

6.)

a.) This is impossible, if  $f_1(n)$  is the upper bound for  $f_2(n)$  than  $f_2(n)$  cannot also be the upper bound for  $f_1(n)$ . BUT

$f_1(n) = O(f_2(n))$  implies  $c_1$  and  $n_0$  that  $0 \leq f_1(n) \leq c_1 f_2(n)$  for all  $n \geq n_0$ .

Above implies  $f_2(n) = O(f_1(n))$

$f_2(n) = O(f_1(n))$  implies  $c_2$  and  $n_1$  that  $0 \leq f_2(n) \leq c_2 f_1(n)$  for all  $n \geq n_1$

By definition  $f_1(n) = O(f_2(n))$  requires  $f_2(n)$  to be greater than  $f_1(n)$  thus this statement is false.

b.) If  $f_1(n) = O(g_1(n))$  and  $f_2(n) = O(g_2(n))$ , then  $f_1(n) \times f_2(n) = O(g_1(n) \times g_2(n))$ .

Proof:

$f_1(n) \leq c_1 g_1(n)$  and  $f_2(n) \leq c_2 g_2(n)$  for large  $n$  thus,

$$f_1(n) \times f_2(n) \leq c_1 g_1(n) \times c_2 g_2(n)$$

$$\leq c_1 \max(g_1(n), g_2(n)) + c_2 \max(g_1(n), g_2(n))$$

$$\leq (c_1 + c_2) \max(g_1(n), g_2(n))$$