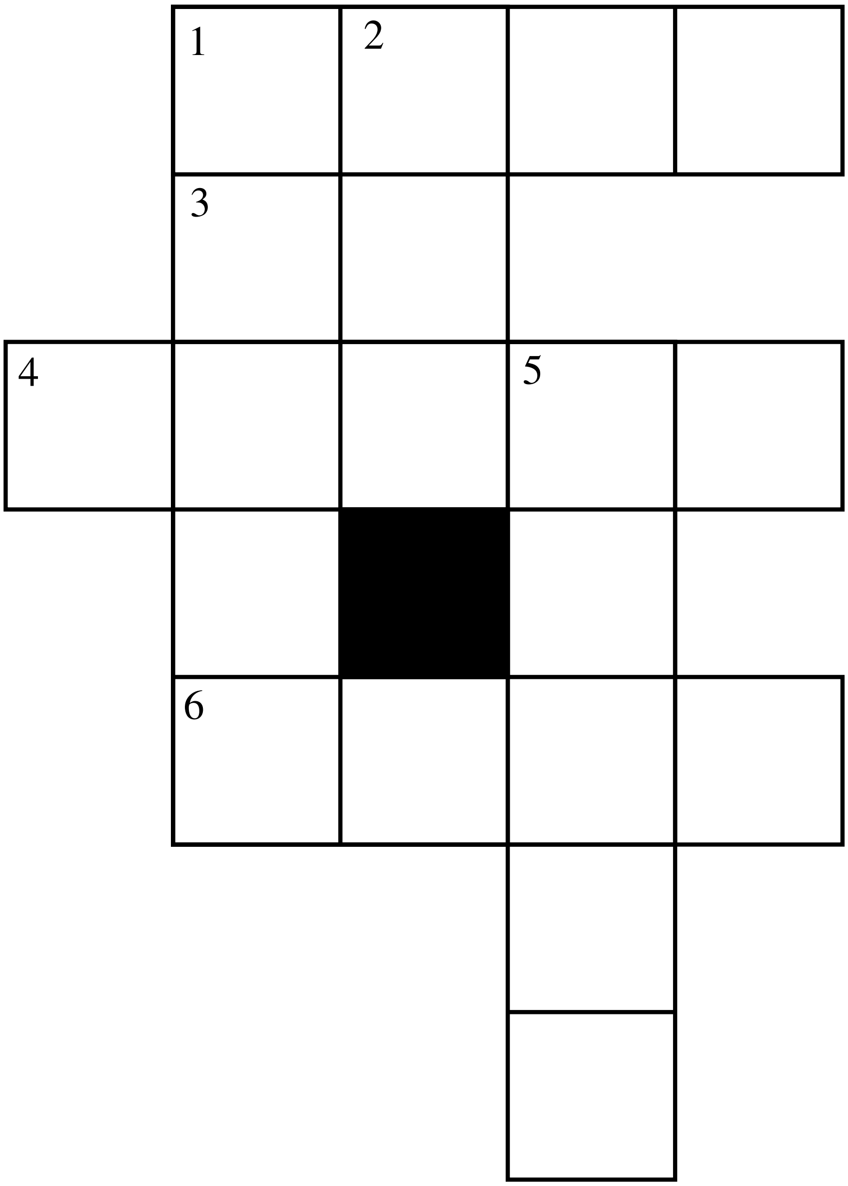
Try to fill the crossword with the words by hand:

Words:

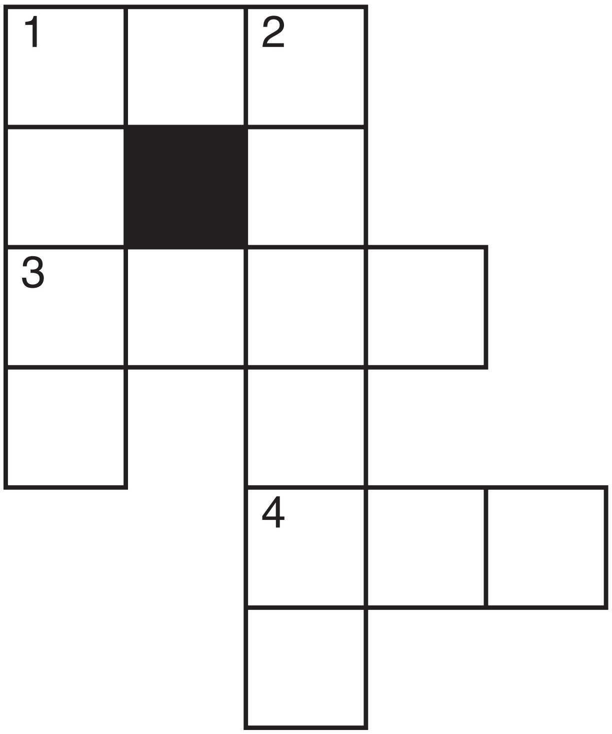
at, eta, be, hat, he, her, it, him, on, one, desk, dance, usage, easy, dove, first, else, loses, fuels, help, haste, given, kind, sense, soon, sound, this, think



Try to fill the crossword with the words using AC-3:

Words:

ant, big, bus, car, has, book, buys, hold, lane, year, beast, ginger, search, symbol, syntax

