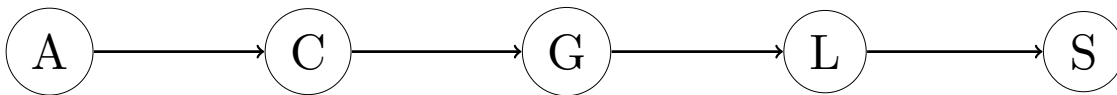# Variable Elimination Examples

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*If Jesse's alarm doesn't go off (A), Jesse probably won't get coffee (C); if Jesse doesn't get coffee, he's likely grumpy (G). If he is grumpy then it's possible that the lecture won't go smoothly L. If the lecture does not go smoothly then the students will likely be sad S.*

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$\mathbf{P(A = true)} = 0.3$

| $\mathbf{P(C = true|A)} =$ | $A$ | $P(C = true|A)$ |
|---|---|---|
| | $t$ | 0.8 |
| | $f$ | 0.15 |

| $\mathbf{P(G = true|C)} =$ | $C$ | $P(G = true|C)$ |
|---|---|---|
| | $t$ | 1.0 |
| | $f$ | 0.2 |

| $\mathbf{P(L = true|G)} =$ | $G$ | $P(L = true|G)$ |
|---|---|---|
| | $t$ | 0.7 |
| | $f$ | 0.2 |

| $\mathbf{P(S = true|L)} =$ | $L$ | $P(S = true|L)$ |
|---|---|---|
| | $t$ | 0.9 |
| | $f$ | 0.3 |

A → C → G → L → S

A=Jesse's alarm doesn't go off
C=Jesse doesn't get coffee
G=Jesse is grumpy
L=lecture doesn't go smoothly
S=students are sad

**Query 1**: $P(S = true)$?
Using simple polytree forward chaining:

$$P(S = true) = \sum_{A,C,G,L} P(S = true, A, C, G, L)$$

$$= \sum_{A,C,G,L} P(A)P(C|A)P(G|C)P(L|G)P(S = true|L)$$

$$= \sum_A P(A) \sum_C P(C|A) \sum_G P(G|C) \sum_L P(L|G)P(S = true|L)$$

as factors

$$f_0(A) = P(A) = \quad
\begin{array}{c|c}
A & f_0(A) \\
\hline
t & 0.3 \\
f & 0.7 \\
\end{array}$$

$$f_1(C, A) = P(C|A) = \quad
\begin{array}{cc|c}
A & C & f_1(C, A) \\
\hline
t & t & 0.8 \\
t & f & 0.2 \\
f & t & 0.15 \\
f & f & 0.85 \\
\end{array}$$

$$f_2(G, C) = P(G|C) = \quad
\begin{array}{cc|c}
C & G & f_2(G, C) \\
\hline
t & t & 1.0 \\
t & f & 0.0 \\
f & t & 0.2 \\
f & f & 0.8 \\
\end{array}$$

$$f_3(L, G) = P(L|G) = \quad
\begin{array}{cc|c}
G & L & f_3(L, G) \\
\hline
t & t & 0.7 \\
t & f & 0.3 \\
f & t & 0.2 \\
f & f & 0.8 \\
\end{array}$$

$$f_4(L) = P(S = true|L) = \quad
\begin{array}{c|c}
L & f_4(L) \\
\hline
t & 0.9 \\
f & 0.3 \\
\end{array}$$

choose ordering $L, G, C, A$:

$$P(S = true) = \sum_{A,C,G,L} f_0(A) f_1(C, A) f_2(G, C) f_3(L, G) f_4(L)$$

$$= \sum_A f_0(A) \sum_C f_1(C, A) \sum_G f_2(G, C) \sum_L f_3(L, G) f_4(L)$$

$$= \sum_A f_0(A) \sum_C f_1(C, A) \sum_G f_2(G, C) f_5(G)$$

where $\quad f_5(G) = \sum_L f_3(L, G) f_4(L) =$

| $G$ | $f_5(G)$ |
|-----|----------|
| $t$ | $0.7 * 0.9 + 0.3 * 0.3 = 0.72$ |
| $f$ | $0.9 * 0.2 + 0.3 * 0.8 = 0.42$ |

