Design of a Coupled-Line Microstrip Butterworth Low Pass Filter

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***Abstract*— A new microstrip Butterworth low pass filter design is proposed using Z-domain representation of coupled-line. A least mean square based error function is minimized through the optimization process to obtain the impedance values of a 5th order Butterworth low pass filter. Highly accurate design having absolute relative error of 0.03 upto the design cut-off frequency is achieved in Z-domain. The proposed filter configuration consists of three serial transmission lines cascaded to a coupled line. It is simulated on a microstrip format using RT/Duroid 5880 substrate having 20 mil thickness and relative permittivity of 2.2. The simulation results for the scattering parameter $S_{21}(f)$ gives the 3 dB cut-off frequency of 2.84 GHz.**

Keywords*— *coupled-line, low-pass filter, microstrip, optimization, serial transmission lines

I. Introduction

In typical wireless communication systems, low pass filter (LPF) is a vital circuit element used for supressing undesired high frequency harmonics, which in turn improves the overall system performance. Achieving sharp roll-off, wide stop-band and compact physical size are some considerable challenges while obtaining various LPFs designs over the past few decades.

Numerous techniques exist in the literature for designing of microwave LPF circuits [1, 2]. Most widely used technique by the researchers in the beginning, incorporated the use of inductor-capacitor elements to form a ladder network termed as stepped impedance method. [3-11]. Use of nature inspired Particle Swarm Optimization technique was also identified to design a least-square based stepped-impedance microwave LPF by Oraizi *et al.* [12]. Various LPF designs were also obtained using open stubs [13-17]. A microstrip LPF consisting of shunt connected open stubs and having coupled slots was proposed by Kim *et al.* [13]. Multiple open stubs and short-end coupled lines were used by Tang and Chen to propose low-pass filter with wide stop-band [14]. A compact LPF with 3 dB cut-off frequency of 2 GHz was proposed using two open stubs along with a coupled-line hairpin unit and a spiral slot [15]. Open stubs are added by Hayati *et al.* [16] to increase stop-band attenuation in a microstip LPF. New class of LPF filter was designed using high-impedance coupled-line section terminated by and open-circuited stubs [17]. Using stepped impedance and open-stub based techniques are the two conventional approaches which usually result in narrow upper stop-band and not much roll-off sharpness [18].

A new Z-domain based approach to design and implement microwave filters was developed by Chang and Hsue [19]. Chain scattering matrices for various line elements were proposed for the design purpose. A low-pass filter and a band-pass filter designs were obtained and implemented on a microstrip format using serial and open-circuited stubs. Based on this similar approach, later, Tsai and Hsue [20] used shunt connected serial lines and coupled-serial lines to propose dual-band filters consisting of a cascaded bandstop and wide-band bandpass structure. Tsai *et al.* [21] have designed a microstrip LPF of order $n=5$ in Z-domain using chain scattering parameters of multi-section serial transmission lines and shunt open-circuited stubs.

Literature survey reveals that the Z-domain representation of chain scattering parameters for coupled lines has not been used for the design of a low-pass filter. In this work, a new design of 5th order Butterworth low pass filter is obtained using chain scattering representation of a low-pass parallel coupled line and serial transmission line sections. Butterworth approximation techniques are widely used for the design of an LPF. For a given filter order, such filters give maximally flat magnitude response in the passband region [22]. Attenuation equal to -20 x n dB/decade is also seen for a Butterworth low pass filter (BLPF). The magnitude response of a n^{th} order ideal BLPF is given as:

$$\left|H_i(\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}} \tag{1}$$

where n represents the order of filter. ω is the angular frequency and ω_c is the cut off frequency having units of radians/second.

Impedance values of all the line elements are obtained through an optimization process based on metaheuristic Colliding Bodies Optimization algorithm. Analysis is done based on error parameters viz., absolute relative error, pass-band error and stop-band error. Finally, the proposed configuration is simulated on a microstrip format in ADS environment. Value of the transmission coefficient $S_{21}(f)$ gives 3 dB cut-off frequency of 2.84 GHz.

This work is organized into the following sections. In section 2, design of the coupled-line Butterworth LPF is discussed. Detailed analysis of CBO is also discussed along with the error analysis using various defined

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parameters. The obtained configuration for microstrip coupled-line BLPF is simulated on ADS in section 3. Finally, the work is concluded in section 4.

