

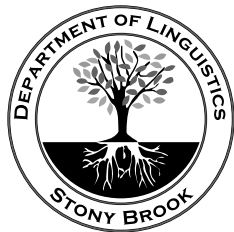
Model Theoretic Phonology and Theory Comparison: Segments, Gestures, and Coupling Graphs

NAPhC 12

Scott Nelson

Stony Brook University

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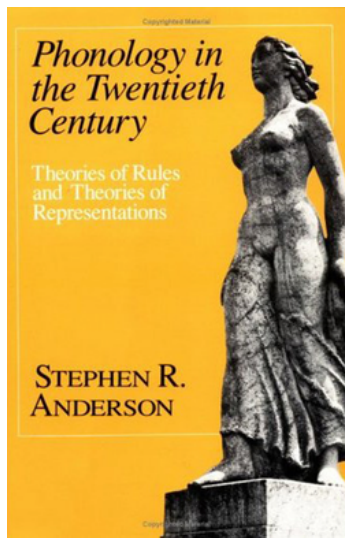
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- ▶ Also available at <http://www.scott-nelson.net>

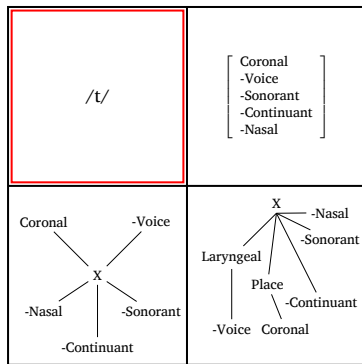
Phonological Representations

- ▶ Representations are central to modern phonological theory.
- ▶ Many different proposals across phonological domains.



Jakobson et al. (1951); Goldsmith (1976); Clements (1985); Browman and Goldstein (1986); Archangeli (1988); Dresher (2009); Backley (2011); Inkelas and Shih (2016); van der Hulst (2020); *among (many)⁺ others*

Phonological Representations



- ▶ Many different proposals throughout the years.
- ▶ Focus of this talk will be on segmental representations.

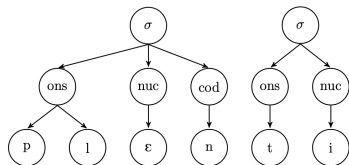
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Phonological Representations

- ▶ Recent work has used model theory to compare different proposed representation schemes.
 - ▶ Syllable Representations
 - ▶ Tonal Geometry
 - ▶ Autosegmental/Q Theory
 - ▶ Feature Systems

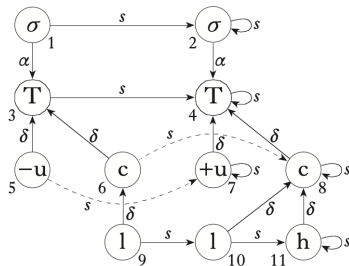
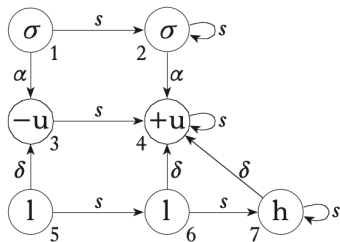
Phonological Representations

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Phonological Representations

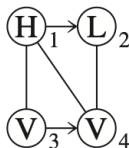
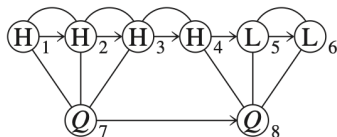
- ▶ Recent work has used model theory to compare different proposed representation schemes.
 - ▶ Syllable Representations
 - ▶ **Tonal Geometry**
 - ▶ Autosegmental/Q Theory
 - ▶ Feature Systems



Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Phonological Representations

- ▶ Recent work has used model theory to compare different proposed representation schemes.
 - ▶ Syllable Representations
 - ▶ Tonal Geometry
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Phonological Representations

- ▶ Recent work has used model theory to compare different proposed representation schemes.
 - ▶ Syllable Representations
 - ▶ Tonal Geometry
 - ▶ Autosegmental/Q Theory
 - ▶ **Feature Systems**

$CPL(\mathcal{M}^v)$	\mathcal{M}_p^v	\mathcal{M}_F^v	\mathcal{M}_C^v
voi	{N,D}	{N,D}	{D}
son	{N}	{N}	{N}
son \wedge voi	{N}	{N}	{}
MISSING	–	{D}, {T}, {D,T}	{T}, {D,T}
EXTRA	–	–	–

$CNPL(\mathcal{M}^v)$	\mathcal{M}_p^v	\mathcal{M}_F^v	\mathcal{M}_C^v
voi	{N,D}	{N,D}	{D}
\neg voi	{T}	{T}	{N,T}
son	{N}	{N}	{N}
\neg son	{D,T}	{D,T}	{D,T}
son \wedge \neg son	{}	{}	{}
son \wedge voi	{N}	{N}	{}
son \wedge \neg voi	{}	{}	{N}
\neg son \wedge voi	{D}	{D}	{D}
\neg son \wedge \neg voi	{T}	{T}	{T}
voi \wedge \neg voi	{}	{}	{}
MISSING	–	–	–
EXTRA	{D}, {T}, {D,T}	–	{N,T}