$$= \sum_A f_0(A) \sum_C f_1(C, A) f_6(C)$$

where $\quad f_6(C) = \sum_G f_2(G, C) f_5(G) =$

| $C$ | $f_6(C)$ |
|-----|----------|
| $t$ | $1.0 * 0.72 + 0 * 0.42 = 0.72$ |
| $f$ | $0.2 * 0.72 + 0.8 * 0.42 = 0.48$ |

$$= \sum_A f_0(A) f_7(A)$$

where $\quad f_7(A) = \sum_C f_1(C, A) f_6(C) =$

| $A$ | $f_7(A)$ |
|-----|----------|
| $t$ | $0.8 * 0.72 + 0.2 * 0.48 = 0.672$ |
| $f$ | $0.15 * 0.72 + 0.85 * 0.48 = 0.516$ |

$$= f_8()$$

where $\quad f_8() = \sum_A f_0(A) f_7(A) = 0.3 * 0.672 + 0.7 * 0.516 = 0.563$

Do the same for $P(S = false)$ and find 0.437, so its already normalized since Bayes' rule was not used.

In variable elimination, we normally would compute both $P(S = true)$ and $P(S = false)$ at the same time as $P(S)$. In this case, $f_0 \ldots f_3$ are the same, but $f_4$ now includes $S$:

$$f_4(S, L) = P(S|L) =$$

| $S$ | $L$ | $f_4(S, L)$ |
|-----|-----|-------------|
| $t$ | $t$ | $0.9$ |
| $t$ | $f$ | $0.3$ |
| $f$ | $t$ | $0.1$ |
| $f$ | $f$ | $0.7$ |

The calculation is similar:

$$P(S) = \sum_{A,C,G,L} f_0(A)f_1(C,A)f_2(G,C)f_3(L,G)f_4(S,L)$$

$$= \sum_A f_0(A) \sum_C f_1(C,A) \sum_G f_2(G,C) \sum_L f_3(L,G)f_4(S,L)$$

$$= \sum_A f_0(A) \sum_C f_1(C,A) \sum_G f_2(G,C)f_5(G,S)$$

where $\quad f_5(G,S) = \sum_L f_3(L,G)f_4(L,S) =$

| S | G | $f_5(G,S)$ |
|---|---|---|
| t | t | $0.7*0.9 + 0.3*0.3 = 0.72$ |
| t | f | $0.9*0.2 + 0.3*0.8 = 0.42$ |
| f | t | $0.7*0.1 + 0.3*0.7 = 0.28$ |
| t | f | $0.1*0.2 + 0.7*0.8 = 0.58$ |

$$= \sum_A f_0(A) \sum_C f_1(C,A)f_6(C,S)$$

where $\quad f_6(C,S) = \sum_G f_2(G,C)f_5(G,S) =$

| S | C | $f_6(C,S)$ |
|---|---|---|
| t | t | $1.0*0.72 + 0*0.42 = 0.72$ |
| t | f | $0.2*0.72 + 0.8*0.42 = 0.48$ |
| f | t | $1.0*0.28 + 0*0.58 = 0.28$ |
| f | f | $0.2*0.28 + 0.8*0.58 = 0.52$ |

$$= \sum_A f_0(A)f_7(A,S)$$

where $\quad f_7(A,S) = \sum_C f_1(C,A)f_6(C,S) =$

| S | A | $f_7(A,S)$ |
|---|---|---|
| t | t | $0.8*0.72 + 0.2*0.48 = 0.672$ |
| t | f | $0.15*0.72 + 0.85*0.48 = 0.516$ |
| f | t | $0.8*0.28 + 0.2*0.52 = 0.328$ |
| t | f | $0.15*0.28 + 0.85*0.52 = 0.484$ |

$$= f_8(S)$$

where $\quad f_8(S) = \sum_A f_0(A)f_7(A,S) =$

| S | $f_8(S)$ |
|---|---|
| t | $0.3*0.672 + 0.7*0.516 = 0.563$ |
| f | $0.3*0.328 + 0.7*0.484 = 0.437$ |

which we can see is normalized, and so is the answer $P(S)$.