II. Design of Coupled-Line BLPF

A. Design Methodology

Chain scattering matrices (T-matrices) for transmission line elements were derived in Z-domain and used for implementing microwave filters [19]. The implemented LPF design was consisting of five open-circuited stubs and five serial transmission line sections, giving the total length of the filter to be 67.4 mm. It is also seen that, a dual-band filter implementation was shown by developing the T-matrix representation of a parallel coupled-line (PCL) [20]. A four-port parallel coupled-line structure can be made to behave as a two-port network by setting port-2 short circuited and port-3 as open-circuited. The resulting structure is shown in Figure 1 which is a low-pass parallel coupled-line (LP-PCL) with port-1 as the input terminal and port-4 as the output terminal [20].

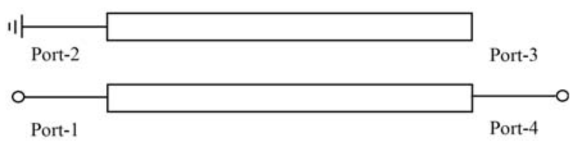

Figure 1. Low-pass parallel coupled-line

A LP-PCL contributes a zero at $z=-1$ ($\omega=\pi$) [20], which emulates the behaviour of a low-pass filter and having the T-matrix given as follows:

$$T_{CL}=\frac{1}{8(b_1^2-b_2^2)Z_0 z^{-1/2}(1-z^{-1})}\begin{bmatrix} p & -s \\ r & \dfrac{4q^2-rs}{p} \end{bmatrix} \tag{2}$$

where $b_1=Z_{ev}+Z_{od}$, $b_2=Z_{ev}-Z_{od}$ and $z=e^{j\beta l}$ with $\omega=\beta l$. Here, Z_{ev} and Z_{od} are the characteristic impedances for even and odd excitation modes of a coupled-line respectively, Z_o is the reference characteristic impedance, β is the propagation constant and l is the physical length. Also, we have:

$$p=[4b_1^2Z_0-2b_2^2Z_0+b_1(b_1^2-b_2^2)+4b_1Z_0^2]+(8b_1^2Z_0-12b_2^2Z_0)z^{-1} +[4b_1^2Z_0-2b_2^2Z_0-b_1(b_1^2-b_2^2)-4b_1Z_0^2]z^{-1} \tag{3}$$

$$q=4(b_1^2-b_2^2)Z_0z^{-1/2}(1+z^{-1}) \tag{4}$$

$$r=[-2b_2^2Z_0+b_1(b_1^2-b_2^2)-4b_1Z_0^2]+4b_2^2Z_0z^{-1} +[-2b_2^2Z_0-b_1(b_1^2-b_2^2)+4b_1Z_0^2]z^{-2} \tag{5}$$

$$s=[2b_2^2Z_0+b_1(b_1^2-b_2^2)-4b_1Z_0^2]-4b_2^2Z_0z^{-1} +[2b_2^2Z_0-b_1(b_1^2-b_2^2)+4b_1Z_0^2]z^{-2} \tag{6}$$

In this paper, we have designed a much simple and compact design of a BLPF of order $n=5$, by cascading serial transmission line sections with a LP-PCL. For a serial transmission line section T-matrix representation is given by [19]:

$$T_{SeL}=\frac{1}{z^{-1/2}(1-V^2)}\begin{bmatrix} 1-V^2z^{-1} & -(V-Vz^{-1}) \\ V-Vz^{-1} & -V^2+z^{-1} \end{bmatrix} \tag{7}$$

where $V=(Z_L-Z_o)/(Z_L+Z_o)$. Here, Z_L is the characteristic impedance of serial transmission line section. The overall transfer function is obtained by computing the product of all the T-matrices of transmission line sections with that of the LP-PCL. Afterwards, $\boldsymbol{T_{11}}$ is calculated for the overall cascaded network which gives the transmission coefficient value as:

$$H_S(z)=\frac{1}{T_{11overall}(z)} \tag{8}$$

Impedances of the line elements are obtained by minimizing an LMS based error function ($e(\omega)$) formed between the transmission coefficient and ideal BLPF magnitude response. It can be defined as:

$$e(\omega)=\sum_{\omega}\{|H_S(\omega)|-|H_i(\omega)|\}^2 \tag{9}$$

For the minimization of above defined error function, CBO has been implemented. CBO in comparison to many existing benchmark nature inspired optimization algorithms is found to give best magnitude response and is computationally superior in achieving fastest convergence of iteration cycles [23].

