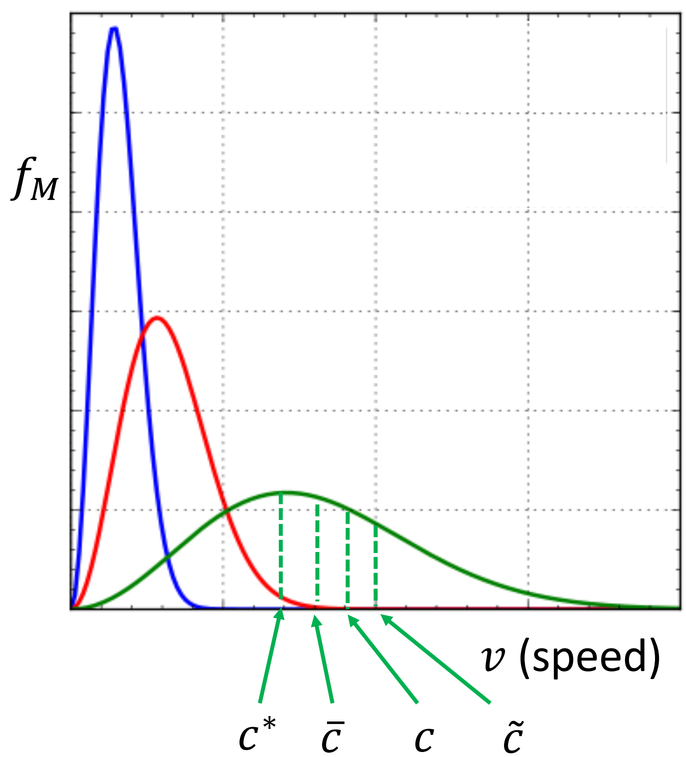
Using our analytical (algebraic) results to demonstrate the ordering

Property		Speed
Most probable	c^*	$\sqrt{\frac{2k_bT}{m}}$
Average	$\bar{c} \equiv < v >$	$\sqrt{\frac{8k_bT}{\pi m}}$
Root-mean-square	$c \equiv < v^2 >^{1/2}$	$\sqrt{\frac{3k_bT}{m}}$

$$\frac{\bar{c}}{c^*} = \frac{\sqrt{\frac{8k_BT}{\pi m}}}{\sqrt{\frac{2k_BT}{m}}} = \sqrt{8/2\pi} > 1$$



But … where did those analytical results come from?

$$< v > = \int_0^\infty v\, f_M(T,v)\, dv \equiv \bar{c}$$

$$< v^2 > = \int_0^\infty v^2\, f_M(T,v)\, dv \equiv c^2$$

$$< v^3 > = \int_0^\infty v^3\, f_M(T,v)\, dv \equiv \tilde{c}^3$$

Gaussian Functions

$$\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x\, e^{-ax^2}\, dx = \frac{1}{2a}$$

$$\int_0^\infty x^2\, e^{-ax^2}\, dx = \frac{1}{4a}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x^3\, e^{-ax^2}\, dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^4\, e^{-ax^2}\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x^5\, e^{-ax^2}\, dx = \frac{1}{a^3}$$

$$\int_0^\infty x^{2n}\, e^{-ax^2}\, dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n+1}a^n}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x^{2n+1}\, e^{-ax^2}\, dx = \frac{n!}{2}\left(\frac{1}{a^{n+1}}\right)$$

For practice, make sure you can derive formulas for $\bar{c}$ and c. Then:

1. Derive a formula for $\tilde{c}$
2. Derive a formula for $< v_x^2 > + < v_y^2 > + < v_z^2 >$. This will require integration of expressions like $\int_{-\infty}^{\infty} v_x^2\, f_B(T, v_x)\, dv_x$.
3. From there, derive a formula for the average kinetic energy of a mole of molecules, $\langle \epsilon \rangle = N_A \times \frac{1}{2} m\, (< v_x^2 > + < v_y^2 > + < v_z^2 >)$.