**Query 2**: With evidence, e.g. $P(S = true | A = true)$

$$f_0(C) = P(C | A = true) =$$

| $C$ | $f_0(C)$ |
|---|---|
| $t$ | 0.8 |
| $f$ | 0.2 |

$$f_1(G, C) = P(G | C) =$$

| $C$ | $G$ | $f_1(G, C)$ |
|---|---|---|
| $t$ | $t$ | 1.0 |
| $t$ | $f$ | 0.0 |
| $f$ | $t$ | 0.2 |
| $f$ | $f$ | 0.8 |

$$f_2(L, G) = P(L | G) =$$

| $G$ | $L$ | $f_2(L, G)$ |
|---|---|---|
| $t$ | $t$ | 0.7 |
| $t$ | $f$ | 0.3 |
| $f$ | $t$ | 0.2 |
| $f$ | $f$ | 0.8 |

$$f_3(L) = P(S = true | L) =$$

| $L$ | $f_3(L)$ |
|---|---|
| $t$ | 0.9 |
| $f$ | 0.3 |

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so that

$$P(S = true | A = true) \propto \sum_{C,G,L} f_0(C) f_1(G,C) f_2(L,G) f_3(L)$$

$$= \sum_C f_0(C) \sum_G f_1(G,C) \sum_L f_2(L,G) f_3(L)$$

$$= \sum_C f_0(C) \sum_G f_1(G,C) f_4(G)$$

where $\quad f_4(G) = \sum_L f_2(L,G) f_3(L) =$

| $G$ | $f_4(G)$ |
|---|---|
| $t$ | $0.7 * 0.9 + 0.3 * 0.3 = 0.72$ |
| $f$ | $0.2 * 0.9 + 0.8 * 0.3 = 0.42$ |

$$= \sum_C f_0(C) f_5(C)$$

where $\quad f_5(C) = \sum_G f_1(G,C) f_4(G) =$

| $C$ | $f_5(C)$ |
|---|---|
| $t$ | $1.0 * 0.72 + 0 * 0.42 = 0.72$ |
| $f$ | $0.2 * 0.72 + 0.8 * 0.42 = 0.48$ |

where $\quad f_7() = \sum_C f_0(C) f_5(C) = 0.8 * 0.72 + 0.2 * 0.48 = 0.672$

Do the same for $P(S = false)$ and find 0.328. Again, Bayes' rule is not used, so normalization is not necessary, but this will not be the case in general.

Similarly to Query 1, we could have asked $P(S|A = true)$ and done both at once.

**Query 3**: Finally, let's try $P(C = true|S = true)$

$$f_0(A) = P(A) = \begin{array}{c|c} A & f_0(A) \\ \hline t & 0.3 \\ f & 0.7 \end{array}$$

$$f_1(A) = P(C = true|A) = \begin{array}{c|c} A & f_1(A) \\ \hline t & 0.8 \\ f & 0.15 \end{array}$$

$$f_2(G) = P(G|C = true) = \begin{array}{c|c} G & f_2(G) \\ \hline t & 1.0 \\ f & 0.0 \end{array}$$

$$f_3(L, G) = P(L|G) = \begin{array}{cc|c} G & L & f_3(L, G) \\ \hline t & t & 0.7 \\ t & f & 0.3 \\ f & t & 0.2 \\ f & f & 0.8 \end{array}$$

$$f_4(L) = P(S = true|L) = \begin{array}{c|c} L & f_4(L) \\ \hline t & 0.9 \\ f & 0.3 \end{array}$$

so that

$$P(C = true|S = true) \propto \sum_{L,G,A} f_0(A)f_1(A)f_2(G)f_3(L, G)f_4(L)$$

$$= \sum_A f_0(A)f_1(A) \sum_G f_2(G) \sum_L f_3(L, G)f_4(L)$$

$$= \sum_A f_0(A)f_1(A) \sum_G f_2(G)f_5(G)$$

where
$$f_5(G) = \sum_L f_3(L, G)f_4(L) = \begin{array}{c|c} G & f_5(G) \\ \hline t & 0.7 * 0.9 + 0.3 * 0.3 = 0.72 \\ f & 0.9 * 0.2 + 0.3 * 0.8 = 0.42 \end{array}$$