B. Colliding Bodies Optimization Algorithm

In 2014 [24], Kaveh and Mahdavai proposed CBO which is a new age nature inspired metaheuristic optimization technique. For finding the best solution to any given optimization problem, a collision happens between a physical pair of objects. Making use of a random search and having simplified formulation, CBO is having a decisive edge over other optimization algorithms in terms of accuracy of results. The procedure of CBO is explained below.

A population of 'n' individual solutions are considered which are called as colliding bodies (CBs) where the i^{th} CB is denoted by $\boldsymbol{X_i}=[x_{1i},x_{2i},\dots,x_{Di}]$ = $[\mathrm{a}_{1i},\mathrm{a}_{2i},\dots,\mathrm{a}_{N+1i},\mathrm{b}_{1i},\mathrm{b}_{2i},\dots,\mathrm{b}_{N+1i}]$ where $\mathrm{i}=1,2,\dots,\mathrm{n}$ and $\mathrm{D}=2\,\mathrm{x}\,(\mathrm{N}+1)$ represents the problem's dimension.

Step 1: The CBs are generated and initialised in an arbitrary fashion within the problem search space given by the equation

$$x_i^0=x_{min}+\mathrm{rand}\,\mathrm{x}\,(x_{max}-x_{min}),\quad \mathrm{i}=1,2,\dots,\mathrm{n} \tag{10}$$

where x_{min} is the minimum impedance which is chosen as $10\ \Omega$ and x_{max} is the maximum impedance chosen as $150\ \Omega$ and **rand** is a number randomly chosen in the range [0,1].
The fitness values are then evaluated and the minimum value is termed $gbest$.
Step 2: The k^{th} CB's magnitude is found out to be

$$m_k = \frac{1/_{fit(k)}}{\Sigma_{i=1}^{n} 1/_{fit(i)}} \quad , \quad \mathrm{i} = 1,2,\ldots\ldots,\mathrm{n} \tag{11}$$

where $fit(i)$ denotes the value of $\boldsymbol{i}$ in the objective function and $\boldsymbol{n}$ shows the size of population.
Step 3: CBs are sorted in ascending order based on their value of fitness function. Two equal groups of these CBs are then formed. Group 1 consists of higher body mass CBs and are considered stationary and group 2 consists of lighter body mass and are termed as moving CBs. Moving CBs are then made to undergo collision in 1-d with respective stationary.
Step 4: Before colliding, the velocities associated with the stationary and moving CBs are given as follows:

- Stationary CBs

$$v_i^s = 0 \quad , \quad \mathrm{i} = 1,2,\ldots.,\mathrm{n}/2 \tag{12}$$

- Moving CBs

$$v_i^m = x_i - x_{i-n/2} \quad , \mathrm{i} = \mathrm{n}/2 + 1,\ldots\ldots,\mathrm{n} \tag{13}$$

where v_i^s is the velocity of the i^{th} stationary CB, v_i^m is the velocity and x_i is the co-ordinate of the i^{th} moving CB.
Step 5: The after collision velocities and position are given as:

- Stationary CBs:

$$v_i^{snew} = \frac{m_{i+(\frac{n}{2})} + \epsilon \times m_{i+(\frac{n}{2})}}{m_i + m_{i+(\frac{n}{2})}} \times v^m_{i+(\frac{n}{2})} \quad , \quad \mathrm{i} = 1,2,\ldots.,\mathrm{n}/2 \tag{14}$$

$$x_i^{snew} = x_i + \mathrm{rand\ x}\ v_i^{snew} \quad , \ i = 1,2,\ldots.,n/2 \tag{2}$$

