Aarhus university

Computer-Science

Introduction to probability and statistics

Handin 6 (Box Edition)

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(a) Plot the probability mass function (PMF) of X, i.e., plot the function P_X.

This PMF is a result of the 'data-frame' created in the code section in Figure 1
The PMF is also rounded down to three decimals.

$$P(X = x) = \begin{cases} 0.049 & x = 0, \\ 0.149 & x = 1, \\ 0.224 & x = 2, \\ 0.224 & x = 3, \\ 0.168 & x = 4, \\ 0.100 & x = 5, \\ 0.050 & x = 6, \\ 0.021 & x = 7, \\ 0.008 & x = 8, \\ 0.002 & x = 9, \\ 0.001 & x = 10, \\ 0 & \text{otherwise.} \end{cases}$$

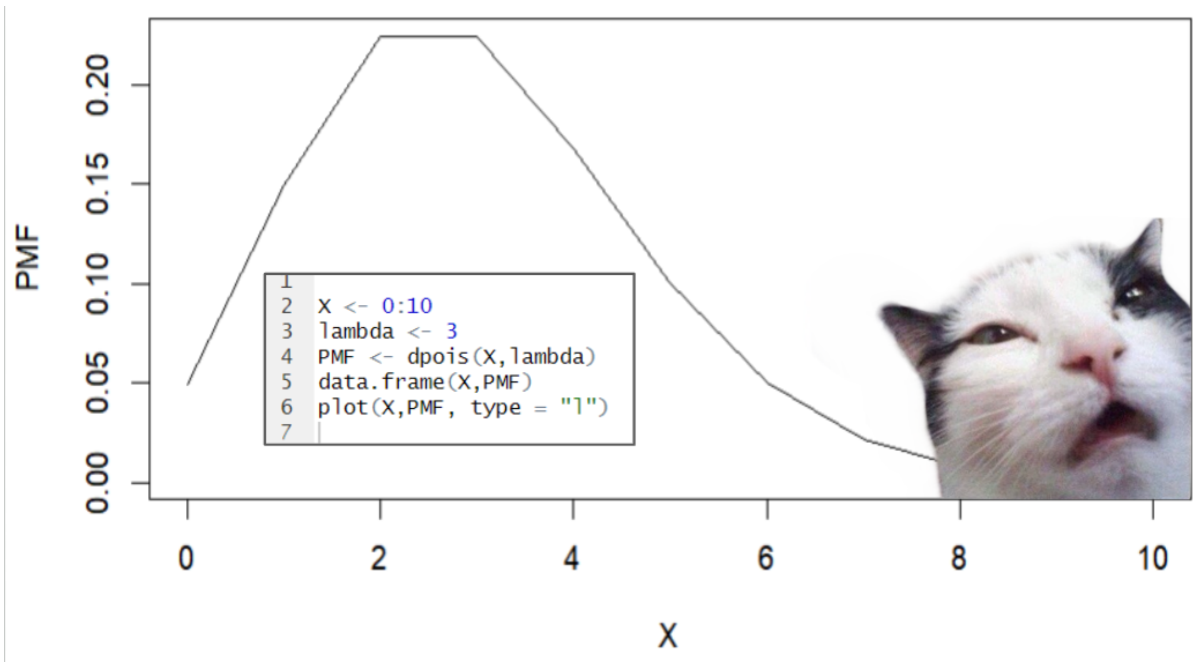

Figure 1: PMF of $X \sim \text{Poisson}(\lambda)$ with $\lambda = 3$.

(b) Simulate the mean (expected value) of X, and compare it with the theoretical result.

For a Poisson distributed random variable $X \sim \text{Poisson}(\lambda)$ the theoretical expected value is $\mathbb{E}[X] = \lambda$ according to example 3.13 [1]. In our case $\lambda = 3$, so $\mathbb{E}[X] = 3$.

