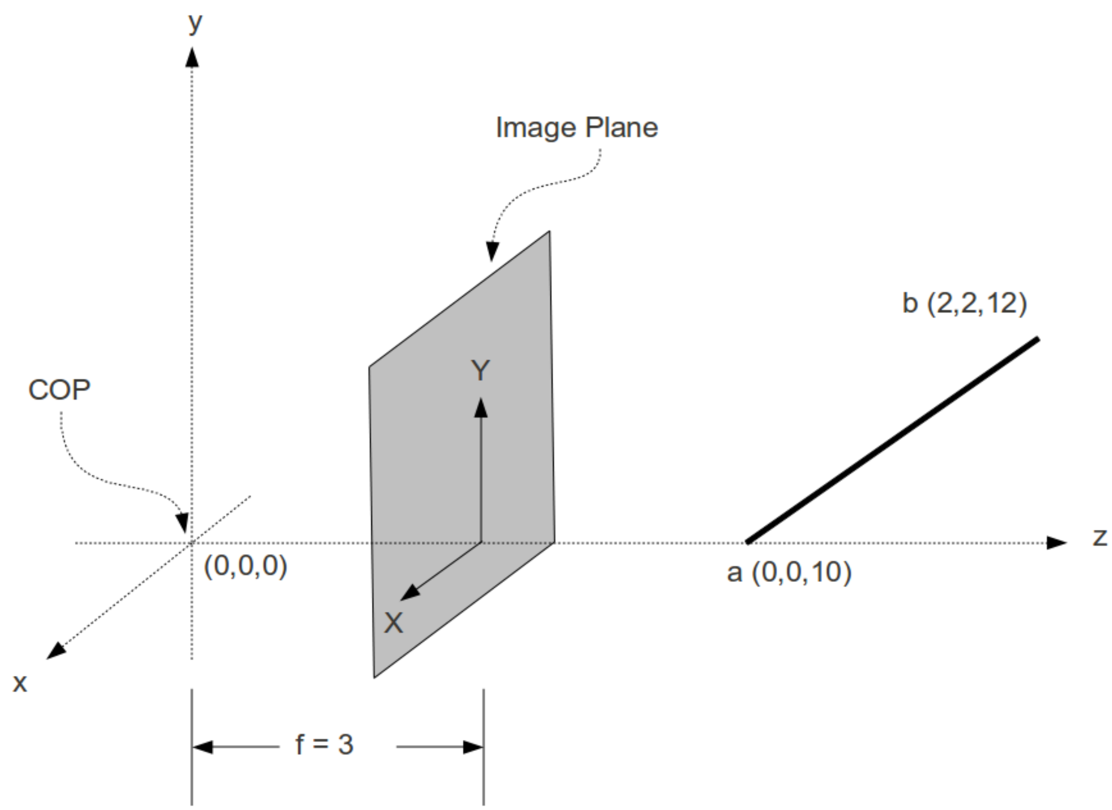

Figure 1: Question 3

Question 5:

1. What is a point at infinity? Provide its homogeneous coordinates representation.
2. How does an Affine transformation affect a point at infinity? Explain.
3. How does a Projective (Homography) transformation affect a point at infinity? Explain.

Question 6: Prove *analytically* that a line in 3D projects as a line in 2D under perspective projection. Start with an equation to a 3D line and then show that the equation corresponding to its projection is a line in 2D.