Aarhus university

Computer-Science

Introduction to probability and statistics

# Handin 1


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August 25, 2025

# Problem 14

Let A and B be two events such that

$$P(A) = 0.4 \quad P(B) = 0.7 \quad P(A \cup B) = 0.9$$

## Question (a) find $P(A \cap B)$

We use the 'inclusion-exclusion-principle'

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$0.2 = 0.4 + 0.7 - 0.9$$

## Question (b) find $P(A^c \cap B)$

$P(A^c \cap B)$ can be rewritten as $P(B) - P(A \cap B)$, this can be found from the equation in figure 1.16, which is values we know

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

$$0.5 = 0.7 - 0.2$$

## Question (c) find $P(A - B)$

$P(A-B)$ is just another way to write $P(A \cap B^c)$, which is noted in figure 1.8 from the book, hence we get

$$P(A - B) = P(A) - P(A \cap B)$$

$$0.2 = 0.4 - 0.2$$

## Question (d) find $P(A^c - B)$

Again another way to write $P(A^c \cap B^c)$.
We can use De-morgans law

$$P(A^c \cap B^c) = P((A \cup B)^c)$$

We can remove the 'complement' by subtracting 1

$$P((A \cup B)^c) = 1 - P(A \cup B)$$

Now we can derive the probability since we already know the probability of $P(A \cup B)$

$$0.1 = 1 - 0.9$$

## Question (e) find $P(A^c \cup B)$

We use the 'inclusion-exclusion-principle' again

$$P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B)$$

$$0.8 = 0.6 + 0.7 - 0.5$$

## Question (f) find $P(A \cap (B \cup A^c))$

We use the distributive law

$$P(A \cap (B \cup A^c)) = P((A \cap B) \cup (A \cap A^c))$$

The set $(A \cap A^c)$ is the empty set $\phi$ so we are left with the set $(A \cap B)$, and we know the probability of this set from (a)

$$P(A \cap (B \cup A^c)) = P(A \cap B) = 0.2$$