$$= f_7()f_6()$$

where
$$f_6() = \sum_G f_2(G)f_5(G) = 1.0 * 0.72 + 0.0 * 0.42 = 0.72$$

and
$$f_7() = \sum_A f_0(A)f_1(A) = 0.3 * 0.8 + 0.7 * 0.15 = 0.345$$

$$= f_8()$$

where
$$f_8() = 0.345 * 0.72 = 0.2484$$

now we compute in the same way $P(C = false | S = true)$ (only $f_1$ and $f_2$ change here)

$$f_1(A) = P(C = false|A) = \begin{array}{c|c} C & f_1(A) \\ \hline t & 0.2 \\ f & 0.85 \end{array}$$

$$f_2(G) = P(G|C = false) = \begin{array}{c|c} G & f_2(G) \\ \hline t & 0.2 \\ f & 0.8 \end{array}$$

so that

$$P(C = false|S = true) \propto \sum_{L,G,A} f_0(A) f_1(A) f_2(G) f_3(L,G) f_4(L)$$

$$= \sum_A f_0(A) f_1(A) \sum_G f_2(G) \sum_L f_3(L,G) f_4(L)$$

$$= \sum_A f_0(A) f_1(A) \sum_G f_2(G) f_5(G)$$

$$\text{where} \quad f_5(G) = \sum_L f_3(L,G) f_4(L) = \begin{array}{c|c} G & f_5(G) \\ \hline t & 0.7 * 0.9 + 0.3 * 0.3 = 0.72 \\ f & 0.9 * 0.2 + 0.3 * 0.8 = 0.42 \end{array}$$

$$= f_7() f_6()$$

$$\text{where} \quad f_6() = \sum_G f_2(G) f_5(G) = 0.2 * 0.72 + 0.8 * 0.42 = 0.48$$

$$\text{and} \quad f_7() = \sum_A f_0(A) f_1(A) = 0.3 * 0.2 + 0.7 * 0.85 = 0.655$$

$$= f_8()$$

$$\text{where} \quad f_8() = 0.655 * 0.48 = 0.3144$$

So

$$P(C = true|S = true) = \frac{P(C = true, S = true)}{P(C = true, S = true) + P(C = false, S = true)} = \frac{0.2484}{0.2484 + 0.3144} = 0.44$$

Note that the normalization factor here is $P(S = true) = P(C = true, S = true) + P(C = false, S = true) = 0.2484 + 0.3144 = 0.563$ which is exactly what we calculated in Query 1.

*COVID is a disease which occurs with probability 0.32 in a certain population (e.g. older adults). A Bayes' net is used to diagnose the presence of covid (C) in such patients by using two binary tests, test A (A) and test B (B) which report the presence or absence of covid. Test A is a simple test the doctor can perform directly on the patient and see the results immediately. Test A has a true positive rate of 0.8 and a false positive rate of 0.15. That is, when covid is present, test A detects it 80% of the time, and when covid is not present, test A reports it 15% of the time. Test B, on the other hand, has greater precision (true positive rate 0.78 and false positive rate 0.044), but requires a complicated machine, which malfunctions (M) with probability 0.08. When the machine malfunctions, test B's true positive rate drops to 0.61 and its false positive rate rises to 0.52. Further, test B's results are not directly available to the doctor. Instead, they are read by a technician who writes them down as a report (R) in a logbook, which is then passed to a data entry person who enters the result in a database (D). The doctor reads the result from the database. The technician and the data entry person sometimes make mistakes, however. The technician's true and false positive rates are 0.98 and 0.01, respectively, while the data entry person's rates are 0.96 and 0.001 (for true and false positive rates, respectively).*
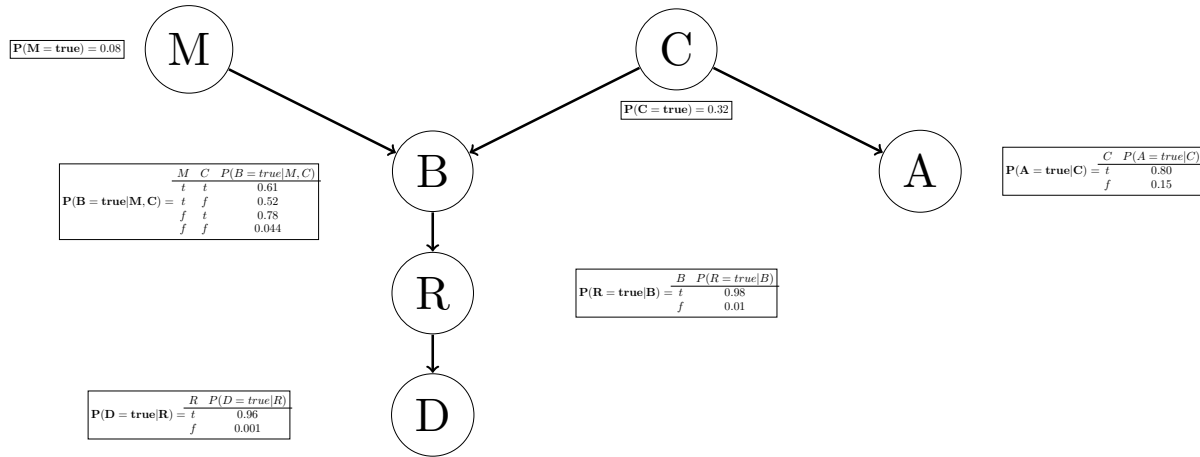


