Topics: Normal distribution, Functions of Random Variables

G.Snehitha
snehitha377@gmail.com

1. **The time required for servicing transmissions is normally distributed with □ = 45 minutesand □ = 8 minutes. The service manager plans to have work begin on the transmission of acustomer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Answer :

X = 60 , Mean = 45+10 = 55 , Std. Deviation = 8

```
from scipy import stats
round(1-stats.norm.cdf(60,loc=55,scale=8),5)
```

Output : 0.26599

Option B is the correct answer.

2. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean □ = 38 and Standard deviation □ =6. For each statement below, please specify True/False. If false, briefly explain why.**

   A. More employees at the processing center are older than 44 than between 38 and 44.

Answer :

False ,

mean = 38 & SD = 6

means that, most of the ages are lying between 32 and 44

```
#Z-score for 44
from scipy import stats
round(1-stats.norm.cdf(44,loc=38,scale=6),4)
```

Output : 0.1587
i.e. 63 employees out of 400

```
#Z-score between 38 and 44
from scipy import stats
round(stats.norm.cdf(44,loc=38,scale=6) - stats.norm.cdf(38,loc=38,scale=6),4)
```

Output : 0.3413
i.e. 137 employees out of 400

therefore , 137 > 63 hence given condition is false.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer :

True ,

```
from scipy import stats
round(stats.norm.cdf(30,loc=38,scale=6),4)
```

Output : 0.0912
i.e. 36 employees out of 400
hence given condition is True.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer :

We know that ,

if $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are two independent random variables then ,

$X_1 + X_2 \sim N(\mu + \mu , \sigma^2 + \sigma^2)$

Similarly if $Z = aX_1 + bX_2$, where X and Y are as defined above, i.e Z is linear combination of X_1 and X_2 , then $Z \sim N(a\mu + b\mu, a^2 \sigma^2 + b^2 \sigma^2)$.

Therefore from the question ,

$2X1 \sim N(2 \mu , 4 \sigma^2)$ &

$X1+X2 \sim N(2 \mu, 2\sigma^2)$

$2X1 - (X1+X2) = N(0,2\sigma^2)$

4. **Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Answer :

Since a and b are symmetric about mean,
It is two tailed test ,
Hence , for 99% , we need to add 0.5% on either side = 0.99+0.005 = 0.995

Z value of 0.005 is ,

```
from scipy import stats
stats.norm.ppf(0.005)
```

Z value of 0.005 = -2.57

Now, Z value of 0.995 is ,

```
from scipy import stats
stats.norm.ppf(0.995)
```

Z value of 0.995 = 2.57
Hence ,

$$Z = \frac{x - Mean}{SD}$$

Hence ,

$$x = SD * Z + Mean$$

$$x = 20*Z + 100$$

therefore ,

$$a = (20 * (-2.57)) + 100$$

$$a = 48.5$$

$$b = (20* 2.57) + 100$$

$$b = 151.5$$

Option D. (48.5, 151.5) is correct answer.

5. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**

   A. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

Answer :

```
import numpy as np
from scipy import stats
mean = 5+7
print('Mean Profit is Rs', mean*45,'Million')
sd = np.sqrt((9)+(16))
print('Standard Deviation is Rs', sd*45, 'Million')
print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')
```

Output :
```
Mean Profit is Rs 540 Million
Standard Deviation is Rs 225.0 Million
Range is Rs (99.00810347848784, 980.9918965215122) in Millions
```

   B. **Specify the 5^{th} percentile of profit (in Rupees) for the company**

Answer :

we know that , Z value for 5^{th} percentile is = -1.645

X = SD*Z + Mean is ,

```
X= 540+(-1.645)*(225)
print('5th percentile of profit is',round(X),'(in Million Rupees)')
```

Output :
```
5th percentile of profit is 170 (in Million Rupees)
```

C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer :

Making loss , i.e X<0

Division 1 :

stats.norm.cdf(0,5,3)

Output : 0.04779035

Division 2 :

stats.norm.cdf(0,7,4)

Output : 0.04005915

Hence ,

Division 2 will face more loss.