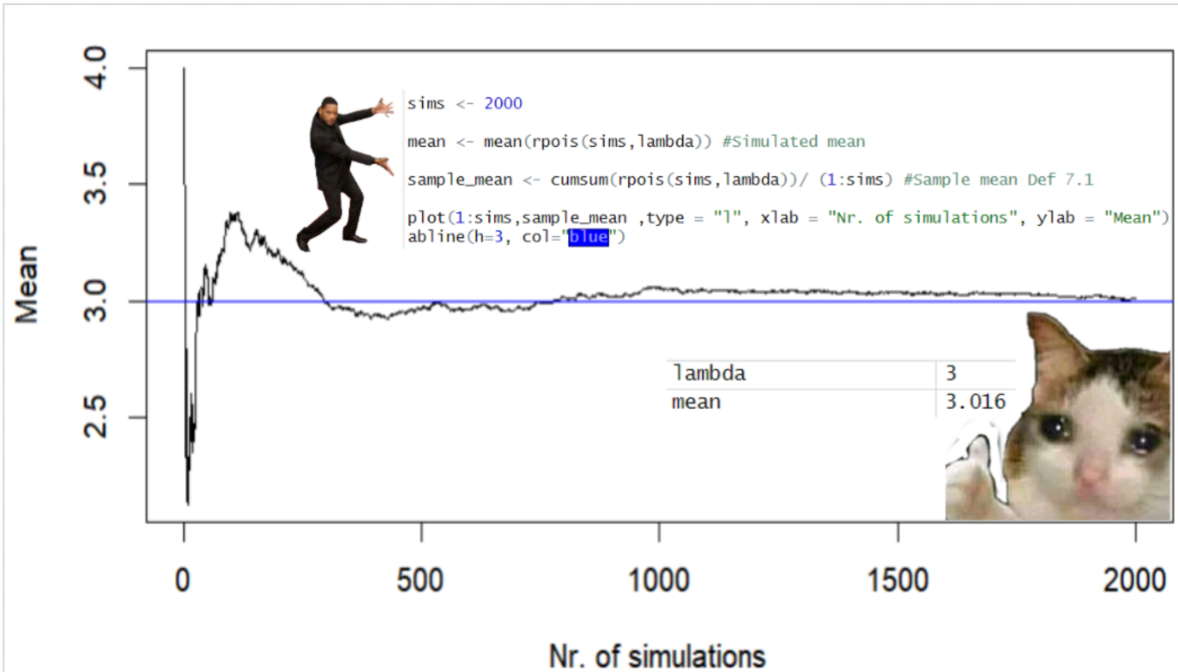

Figure 2: Convergence of the simulated mean of X compared with the theoretical mean ($\lambda = 3$).

It's seen that for high enough number of samples, the simulated mean converges to the theoretical mean of 3, as shown in Figure 2.

(c) Simulate the mean of $\sqrt{X}$.

Since it's hard to simplify and calculate the theoretical $\mathbb{E}[\sqrt{X}]$ for a Poisson distribution, we will use Monty Carlo simulations to approximate it.