M=Machine malfunctions
C=Patient has covid
A=Test A reads positive
B=Test B reads positive
R=Report is positive
D=Database entry is positive

**Query 1**: $P(C = true)$?
There are no relevant variables, since there is no evidence and $C$ has no parents. Thus,

$$f_0(C) = P(C) = \begin{array}{c|c} C & f_0(C) \\ \hline t & 0.32 \\ f & 0.68 \end{array}$$

so that

$$P(C = true) = f_0(C)$$

**Query 2**: $P(C|D = true)$?
Only $A$ is irrelevant

$$f_0(C) = P(C) = \begin{array}{c|c} C & f_0(C) \\ \hline t & 0.32 \\ f & 0.68 \end{array}$$

$$f_1(B, M, C) = P(B|M, C) = \begin{array}{ccc|c} M & C & B & f_1(B, M, C) \\ \hline t & t & t & 0.61 \\ t & t & f & 0.39 \\ t & f & t & 0.52 \\ t & f & f & 0.48 \\ f & t & t & 0.78 \\ f & t & f & 0.22 \\ f & f & t & 0.044 \\ f & f & f & 0.956 \end{array}$$

$$f_2(R, B) = P(R|B) = \begin{array}{cc|c} R & B & f_2(R, B) \\ \hline t & t & 0.98 \\ t & f & 0.01 \\ f & t & 0.02 \\ f & f & 0.99 \end{array}$$

$$f_3(R) = P(D = true|R) = \begin{array}{c|c} R & f_3(R) \\ \hline t & 0.96 \\ f & 0.001 \end{array}$$

$$f_4(M) = P(M) = \begin{array}{c|c} M & P(M) \\ \hline t & 0.08 \\ f & 0.92 \end{array}$$

so that

$$P(C|D = true) \propto \sum_{M,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)$$

$$= f_0(C)\sum_M f_4(M)\sum_B f_1(B,M,C)\sum_R f_2(R,B)f_3(R)$$

$$= f_0(C)\sum_M f_4(M)\sum_B f_1(B,M,C)f_5(B)$$

where $\quad f_5(B) = \sum_R f_2(R,B)f_3(R) =$

| $B$ | $f_5(B)$ |
|---|---|
| $t$ | $0.98 * 0.96 + 0.02 * 0.001 = 0.941$ |
| $f$ | $0.01 * 0.96 + 0.99 * 0.001 = 0.011$ |