Phonological Representations

- ▶ All of the previous work has looked at representations within the larger tradition of generative phonology
 - ▶ roughly: input/output mappings described by symbolic changes
- ▶ Articulatory Phonology is a theory of phonological representations based in nonlinear dynamics.
 - ▶ crucially: no input/output mappings
- ▶ **addendum:** many researchers have used gestural representations within generative phonology, but here I am focused on Articulatory Phonology as a theory of phonology that **does not** have a generative element as it offers a more interesting comparison case.

Browman and Goldstein (1986, 1992); McMahon et al. (1994); Browman and Goldstein (1995); Zsiga (1997); Gafos (2002); Hall (2003); Davidson (2004); Bateman (2007); Nam (2007); Goldstein (2011); Friedman and Visser (2014)

Overview of Talk

- ▶ In this talk I will use model theory and logic to show the *bi-interpretability* of segmental strings and coupling graphs which are the lexical representations used in Articulatory Phonology.

- ▶ **Translation:** Any rule/constraint written about segments can be directly translated into a rule/constraint about coupling graphs in a computationally restricted way.

Model Theoretic Phonology

Model Theoretic Phonology

- ▶ What is (finite) model theory?
 - ▶ In short: a branch of logic used to study finite structures.
- ▶ Why use model theory to study phonology?
 - ▶ Allows for flexibility in choices of representation.
 - ▶ Gives an idea of the computational complexity required to implement a mapping.
 - ▶ Provides a general framework within which specific phonological theories can be studied and compared.

Model Theoretic Phonology - Structure

A model is a tuple $\mathcal{M} = \langle \mathcal{D}, \mathcal{C}, \mathcal{P}, \mathcal{F} \rangle$ where

- ▶ \mathcal{D} is a set of domain elements
- ▶ \mathcal{C} is a set of constants (not regularly used in phonological models)
- ▶ \mathcal{R} is a set of predicates/relations
- ▶ \mathcal{F} is a set of functions

Model Theoretic Phonology - Structure

- ▶ It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.

Model Theoretic Phonology - Structure

- ▶ It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.
- ▶ \mathcal{M} - Whatever representational framework we are working under such as binary features, feature geometry, elements, ...

Model Theoretic Phonology - Structure

- ▶ It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.
- ▶ \mathcal{D} - Identification markers that differentiate the segments, feature bundles, X-slots, and other positional elements in a representation.

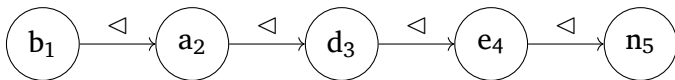
Model Theoretic Phonology - Structure

- ▶ It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.
- ▶ \mathcal{R}/\mathcal{F} - They can be segment labels (IPA symbols) or they can be different phonological or phonetic features. They can also include syllabic, prosodic, and any other necessary information. These also determine ordering relationships between domain elements.

Model Theoretic Phonology - Structure

- ▶ A specific collection of domain elements, predicates, and functions is called a **model signature**. One model signature for strings is:
 - ▶ $\langle \mathcal{D}, \mathcal{P}_{\triangleleft}, \mathcal{P}_{\sigma} \mid \sigma \in \Sigma \rangle$ (successor relation model)
 - ▶ Σ is called the **alphabet** and contains the base set of symbols used. (e.g. - $\Sigma = \{a, b\}$ or $\Sigma = \{+voice, +syl, -sonorant, \dots\}$)

Model Theoretic Phonology - Structure



$$\begin{aligned} \langle \mathcal{D} &= \{1, 2, 3, 4, 5\}, \\ \triangleleft &= \{(1, 2), (2, 3), (3, 4), (4, 5)\}, \\ \mathbf{b} &= \{1\}, \mathbf{a} = \{2\}, \mathbf{d} = \{3\}, \mathbf{e} = \{4\}, \mathbf{n} = \{5\} \end{aligned}$$

Model Theoretic Phonology - Translations

- ▶ Given a specific model, it is possible to define a **logical translation** from one structure into another structure.
- ▶ Translating one structure into another structure is exactly what phonology does.
- ▶ Therefore, these logical translations can be thought of as declarative statements that describe phonological maps.

Model Theoretic Phonology - Translations

- ▶ Translation is done by defining output properties of a structure in terms of input properties.
 - ▶ Formulae such as $\phi_P(x) = Q(x)$ are interpreted as “domain element x has property P in the output structure if it has property Q in the input structure”.
 - ▶ Every predicate/function gets an output formula
 - ▶ Additionally, one must specify how many copies of the input domain are needed (> 1 if epenthesis or reduplication; else $= 1$) and which copies are licensed in the output (TRUE unless deletion occurs).