- Moving CBs:

$$v_i^{mnew} = \frac{m_i - \epsilon \times m_{i-(\frac{n}{2})}}{m_i + m_{i-(\frac{n}{2})}} \times v^m_{i+(\frac{n}{2})} \quad ,$$

$$\mathrm{i} = \mathrm{n}/2 + 1,\ldots\ldots.,\mathrm{n} \tag{3}$$

$$\mathrm{x}_i^{mnew} = x_{i-\frac{n}{2}} + \mathrm{rand\ x}\ \mathrm{v}_i^{mnew} \quad \mathrm{i} = \mathrm{n}/2 + 1,\ldots\ldots.,\mathrm{n} \tag{4}$$

where v_i^{mnew}, x_i^{mnew} and v_i^{snew}, x_i^{snew} are the velocities and positions of the moving and stationary CB, m_i post collision, and the COR, $\epsilon = 1 - \frac{presiter}{maxiter}$. Here, presiter is the current iteration number and maxiter is the set number of maximum iterations.
Step 6: The fitness values with the updated positions of all the CBs (PCl and serial transmission line) are calculated and $gbest$ is revised if necessary.

Step 7: Before declaring $gbest$ as the most optimal design, repeat the process from Step 2 until the iterations complete.

C. Result Analysis

Simulations are carried out on MATLAB to find the impedance values for an LP-PCL and serial transmission line sections. The error function defined in equation (9) is minimized using CBO algorithm which is made to run for 500 iterations. $\omega_c = 1$ rad/s is chosen as the filter cut-off frequency for the design purpose. Different designs obtained by increasing the number of serial transmission line sections in cascade to an LP-PLC are given in Table (1).

Table 1. Different designs obtained by increasing the number of serial transmission line sections in cascade to an LP-PLC

Design	No. of Serial Transmission lines	Impedance Values (Ω)				
		Z_{ev}	Z_{od}	Z_{L1}	Z_{L2}	Z_{L3}
$D1$	1	140	140	10	---	---
$D2$	2	15.32	10	120.35	12.10	---
$D3$	3	22.41	20.61	136.47	12.54	137.52

Superiority of these designs is judged by calculating the value of absolute relative error given by the formula:

$$e_{ARE} = \left| \frac{|H_S(\omega)| - |H_i(\omega)|}{|H_i(\omega)|} \right| \tag{18}$$

Figure 2 shows the ARE plot for different BLPF designs. It is clearly seen that the BLPF configuration with three serial transmission line sections give highly accurate error performance with ARE value of 0.03 upto the cut-off frequency.

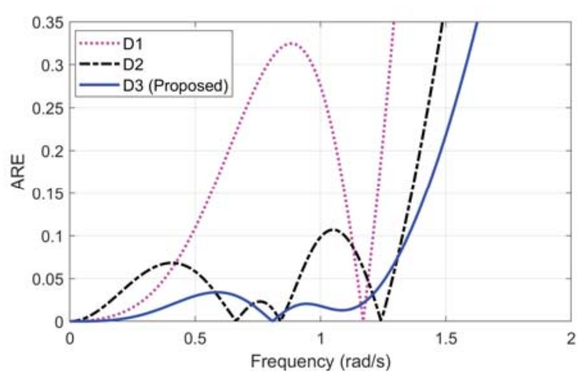

Figure 2. ARE plot for different BLPF designs

To further evaluate the obtained designs, values of pass-band error (e_p) and stop-band error (e_s) are calculated using the equations:

$$e_p = 20\log_{10}\sqrt{\frac{\sum_{\omega}\{|H_S(\omega)| - |H_i(\omega)|\}^2}{k}} \quad (19)$$

where $k = 500$ and $0 \le \omega \le 1$.

$$e_s = 20\log_{10}\sqrt{\frac{\sum_{\omega}\{|H_S(\omega)| - |H_i(\omega)|\}^2}{k}} \quad (20)$$

where $k = 500$ and $1 < \omega \le \pi$.

Values of e_p and e_s for the obtained BLPF designs listed in Table 2 shows the least values of pass-band and stop-band error for the pr0posed BLFP design with three serial transmission line sections cascaded with an LP-PCL ($D3$) . Also, the magnitude response comparison of different BLPF designs shown in Figure 3. It is seen that D_3 closely follows the ideal behavior over the entire pass-band frequency region.