$$= f_0(C)\sum_M f_4(M)f_6(M,C)$$

where $\quad f_6(M,C) = \sum_B f_1(B,M,C)f_5(B) =$

| $M$ | $C$ | $f_6(M,C)$ |
|---|---|---|
| $t$ | $t$ | $0.61 * 0.941 + .39 * 0.011 = .578$ |
| $t$ | $f$ | $0.52 * 0.941 + 0.48 * 0.011 = 0.495$ |
| $f$ | $t$ | $0.78 * 0.941 + 0.22 * 0.011 = 0.736$ |
| $f$ | $f$ | $0.044 * 0.941 + 0.956 * 0.011 = 0.052$ |

$$= f_0(C)f_7(C)$$

where $\quad f_7(C) = \sum_M f_4(M)f_6(M,C) =$

| $C$ | $f_7(C)$ |
|---|---|
| $t$ | $0.08 * 0.578 + 0.92 * 0.736 = 0.723$ |
| $f$ | $0.08 * 0.495 + 0.92 * 0.052 = 0.087$ |

$$= f_8(C)$$

where $\quad f_8(C) = f_0(C)f_7(C) =$

| $C$ | $f_8(C)$ |
|---|---|
| $t$ | $0.32 * 0.723 = 0.231$ |
| $f$ | $0.68 * 0.087 = 0.059$ |

and so,

$$P(C|D = true) =$$

| $C$ | $P(C|D = true)$ |
|---|---|
| $t$ | $0.231/(0.231 + 0.059) = 0.797$ |
| $f$ | $0.059/(0.231 + 0.059) = 0.203$ |

**Query 3**: $P(C|D = true, A = true)$?

No irrelevant variables, and only one additional factor is added, which we will call $f_9(C)$ for clarity ($f_0 \dots f_4$ are the same as in Query 2).

$$f_9(C) = P(A = true|C) =$$

| $C$ | $f_9(C)$ |
|---|---|
| $t$ | $0.80$ |
| $f$ | $0.15$ |

so that

$$P(C|D = true, A = true) \propto \sum_{M,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)f_9(C)$$

$$= f_9(C)f_0(C)\sum_M f_4(M)\sum_B f_1(B,M,C)\sum_R f_2(R,B)f_3(R)$$

$$= \ldots \text{same calculation as Query 3 up to } f_8(C)$$

$$= f_9(C)f_8(C)$$

where $f_8(C) =$

| $C$ | $f_8(C)$ |
|---|---|
| $t$ | 0.231 |
| $f$ | 0.059 |

$$= f_{10}(C)$$

where $f_{10}(C) = f_9(C)f_8(C) =$

| $C$ | $f_{10}(C)$ |
|---|---|
| $t$ | $0.8 * 0.231 = 0.185$ |
| $f$ | $0.15 * 0.059 = 0.0089$ |

and so,

$$P(C|D = true, A = true) =$$

| $C$ | $P(C|D = true, A = true)$ |
|---|---|
| $t$ | $0.185/(0.185 + 0.0089) = 0.954$ |
| $f$ | $0.0089/(0.185 + 0.0089) = 0.046$ |

**Query 4**: $P(C|D = true, A = false)$?
Same as previous calculation up to the final step, where $f_9(C)$ is replaced with $f_{11}(C)$:

$$f_{11}(C) = P(A = false|C) =$$

| $C$ | $f_{11}(C)$ |
|---|---|
| $t$ | 0.2 |
| $f$ | 0.85 |

so that

$$P(C|D = true, A = false) \propto \sum_{M,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)f_{11}(C)$$

$$= f_{11}(C)f_0(C)\sum_M f_4(M)\sum_B f_1(B,M,C)\sum_R f_2(R,B)f_3(R)$$

$$= \ldots \text{same calculation as Query 3 up to} f_8(C)$$

$$= f_{12}(C)$$

where $f_{12}(C) = f_{11}(C)f_8(C) =$

| $C$ | $f_{11}(C)$ |
|---|---|
| $t$ | $0.2 * 0.231 = 0.0462$ |
| $f$ | $0.85 * 0.059 = 0.0502$ |

and so,

$$P(C|D = true, A = false) = \begin{array}{c|c} C & P(C|D = true, A = false) \\ \hline t & 0.0462/(0.0462 + 0.0502) = 0.48 \\ f & 0.0502/(0.0462 + 0.0502) = 0.52 \end{array}$$