Model Theoretic Phonology - Translations

- ▶ Given a rule $a \rightarrow b/c_d$
- ▶ $\phi_b(x) \stackrel{\text{def}}{=} b(x) \vee [a(x) \wedge c(p(x)) \wedge d(s(x))]$
 - ▶ $\phi_b(x) \stackrel{\text{def}}{=} \dots$ “make domain element x a b on the output if...”
 - ▶ $b(x) \vee \dots$ “ x is a b on the input or...”
 - ▶ $[a(x) \wedge c(p(x)) \wedge d(s(x))]$... “ x is an a on the input and preceded by a c and followed by a d .”

$s(x)$ and $p(x)$ are successor and predecessor **functions** with types $\mathcal{D} \rightarrow \mathcal{D}$.

Model Theoretic Phonology - Translations

- ▶ Some actual linguistic data...aka the “Heather slide”.
- ▶ German final devoicing.
 - 1 /bad + en/ → [baden] ‘to bathe’
 - 2 /bad/ → [bat] ‘bath’
 - 3 bat + en/ → [baten] ‘asked’
 - 4 /bat/ → [bat] ‘ask’

Model Theoretic Phonology - Translations

- The following set of formulae describe the final devoicing mapping.

$$\text{final}(x) \stackrel{\text{def}}{=} s(x) = x$$

$$\phi_{\text{domain}} \stackrel{\text{def}}{=} \text{TRUE}$$

$$C \stackrel{\text{def}}{=} \{1\}$$

$$\phi_{\text{license}} \stackrel{\text{def}}{=} \text{TRUE}$$

$$\phi_a(x) \stackrel{\text{def}}{=} a(x)$$

$$\phi_e(x) \stackrel{\text{def}}{=} e(x)$$

$$\phi_n(x) \stackrel{\text{def}}{=} n(x)$$

$$\phi_b(x) \stackrel{\text{def}}{=} b(x) \wedge \neg \text{final}(x)$$

$$\phi_p(x) \stackrel{\text{def}}{=} p(x) \vee (b(x) \wedge \text{final}(x))$$

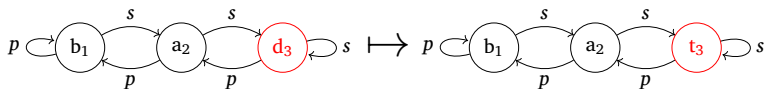
$$\phi_d(x) \stackrel{\text{def}}{=} d(x) \wedge \neg \text{final}(x)$$

$$\phi_t(x) \stackrel{\text{def}}{=} t(x) \vee (d(x) \wedge \text{final}(x))$$

$$\phi_s(x) \stackrel{\text{def}}{=} s(x)$$

$$\phi_p(x) \stackrel{\text{def}}{=} p(x)$$

Model Theoretic Phonology - Translations

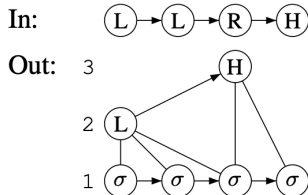


- ▶ Domain element 3 gets changed from **d** to **t** because it satisfies this formula.
- ▶ $\phi_t(x) \stackrel{\text{def}}{=} t(x) \vee (d(x) \wedge \text{final}(x))$

Model Theoretic Phonology - Translations

- ▶ There is no limitation on what type of structures one can translate between.
- ▶ In other words, the same technique can be used to translate between different representation schemes.
- ▶ This allows for direct comparisons of equality and expressivity.

Model Theoretic Phonology - Translations



$$\varphi_{\mathcal{A}}^{1,1}(x, y) \stackrel{\text{def}}{=} x \triangleleft y$$

$$\varphi_{\mathcal{A}}^{2,2}(x, y) \stackrel{\text{def}}{=} x \prec y \wedge$$

$$\text{spanfirst}(x) \wedge \text{spanfirst}(y) \wedge (\forall z)[x \prec z \prec y \rightarrow \text{span}(x, z)]$$

$$\varphi_{\mathcal{A}}^{2,3}(x, y) \stackrel{\text{def}}{=}$$

$$(\text{spanfirst}(x) \wedge (F(y) \vee R(y)) \wedge x = y) \vee (\text{spanfirst}(x) \wedge (F(x) \vee R(x)) \wedge \text{span}(x, y))$$

$$\varphi_{\mathcal{A}}^{3,2}(x, y) \stackrel{\text{def}}{=} (F(x) \vee R(x)) \wedge \text{spanfirst}(y) \wedge \text{span}(x, y)$$

$$\varphi_{\mathcal{A}}^{3,3}(x, y) \stackrel{\text{def}}{=} (F(x) \vee R(x)) \wedge (F(y) \vee R(y)) \wedge \text{span}(x, y)$$

- ▶ “The second, positive, result is that [Autosegmental Representation]s are [First Order]-definable from strings, showing that they do not significantly increase the expressive power of phonotactic grammars. It is thus also likely that they do not significantly increase the expressive power of string mappings, although the logical study of phonological transformations is still ongoing [SN 2023 update: they don’t].”

