AARHUS UNIVERSITY

COMPUTER-SCIENCE

INTRODUCTION TO PROBABILITY AND STATISTICS

Handin 3

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Problem 7 (Exam Winter 2017/2018)

It is given that 10% of the population has influenza in a given period.

1. Suppose that 20 people are chosen randomly and independently of each other. Find the probability that at least one of the 20 selected people has influenza.

   When handling cases with 'Atleast one' it can be an advantage to look at the opposite:
   $$P(\text{Atleast one infected}) = 1 - P(\text{No infected})$$
   Since we are looking at independant trials when picking from the population, we can multiply the propability of each trials to get their intersected probability [1.4.1][1].
   Let:
   $$P(\text{Is infected}) = P(B) = 0.1 \qquad P(\text{Not infected}) = P(B^c) = 0.9$$
   We are picking 20 people which is 20 independant trials hence our probability is:
   $$P(\text{No infected}) = 0.9^{20} = 0.121$$
   Now we can look back at the initial proposition
   $$P(\text{Atleast one infected}) = 1 - 0.121 = \underline{0.879}$$
   We Did it!

   A person can be tested for influenza using a test, which does not always provide the correct answer.
   The test gives a positive result for influenza 85% of the time for people who have influenza, and a negative result for influenza 95% of the time for people who do not have influenza.

2. Find the probability of having influenza given that a person has tested positive for influenza.

   We are looking for $P(B|A)$ where B is the event that a person has influenza(Is infected) and A is the event that a person tests positive.
   For this we can use Bayes' Theorem [1.4.3][1], that is used to invert conditional probabilities:
   $$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

But we don't know $P(A)$, but Bayes' Theorem can be manipulated to rewrite $P(A)$ [1.4.3][1]:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

Now we can insert our known values:

$$P(A|B) = 0.85 \qquad P(B) = 0.1 \qquad P(A|B^c) = 0.05 \qquad P(B^c) = 0.9$$

And we get:

$$P(B|A) = \frac{0.85 \cdot 0.1}{0.85 \cdot 0.1 + 0.05 \cdot 0.9} = \underline{0.654}$$

References

[1] Hossein Pishro-Nik. *Introduction to Probability ,Statistics and Random Processes*. Kappa Research, LLC, 2014.