Aarhus university

Computer-Science

Introduction to probability and statistics

Handin 2

Author
Søren M. Damsgaard

Student number
202309814

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Problem 28: In a factory there are 100 units of a certain product, 5 of which are defective. We pick three units from the 100 units at random. What is the probability that exactly one of them is defective?

Explanation

This is a typical problem that can be solved using hypergeometric distribution. It's assumed that we are chooosing 3 units at random without replacement.
we will use the formula for hypergeometric distribution given in definition [3.8][1].

$$\frac{\binom{b}{x}\binom{r}{k-x}}{\binom{b+r}{k}}$$

This distribution uses what we call binomial coefficients, or "n choose k". It's the number of ways to choose/distribute 'k' in 'n' objects [2.1.3][1].
The notation used for this is:

$$\binom{n}{k}$$

It also uses the classical probability formula for finite sample spaces [1.3.4][1]:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Since all the ways to choose 1 defect unit and all the ways to choose 2 functional units, combined, is our favorable outcome, while all the ways to choose 3 units out of a 100 regardless of condition, the total number of outcomes.

This problem can be seen as 'Unordered sampling without replacement'. This means we use the binomial coefficient formula for unordered sampling without replacement given in chapter [2.1.3][1].
Please note that n and k are dummy variables and have no relation to the n and k we using in the next step (Calculations).

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Calculations

Now that we understand ALL THE MATH!
Let's look at the numbers we have.

$$\begin{aligned}&\text{Let our total number of units be } N = 100\\&\text{Let our defects be } b = 5\\&\text{Let the number of units we pick be } k = 3\\&\text{Let the number of defects we want be } x = 1\\&\text{Let our functional units be } r = 95\end{aligned}$$

throw the nummbers into our formula for 'hypergeometric distribution'

$$\frac{\binom{5}{1}\binom{95}{2}}{\binom{100}{3}}$$

We have 3 binomial coefficients we have to calculate, let's get to it!.

$$\binom{5}{1} = \frac{5!}{1!(5-1)!} = \frac{5!}{1!4!} = \frac{120}{24} = 5$$

For the next two we will let some of the factorials cancel out to make it easier for the soul.

$$\binom{95}{2} = \frac{95!}{2!(95-2)!} = \frac{95!}{2!93!} = \frac{95 \cdot 94}{2} = 4465$$

$$\binom{100}{3} = \frac{100!}{3!(100-3)!} = \frac{100!}{3!97!} = \frac{100 \cdot 99 \cdot 98}{6} = 161700$$

Putting it all together we get

$$\frac{\binom{5}{1}\binom{95}{2}}{\binom{100}{3}} = \frac{5 \cdot 4465}{161700} = \frac{22325}{161700} \approx 0.138$$

The probability of getting exactly one defective unit when picking 3 units is something along the lines of 13.8%.
Obligatory victory cry! HUZZAR!

References

[1] Hossein Pishro-Nik. *Introduction to Probability ,Statistics and Random Processes*. Kappa Research, LLC, 2014.