Articulatory Phonology

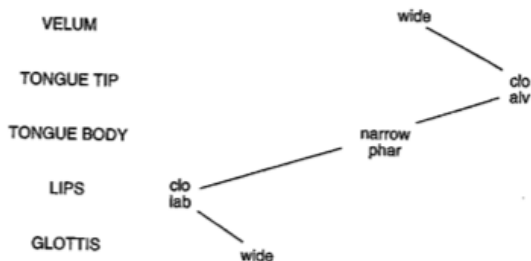
Gestural Representations

- ▶ A **gesture** is “a characteristic pattern of movement of an articulator (or of an articulatory subsystem) through space, over time” (p. 237)
- ▶ “We take, then, as a first hypothesis that gestures can be characterised in terms of a dynamical system and its associated motion variables and parameter values [e.g. constriction location/degree]...” (p. 240).
- ▶ $m\ddot{x} + k(x - x_0) = 0$

Gestural Representations

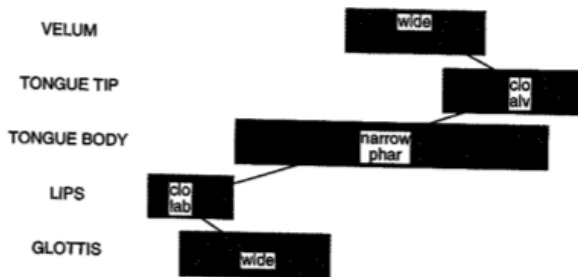
- ▶ “...a **constellation** of gestures is a set of gestures that are coordinated with one another by means of phasing...” (p. 185).
- ▶ “Each gesture is assumed to be active for a fixed proportion of its virtual cycle...The linguistic gestural model uses this proportion, along with the stiffness of each gesture and the phase relations among the gestures, to calculate a *gestural score* that specifies the temporal activation intervals for each each gesture in an utterance” (p. 187).
- ▶ “The parameter value specifications and activation intervals from the gestural score are input to the task-dynamical model..., which calculates the time-varying response of the tract variable and component articulators to the imposition of the dynamical regimes defined by the gestural score” (p. 188).

Gestural Representations



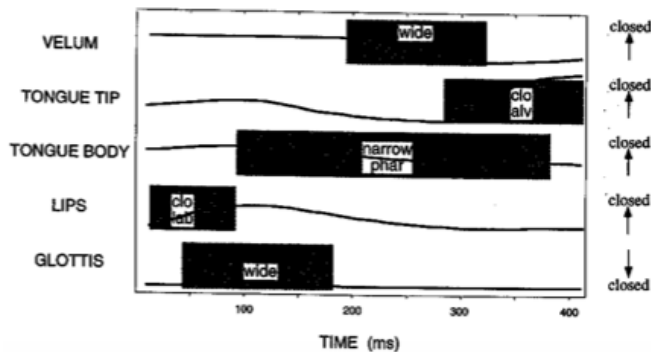
- ▶ Representations of [pan]
 - ▶ Constellation
 - ▶ Gestural Score
 - ▶ Trajectories

Gestural Representations



- ▶ Representations of [pan]
 - ▶ Constellation
 - ▶ **Gestural Score**
 - ▶ Trajectories

Gestural Representations

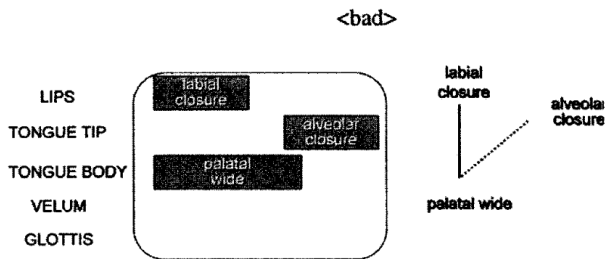


- ▶ Representations of [pan]
 - ▶ Constellation
 - ▶ Gestural Score
 - ▶ Trajectories

Coupling Graphs as Lexical Representations

*“In previous work, the gestural scores were constructed using rules that specified the relative phase of pairs of gestures...In this model, planning oscillators associated with the set of gestures in a given utterance are coupled in a pairwise, bidirectional manner specified in a coupling graph (or structure) that is part of **the lexical specification of a word**” (p. 38).*

Coupling Graphs as Lexical Representations



- ▶ Solid line is in-phase (0 degrees; simultaneous)
- ▶ Dashed line is anti-phase (180 degrees; opposite)