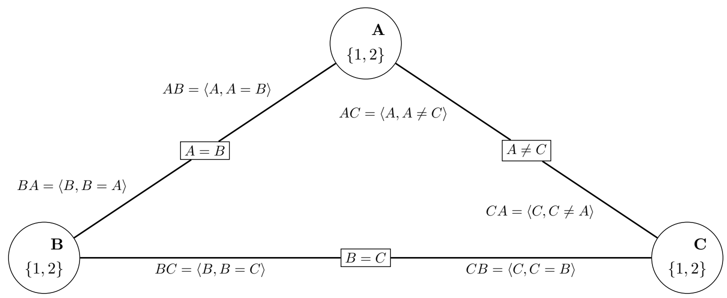


- **Variables**: let W_{ix} be the word at position ix where $i \in \{1, 2, \ldots\}$ and $x \in \{a, d\}$. Thus, for the small example above, the list of variables is $\{W_{1a}, W_{2d}, W_{1d}, W_{3a}, W_{4a}\}$. Let $|W_{ix}|$ be the length of the word ix. Also, let W_{ix_j} be the j^{th} letter of word W_{ix}, e.g. W_{1a_2} is the second letter of word 1-across. We could also use a predicate $letter(W_{ix}, j)$ that returns the j^{th} letter of word W_{ix}.
- **Domains**: Dictionary of words $\{w_1, w_2, w_3, \ldots, w_{15}\}$ in the order above (e.g. $w_1 = ant, w_2 = big, \ldots$). Let $|w_j|$ be the length of the word w_j.
- **Constraints**:
  - **domain:** $W_{ix} \neq w_j \quad \forall\ j \quad \text{s.t.} \quad |w_j| \neq |W_{ix}|$ (eliminate all words that are not the correct length). For the example above, the domain of W_{1a} is therefore only the 3-letter words $\{ant, big, bus, car, has\}$
  - **binary:** $W_{ia_j} = W_{kd_l} \quad \forall\ i, k \quad \text{that intersect at} \quad j, l$. For the example above, $W_{1a_1} = W_{1d_1}$ and $W_{1a_3} = W_{2d_1}$. Using the predicate this would be $letter(W_{ix}, j) = letter(W_{kd}, l)$.

This “tabular” format shows the solution. A ✓ means the arc is *consistent* (i.e. *not* on the TDA). Domains are only shown when changes are made. The arcs are chosen from left-to-right: always pick the leftmost inconsistent (no checkmark) arc in the table to make consistent next (this is a convention only). The final row shows the final state of AC-3, meaning we do not know if the problem has a solution yet. One domain needs to be split in two, and the two problems solved recursively to show the two possible solutions.

W_{1a}	W_{1d}	W_{2d}	W_{3a}	W_{4a}	$\langle W_{2d}, W_{2d}^3 = W_{3a}^3 \rangle$	$\langle W_{3a}, W_{3a}^3 = W_{2d}^3 \rangle$	$\langle W_{1a}, W_{1a}^3 = W_{2d}^1 \rangle$	$\langle W_{2d}, W_{2d}^1 = W_{1a}^3 \rangle$	$\langle W_{1a}, W_{1a}^1 = W_{1d}^1 \rangle$	$\langle W_{1d}, W_{1d}^1 = W_{1a}^1 \rangle$	$\langle W_{3a}, W_{3a}^1 = W_{1d}^3 \rangle$	$\langle W_{1d}, W_{1d}^3 = W_{3a}^1 \rangle$	$\langle W_{2d}, W_{2d}^5 = W_{4a}^1 \rangle$	$\langle W_{4a}, W_{4a}^1 = W_{2d}^5 \rangle$
ant, big, bus, car, has	book, buys, hold, lane, year	ginger, search, syntax, symbol	book, buys, hold, lane, year	ant, big, bus, car, has										
		ginger, search, syntax			✓									
			year, lane		✓	✓								
bus, has, big					✓	✓	✓							
					✓	✓	✓	✓						
					✓	✓	✓	✓	✓					
	book, buys, hold				✓	✓	✓	✓	✓	✓				
					✓	✓	✓	✓	✓	✓	✓			
	buys, hold				✓	✓	✓	✓		✓	✓	✓		
					✓	✓	✓	✓	✓	✓	✓	✓		
		search, syntax			✓			✓	✓	✓	✓	✓	✓	
					✓	✓		✓	✓	✓	✓	✓	✓	
bus, has					✓	✓	✓	✓	✓		✓	✓	✓	
					✓	✓	✓	✓	✓	✓	✓	✓	✓	
bus, has	buys, hold	search, syntax	year, lane	ant, car	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

The following is a simple example to show how AC-3 can terminate in the "third" condition (where some domain has more than one value) but there is still no solution. AC-3 only enforces local constraints, but there still may be no global solution



I abbreviate $\langle A, A = B\rangle$ in the following as AB

A	B	C	AB	BA	AC	CA	BC	CB
1,2	1,2	1,2						
1,2	1,2	1,2	$\checkmark$					
1,2	1,2	1,2	$\checkmark$	$\checkmark$				
1,2	1,2	1,2	$\checkmark$	$\checkmark$	$\checkmark$			
1,2	1,2	1,2	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$		
1,2	1,2	1,2	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	
1,2	1,2	1,2	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

AC-3 completes without any domain changes, and so we must split a domain to continue. Any split will yield the "first" termination (where all domains are empty). Splitting A by removing the value 2:

A	B	C	AB	BA	AC	CA	BC	CB
1	1,2	1,2	$\checkmark$		$\checkmark$		$\checkmark$	$\checkmark$
1	1	1,2	$\checkmark$	$\checkmark$	$\checkmark$		$\checkmark$	$\checkmark$
1	1	2	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$		$\checkmark$
1		2		$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
		2	$\checkmark$	$\checkmark$	$\checkmark$		$\checkmark$	$\checkmark$
			$\checkmark$	$\checkmark$		$\checkmark$		$\checkmark$
			$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$		$\checkmark$
			$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$