First, we need to prove that $Fib(n) = \dfrac{\phi^n - \psi^n}{\sqrt{5}}$.
Base case:

$$
\begin{align}
Fib(0) &= \frac{\phi^0 - \psi^0}{\sqrt{5}} \tag{1}\\
&= \frac{0}{\sqrt{5}} \tag{2}\\
&= 0 \tag{3}
\end{align}
$$

$$
\begin{align}
Fib(1) &= \frac{\phi^1 - \psi^1}{\sqrt{5}} \tag{4}\\
&= \frac{\frac{1}{2}\left(1+\sqrt{5}-1+\sqrt{5}\right)}{\sqrt{5}} \tag{5}\\
&= 1 \tag{6}
\end{align}
$$

Assuming $n = k$ such that $Fib(0), Fib(1), ..., Fib(k) = \dfrac{\phi^k - \psi^k}{\sqrt{5}}$ true when $k \geq 1$, we want to show that $n = k+1$ such that $Fib(0), Fib(1), ..., Fib(k+1) = \dfrac{\phi^{k+1} - \psi^{k+1}}{\sqrt{5}}$ is also true when $k \geq 1$.

$$
\begin{align}
Fib(k+1) &= \frac{\phi^{k+1} - \psi^{k+1}}{\sqrt{5}} \tag{7}\\
Fib(k+1) &= Fib(k) + Fib(k-1) \tag{8}\\
&= \frac{\phi^k - \psi^k + \phi^{k-1} - \psi^{k-1}}{\sqrt{5}} \tag{9}\\
&= \frac{\phi^{k-1}(\phi+1) - \psi^{k-1}(\psi+1)}{\sqrt{5}} \tag{10}\\
&= \frac{\phi^{k-1}(\frac{1+\sqrt{5}+2}{2}) - \psi^{k-1}(\frac{1-\sqrt{5}+2}{2})}{\sqrt{5}} \tag{11}\\
&= \frac{\phi^{k-1}(\frac{6+2\sqrt{5}}{4}) - \psi^{k-1}(\frac{6-2\sqrt{5}}{4})}{\sqrt{5}} \tag{12}\\
&= \frac{\phi^{k-1}(\phi^2) - \psi^{k-1}(\psi^2)}{\sqrt{5}} \tag{13}\\
&= \frac{\phi^{k+1} - \psi^{k+1}}{\sqrt{5}} \tag{14}
\end{align}
$$

So the statement is proven true.
Finally, we are going to prove that $Fib(n)$ is the closest integer to $\dfrac{\phi^n}{\sqrt{5}}$.

Notice that since $|\psi| < 1$ and $\sqrt{5} > 2$, one has $\left|\frac{\psi^n}{\sqrt{5}}\right| < \frac{1}{2}$.
Thus the integer closest to $Fib(n) + \frac{\psi^n}{\sqrt{5}} = \frac{\phi^n}{\sqrt{5}}$ is $Fib(n)$.