Model Theoretic Representations

Coupling Graph Models

Relation	Label
\diamond	In-phase
\triangleleft_{180}	Anti-phase
\triangleleft_{60}	Abutting
\triangleleft_{30}	Eccentric

- ▶ 4 binary relations based on common phase relations in Articulatory Phonology

Coupling Graph Models

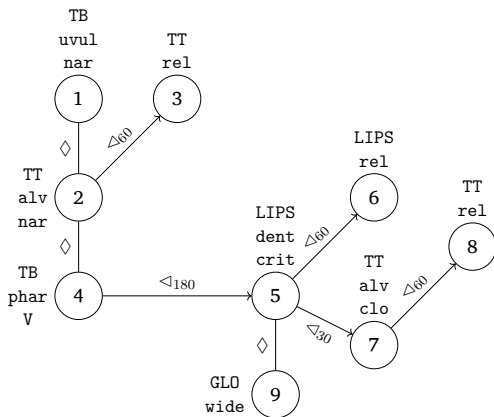
Relation	Label	Relation	Label
LIPS	Labial Articulator	rel	Constriction Degree: release
TT	Tongue Tip Articulator	pro	Constriction Location: protruded
TB	Tongue Body Articulator	dent	Constriction Location: dental
VEL	Velum Articulator	alv	Constriction Location: alveolar
GLO	Glottis Articulator	palv	Constriction Location: postalveolar
clo	Constriction Degree: closed	pal	Constriction Location: palatal
crit	Constriction Degree: critical	vel	Constriction Location: velar
nar	Constriction Degree: narrow	uvul	Constriction Location: uvular
V	Constriction Degree: vowel	phar	Constriction Location: pharyngeal
wide	Constriction Degree: wide		

- Unary labeling relations.

Coupling Graph Model: [læft]

$\mathcal{D} := \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$	$\text{dent} := \{5\}$
$\diamond := \{(1, 2), (2, 4), (5, 9)\}$	$\text{alv} := \{2, 7\}$
$\triangleleft_{180} := \{(4, 5)\}$	$\text{uvul} := \{1\}$
$\triangleleft_{60} := \{(2, 3), (5, 6), (7, 8)\}$	$\text{phar} := \{4\}$
$\triangleleft_{30} := \{(5, 7)\}$	$\text{clo} := \{7\}$
$\text{LIPS} := \{5, 6\}$	$\text{crit} := \{5\}$
$\text{TT} := \{2, 3, 7, 8\}$	$\text{nar} := \{1, 2\}$
$\text{TB} := \{1, 4\}$	$\text{wide} := \{9\}$
$\text{GLO} := \{9\}$	$\text{rel} := \{3, 6, 8\}$
	$\text{V} := \{4\}$

Coupling Graph Model: [læft]



String Model: [læft]

Relation	Label
\triangleleft	Successor
$\sigma (\forall \sigma \in \Sigma)$	Segment

$\langle \mathcal{D} := \{1, 2, 3, 4\}$

$\triangleleft := \{(1, 2), (2, 3), (3, 4)\}$

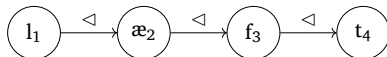
$\mathfrak{a} := \{2\}$

$\mathfrak{f} := \{3\}$

$\mathfrak{l} := \{1\}$

$\mathfrak{t} := \{4\}$

$\sigma := \{\}; \sigma \in \Sigma \setminus \{\mathfrak{a}, \mathfrak{f}, \mathfrak{l}, \mathfrak{t}\}$

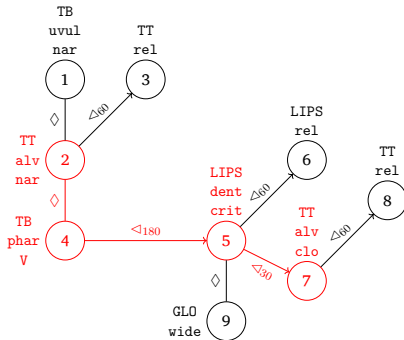


Translations

Translating between Structures

- ▶ Here we'll define two translations:
 - ▶ Coupling graph to string: Γ^{sg}
 - ▶ String to coupling graph: Γ^{gs}

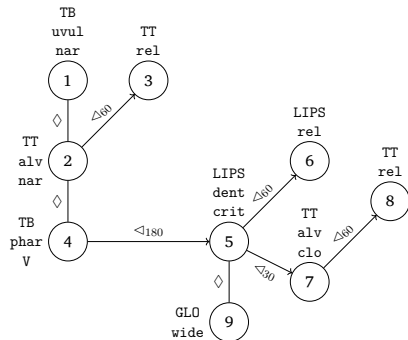
Identifying the “spine”