**Query 5**: $P(M|D = true, A = false)$?
Everything is relevant, and we will use the same factors as for Query 4, except for the sums are different now, as we sum over $C$ rather than over $M$:

$$P(M|D = true, A = false) \propto \sum_{C,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)f_{11}(C)$$

$$= f_4(M) \sum_C f_{11}(C)f_0(C) \sum_B f_1(B,M,C) \sum_R f_2(R,B)f_3(R)$$

$$= \ldots \text{same calculation as Query 2 up to} f_6(C)$$

$$= f_4(M) \sum_C f_{11}(C)f_0(C)f_6(M,C)$$

$$\text{where} \quad f_6(M,C) = \begin{array}{c c|c} M & C & f_6(M,C) \\ \hline t & t & 0.578 \\ t & f & 0.495 \\ f & t & 0.736 \\ f & f & 0.052 \end{array}$$

$$= f_4(M)f_{13}(M)$$

$$\text{where} \quad f_{13}(M) = \sum_C f_{11}(C)f_0(C)f_6(M,C) = \begin{array}{c|c} C & f_{13}(C) \\ \hline t & 0.2*0.32*0.578 + 0.85*0.68*0.495 = 0.323 \\ f & 0.2*0.32*0.736 + 0.85*0.68*0.052 = 0.077 \end{array}$$

$$= f_{14}(M)$$

$$\text{where} \quad f_{14}(M) = f_4(M)f_{13}(M) = \begin{array}{c|c} M & f_{14}(M) \\ \hline t & 0.08*0.323 = 0.0258 \\ f & 0.92*0.077 = 0.0708 \end{array}$$
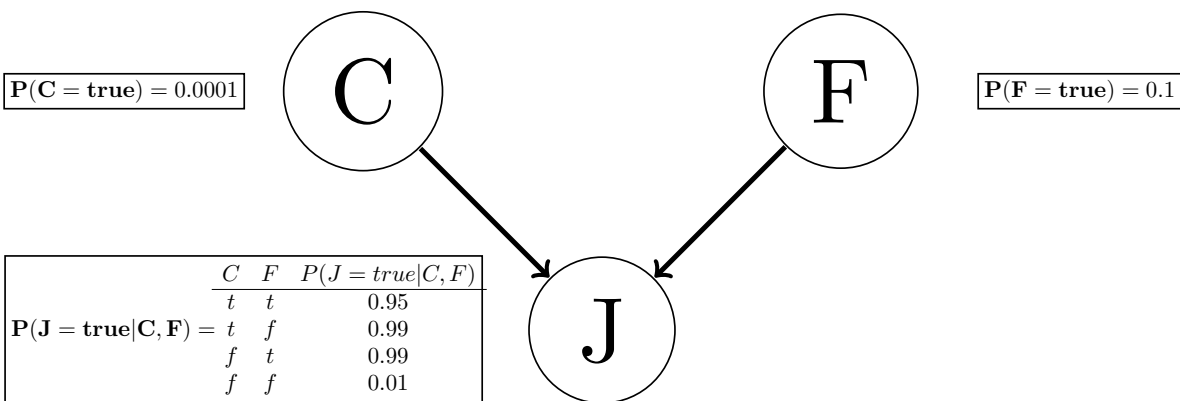
and so,

$$P(M|D = true, A = false) = \begin{array}{c|c} M & f_{14}(M) \\ \hline t & 0.0258/(0.0708 + 0.0258) = 0.27 \\ f & 0.0708/(0.0708 + 0.0258) = 0.73 \end{array}$$

*The final example is based on the XKCD comic shown above taken from* `http://imgs.xkcd.com/comics/bridge.png`. *All the guy's friends jump off a bridge (J), and it might be either because they are all crazy (C) or because the bridge is on fire (F). Let's say the probability that all his friends are simultaneously crazy is* 0.0001 *(this is product of N probabilities of any individual friend being crazy independently), and the probability the bridge is on fire is* 0.1. *All friends will jump if the bridge is on fire with probability* 0.99 *if they are not crazy and probability* 0.95 *if they are crazy. If the bridge is not on fire, then all friends jump with probability* 0.99 *if they are crazy and* 0.01 *if they are not[a].*

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[a]Note we are not considering the case that some fraction of friends are crazy (as in the comic "many of whom are levelheaded"...or that some fraction jump regardless of whether they are crazy or not. You could, however, work out how many friends you'd need to see jump off before your belief that the bridge is on fire becomes greater than 0.5, given that you know how likely each friend is to be crazy, and how likely each friend is to jump off given their state of craziness and whether the bridge is on fire, and that all friends state of craziness and their decisions to jump are independent.

