Aarhus university

Computer-Science

Introduction to probability and statistics

Handin 9

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Let X_1 and X_2 denote two independent random variables such that $X_1 \sim \text{Exponential}(\lambda_1)$ and $X_2 \sim \text{Exponential}(\lambda_2)$, where $\lambda_1, \lambda_2 > 0$. Find the distribution of $Y := \min\{X_1, X_2\}$.

We start by understanding the meaning of life...42.
And then we look at the CDF of Y, which by the definition of CDF is given by:

$$F_Y(y) = P(Y \leq y) = P(\min\{X_1, X_2\} \leq y).$$

this can also be written as:

$$P(\min\{X_1, X_2\} \leq y) = P(X_1 \leq y \cup X_2 \leq y). \tag{1}$$

Which gives us the opportunity to use the inclusion-exclusion principle to calculate the CDF of Y tha way:

$$F_Y(y) = P(X_1 \leq y) + P(X_2 \leq y) - P(X_1 \leq y \cap X_2 \leq y).$$

Since X_1 and X_2 are independent we can write:

$$P(X_1 \leq y \cap X_2 \leq y) = P(X_1 \leq y)P(X_2 \leq y).$$

which gives us:

$$F_Y(y) = P(X_1 \leq y) + P(X_2 \leq y) - P(X_1 \leq y)P(X_2 \leq y).$$

But that's a lot of work that i don't wanna do... `Insert tired spongebob meme here`
Instead, we will do it the lazt way!

Let's do some math magic on Equation 1 also know as taking the complement.

$$P(X_1 \leq y \cup X_2 \leq y) = 1 - P(X_1 > y \cap X_2 > y).$$

Again, since X_1 and X_2 are independent we can write:

$$P(X_1 > y \cap X_2 > y) = P(X_1 > y)P(X_2 > y).$$

Thus we have:

$$F_Y(y) = 1 - P(X_1 > y)P(X_2 > y). \tag{2}$$

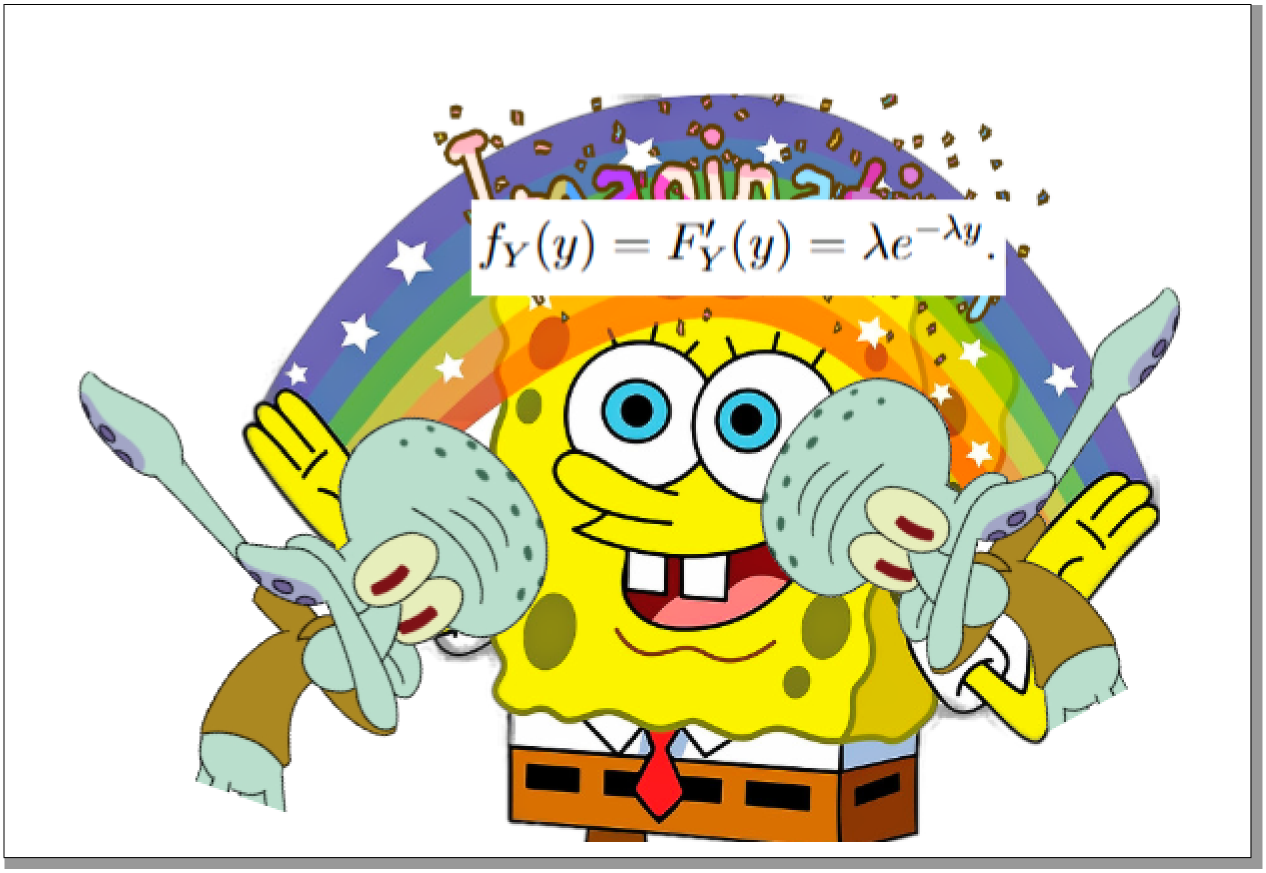
We are smart and see that it looks like the memoryless property of the exponential distribution from chapter 4.2.2, which means we can use the expression:

$$P(X > x) = e^{-\lambda x}$$

We can then insert that into Equation 2:

$$F_Y(y) = 1 - (e^{-\lambda y})(e^{-\lambda y}) = 1 - e^{-(\lambda_1 + \lambda_2)y}.$$

Then to finish, we let $\lambda = \lambda_1 + \lambda_2$ and differentiate the CDF to get the PDF:



Thus we see that $Y \sim \text{Exponential}(\lambda_1 + \lambda_2)$.