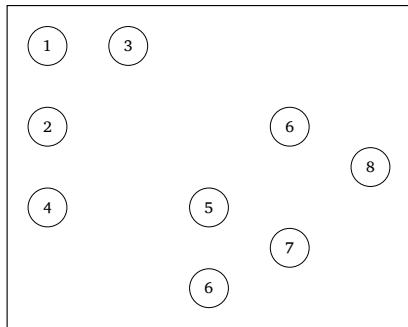
- ▶ We can identify the *spine* of a coupling graph by looking at the subgraph that does not include:
 - ▶ Secondary articulations
 - ▶ Release Gestures
 - ▶ Glottal Gestures
 - ▶ Velum Gestures

Translating from coupling graph to string

Input



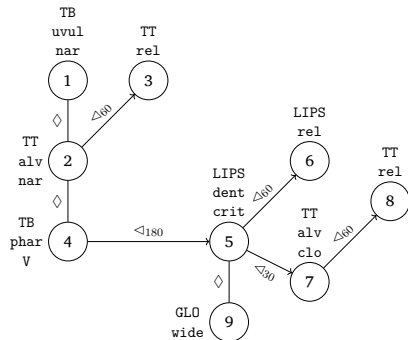
Workspace



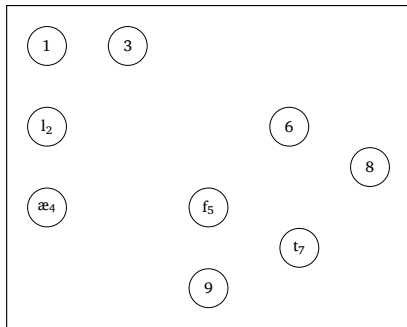
► $C := \{1\}$

Translating from coupling graph to string

Input



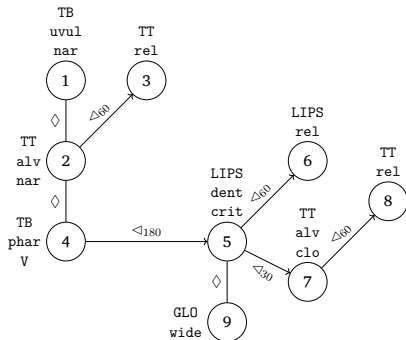
Workspace



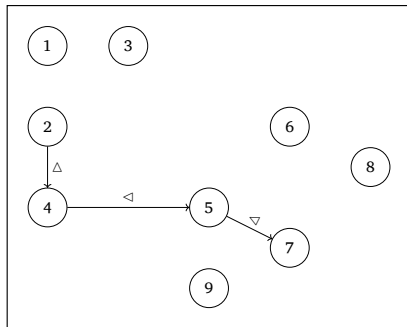
- ▶ $\varphi_1(x) := TT(x) \wedge alv(x) \wedge nar(x) \wedge \exists y[x \diamond y \wedge voc(y) \wedge TB(y) \wedge uvul(y)]$
- ▶ $\varphi_{\text{æ}}(x) := TB(x) \wedge phar(x) \wedge V(x)$
- ▶ $\varphi_f(x) := LIPS(x) \wedge dent(x) \wedge crit(x) \wedge \exists y[x \diamond y \wedge GLO(y) \wedge wide(y)]$
- ▶ $\varphi_t(x) := TT(x) \wedge alv(x) \wedge clo(x) \wedge ((\exists yz[y \triangleleft_{30} x \Rightarrow (y \diamond z \wedge GLO(z))]) \vee (\exists y[x \diamond y \wedge GLO(y) \wedge wide(y)]))$

Translating from coupling graph to string

Input



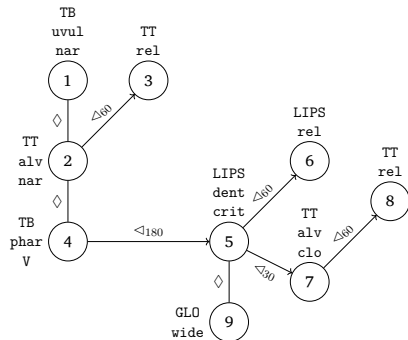
Workspace



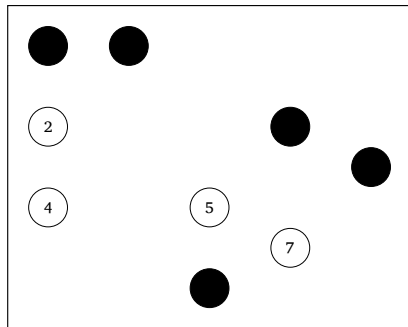
- ▶ Onset Cs are in phase with V and anti-phase with preceding C.
- ▶ First coda C is anti-phase with V; all other Cs eccentric with preceding C.
- ▶ $\varphi_{\triangleleft}(x, y) := (x \triangleleft_{180} y) \vee (x \triangleleft_{30} y) \vee (x \diamond y \wedge V(y) \wedge \neg \exists z [x \triangleleft_{180} z])$