$P(C = true) = 0.0001$

$P(F = true) = 0.1$

| $P(J = true|C, F) =$ | $C$ | $F$ | $P(J = true|C, F)$ |
|---|---|---|---|
| | $t$ | $t$ | 0.95 |
| | $t$ | $f$ | 0.99 |
| | $f$ | $t$ | 0.99 |
| | $f$ | $f$ | 0.01 |

C=All friends are crazy
F=Bridge is on fire
J=All friends jump off the bridge

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**Query 1**: $P(F|J = true)$? How likely is it that the bridge is on fire given that all my friends just jumped off the bridge?

All variables are relevant since $J$ in evidence means $C$ is relevant

Using the simple polytree algorithm:

$$P(F|J = true) \propto P(F, J = true)$$

$$= \sum_C P(J = true, C, F)$$

$$= \sum_C P(J = true|C, F)P(C|F)P(F)$$

$$= \sum_C P(J = true|C, F)P(C)P(F) \quad \text{using independence of C on F}$$

$$= P(F) \sum_C P(J = true|C, F)P(C)$$

which, for F=true is:

$$= 0.1 * [0.95 * 0.0001 + 0.99 * 0.9999] = 0.099$$

and, for F=false is:

$$= 0.9 * [0.99 * 0.0001 + 0.01 * 0.9999] = 0.0091$$

so that, after normalization, $P(F = true|J = true) = \frac{0.099}{0.099+0.0091} = 0.916$

Using the variable elimination algorithm

$$f_0(C) = P(C) = \begin{array}{c|c} C & f_0(C) \\ \hline t & 0.0001 \\ f & 0.9999 \end{array}$$

$$f_1(F) = P(F) = \begin{array}{c|c} F & f_0(F) \\ \hline t & 0.1 \\ f & 0.9 \end{array}$$

$$f_2(C, F) = P(J = true|C, F) = \begin{array}{cc|c} C & F & f_2(C, F) \\ \hline t & t & 0.95 \\ t & f & 0.99 \\ f & t & 0.99 \\ f & f & 0.01 \end{array}$$

$$P(F|J = true) \propto f_1(F) \sum_C f_0(C) f_2(C, F)$$

$$= f_1(F) f_3(F)$$

where  $f_3(F) = \sum_C f_0(C) f_2(C, F) =$

| $F$ | $f_3(F)$ |
|---|---|
| $t$ | $0.0001 * 0.95 + 0.9999 * 0.99 = 0.989996$ |
| $f$ | $0.0001 * 0.99 + 0.9999 * 0.01 = 0.010098$ |

$$= f_4(F)$$

where  $f_4(F) = f_1(F) f_3(F) =$

| $F$ | $f_4(F)$ |
|---|---|
| $t$ | $0.989996 * 0.1 = 0.0989996$ |
| $f$ | $0.0101 * 0.9 = 0.0091$ |

So that

| $F$ | $P(F|J = true)$ |
|---|---|
| $P(F|J = true) =$ $t$ | $0.0989996/(0.0989996 + 0.0091) = 0.916$ |
| $f$ | $0.0091/(0.0989996 + 0.0091) = 0.0894$ |

So, again, we find that the probability the bridge is on fire is about 0.92 ... JUMP!!![1]



(from `kwc.org`)

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[1]Disclaimer: do not follow this advice directly. If you are in this situation and decide to jump off the bridge, I take no responsibility for your actions due to my inferences based on *my beliefs*. You should re-do the queries after first replacing the probabilities in the Bayesian network with *your own beliefs* about your friends being crazy, bridges being on fire, and the relative effects of these two things on people jumping from bridges.