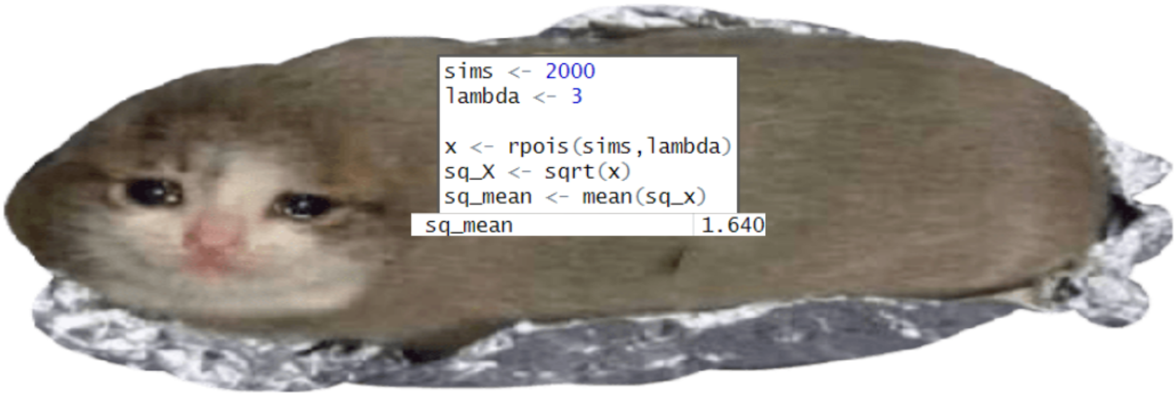

Figure 3: Simulated mean of $\sqrt{X}$ for $X \sim \text{Poisson}(3)$.

(d) Simulate the standard deviation of X, and compare it with the theoretical result.

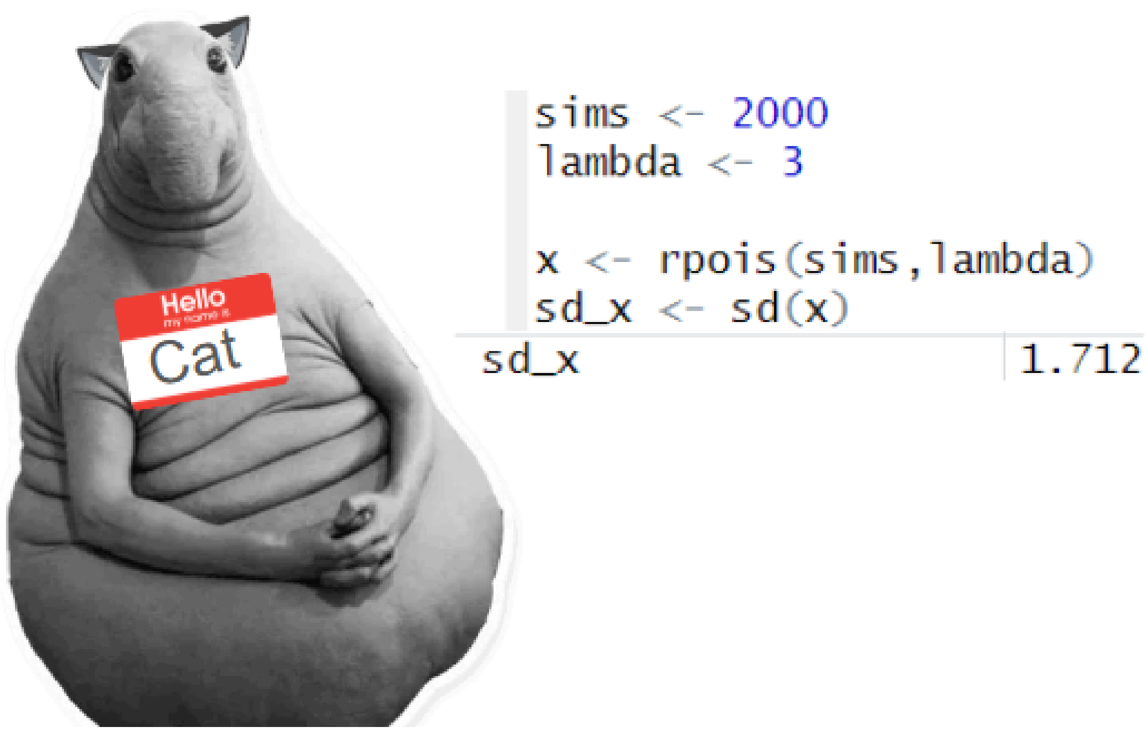

Figure 4: Simulated standard deviation of $X \sim \text{Poisson}(3)$.

According to problem 8 from chapter 3 [1] we know for the Poisson distribution that $\text{Var}(X) = \lambda$.

And for our $\lambda = 3$, we have $SD(X) = \sqrt{3} \approx 1.732$. Which compared to the simulated value in Figure 4 shows a pretty good convergence.

References

[1] Hossein Pishro-Nik. *Introduction to Probability ,Statistics and Random Processes.* Kappa Research, LLC, 2014.