Translating from coupling graph to string

Input



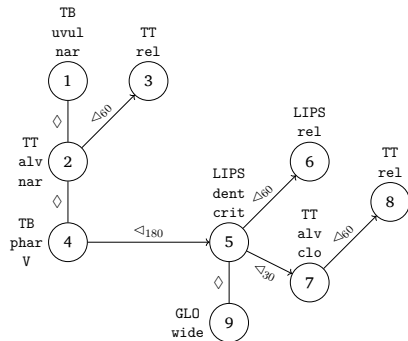
Workspace



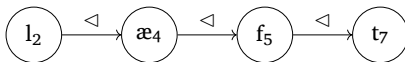
- ▶ “spine” identification.
- ▶ $\varphi_{\text{license}}(x) := \neg \text{rel}(x) \wedge \neg \text{GLO}(x) \wedge ((\text{TB}(x) \wedge \neg \text{V}(x)) \Rightarrow \neg \exists y [\text{TT}(y) \wedge x \diamond y])$

Translating from coupling graph to string

Input



Output

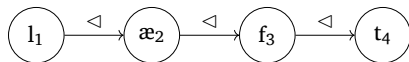


Expansion

- ▶ Going from coupling graph to string removes information.
- ▶ What happens when we have to expand the representation and add more information by going from a string to a coupling graph?
- ▶ **Spoiler:** no real problems arise

Translating from string to coupling graph

Input



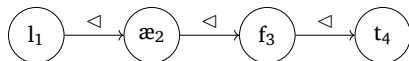
Workspace



- ▶ $C := \{1, 2, 3, 4\}$
- ▶ Unique copy sets for primary gesture, release gesture, secondary gestures, glottal/nasal gesture

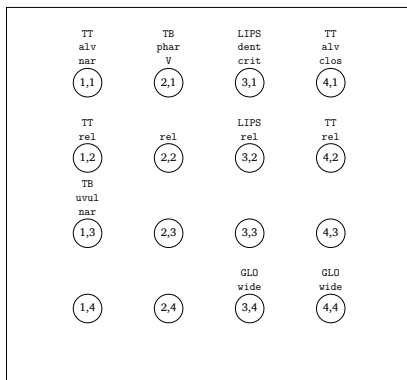
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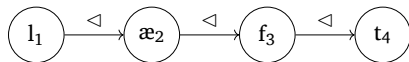
$$\begin{aligned}
 \varphi_{\text{LIPS}}^1(x) &:= f(x) & \varphi_{\text{phar}}^1 &:= \text{æ}(x) \\
 \varphi_{\text{LIPS}}^2(x) &:= \varphi_{\text{LIPS}}^1(x) & \varphi_{\text{uvul}}^3 &:= l(x) \\
 \varphi_{\text{TT}}^1(x) &:= t(x) \vee l(x) & \varphi_{\text{clo}}^1 &:= t(x) \\
 \varphi_{\text{TT}}^2(x) &:= \varphi_{\text{TT}}^1(x) & \varphi_{\text{crit}}^1 &:= f(x) \\
 \varphi_{\text{TB}}^1(x) &:= \text{æ}(x) & \varphi_{\text{v}}^1 &:= \text{æ}(x) \\
 \varphi_{\text{TB}}^3(x) &:= l(x) & \varphi_{\text{nar}}^1 &:= l(x) \\
 \varphi_{\text{GLO}}^4(x) &:= t(x) \vee f(x) & \varphi_{\text{nar}}^3 &:= l(x) \\
 \varphi_{\text{dent}}^1 &:= f(x) & \varphi_{\text{wide}}^4 &:= t(x) \vee f(x) \\
 \varphi_{\text{alv}}^1 &:= t(x) & &
 \end{aligned}$$

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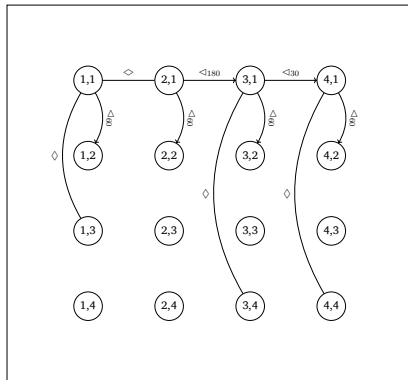


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$$\varphi_{\diamond}^{1,1}(x, y) := x \triangleleft y \wedge \text{æ}(y) \wedge \neg \text{æ}(x)$$

$$\varphi_{\diamond}^{1,3}(x, y) := (x = y) \wedge l(x)$$

$$\varphi_{\diamond}^{1,4}(x, y) := (x = y) \wedge t(x) \vee f(x)$$

$$\varphi_{\triangleleft_{180}}^{1,1}(x, y) := x \triangleleft y \wedge \text{æ}(x) \wedge \neg \text{æ}(y)$$