Table 2. Values of e_p and e_s for the obtained BLPF designs

Design	$D1$	$D2$	$D3$
e_p	−22.33	−34.60	−41.68
e_s	−20.36	−30.00	−35.55

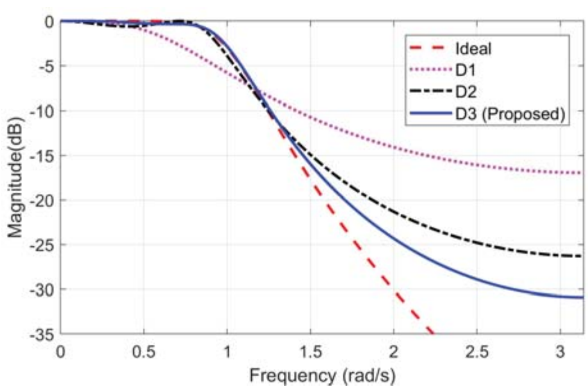

Figure 3. Magnitude response comparison of different BLPF designs

III. MICROSTRIP BLPF

The proposed BLPF configuration is simulated in the ADS environment on a microstrip format. The microstrips are assumed to be dispersionless and lossless. RT/Duroid 5880 is used as the substrate which is having 20 mil thickness, loss tangent value of 0.0009 and relative permittivity value of 2.2. 90° is taken as equal-electrical length of each transmission line element and the physical length is considered to be λ/4 where λ is wavelength at 10 GHz normalizing frequency. Table 3 gives the physical dimensions of the proposed filter.

The prototype layout for the proposed microstrip BLPF is shown in Figure 4. Short circuit in LP-PCL is achieved by creating via-holes. Figure 5 shows the magnitude responses for the scattering parameters S21 (f) and S11 (f). The 3 dB cut-off frequency for the proposed microstrip BLPF is found to be 2.84 GHz which is in close approximation to the considered theoretical cut-off frequency.

Table 3. Physical dimensions of the proposed filter

Line Element	Impedance (Ω)	Width (mm)	Length (mm)
Serial Transmission Line Section	$Z_{L1} = 136.47$	0.163	5.80
	$Z_{L2} = 12.54$	9.020	5.18
	$Z_{L3} = 137.52$	0.159	5.80
Coupled Line	$Z_{ev} = 22.41$	4.810	5.28
	$Z_{od} = 20.61$	Spacing between Coupled Line = 1.35 mm	

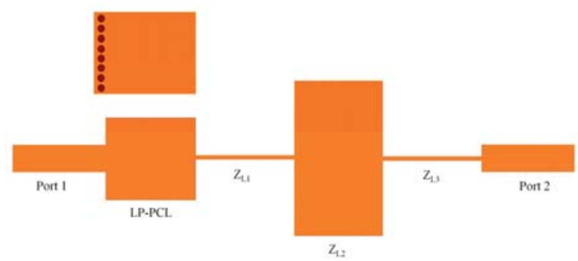

Figure 4. Prototype layout for the proposed microstrip BLPF

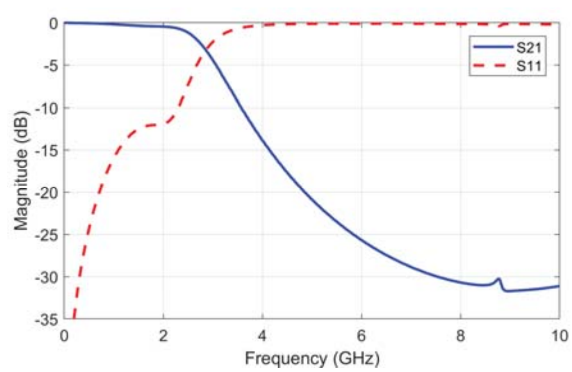

Figure 5. Magnitude responses for the scattering parameters S21 (f) and S11 (f)

IV. CONCLUSION

Butterworth low pass filter design is obtained in Z-domain using chain scattering parameters. Serial transmission lines are cascaded with a low-pass parallel coupled line to formulate an LMS based error function. The opmization process based on metaheuristic Colliding Bodies Optimization algorithm yields highly accurate design having pass-band error and stop-band error values of -41.68 dB and -35.55 dB respectively. The proposed configuration consisting of three transmission lines in cascade to an LP-PCL is simulated on a microstrip format.

The simulation results for the scattering parameter $S_{21}(f)$ gives the 3 dB cut-off frequency of 2.84 GHz. The proposed design is suitable for many wireless applications including Bluetooth, navigation systems like GNSS, GPS and GLONASS, various biomedical applications and designing transceivers of various communication systems.

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