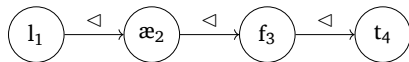
$$\varphi_{\triangleleft_{60}}^{1,2}(x, y) := (x = y)$$

$$\varphi_{\triangleleft_{30}}^{1,1}(x, y) := \neg \text{æ}(x) \wedge \neg \text{æ}(y) \wedge$$

$$\exists z[z \triangleleft x \wedge \text{æ}(z)]$$

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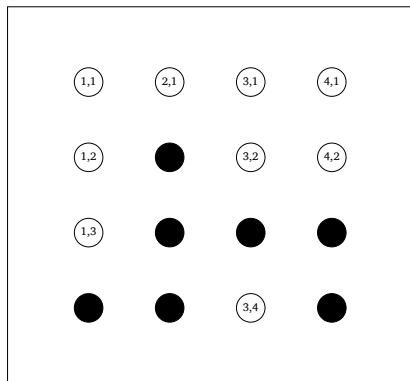
$$\varphi_{\text{license}}^1(x) := \text{True}$$

$$\varphi_{\text{license}}^2(x) := f(x) \vee t(x) \vee l(x)$$

$$\varphi_{\text{license}}^3(x) := l(x)$$

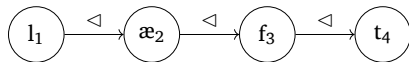
$$\begin{aligned} \varphi_{\text{license}}^4(x) &:= t(x) \vee f(x) \wedge \\ &\quad \neg \exists y[y \triangleleft x \wedge f(y) \vee t(y)] \end{aligned}$$

Workspace

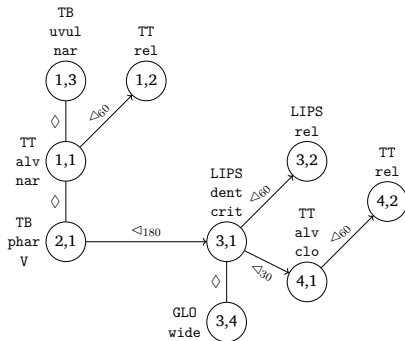


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Definition:

We note that an interpretation $K : U \rightarrow V$ gives us a construction of an internal model $\tilde{K}(\mathcal{M})$ of U from a model \mathcal{M} of V . We find that U and V are bi-interpretable iff, there are interpretations $K : U \rightarrow V$ and $M : V \rightarrow U$ and formulas F and G such that, for all models \mathcal{M} of V , the formula F defines an isomorphism between \mathcal{M} and $\tilde{M}\tilde{K}(\mathcal{M})$, and, for all models \mathcal{N} of U , the formula G defines an isomorphism between \mathcal{N} and $\tilde{K}\tilde{M}(\mathcal{N})$.

Bi-Interpretability

\mathcal{M}^s	string model of <i>laughed</i>
\mathcal{M}^g	coupling graph model of <i>laughed</i>
Γ^{sg}	string to coupling graph transduction
Γ^{gs}	coupling graph to string transduction

- ▶ $\mathcal{M}^s \equiv \Gamma^{gs}(\Gamma^{sg}(\mathcal{M}^s))$
- ▶ $\mathcal{M}^g \equiv \Gamma^{sg}(\Gamma^{gs}(\mathcal{M}^g))$
- ▶ This indicates the string and coupling graph models are **bi-interpretable**

Conclusion

It's symbols all the way down

*Thus we are referring to the same set of dynamically specified gestures, but this time using symbols which serve as indices to entire dynamical systems. **These symbolic descriptions highlight those aspects of the gestural structures that are relevant for contrast among lexical items**" (p. 241).*

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Note: while the other models represented by boxes in Figure 1 (**Coupled oscillator model of inter-gestural coordination** and **Task dynamic model of inter-articulator coordination**) are meant to be part of a model of the human speech production process, the method used for automatic generation of coupling graphs is a heuristic that is not meant to model how a speaker would go about constructing a coupling graph for an arbitrary form. Coupling graphs could simply be stored by speakers in the lexicon. The automatic computation has two major benefits: (1) It represents in compact form generalizations about the set of coupling graphs that speakers use (in English, at least) and their relation to more conventional phonological representations (segments, features, syllable structure). (2) It allows the later stages of the model to be tested, by allowing automatic generation of a variety of input files.

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- ▶ But the coupling graph to string translation is novel (Jason Shaw, p.c.).

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- ▶ Can we change the representations slightly to write a QF transduction?

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- ▶ This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.
 - ▶ AP coupling graphs already contain all the necessary information for phonetic implementation.
 - ▶ Strings must be further transformed somehow (but we've seen that's not too difficult to do 😎).

Thank You!

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