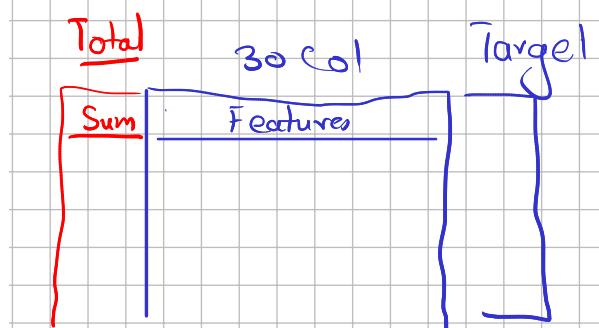
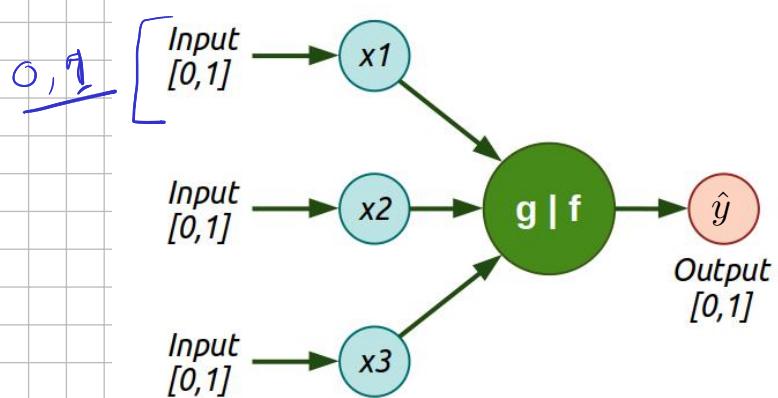


MP Neurons



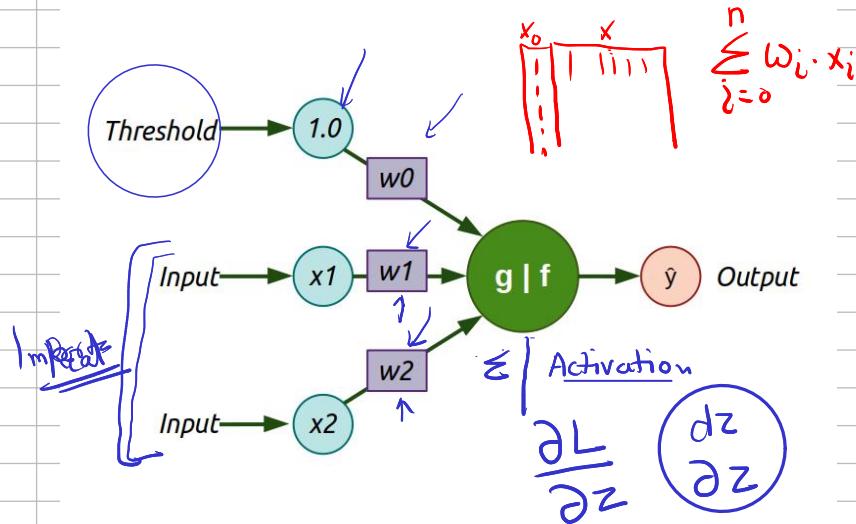
$$z = g(x) = \sum_{i=1}^n (x_i)$$

$$\hat{y} = a = f(z)$$

$$\hat{y} = a = 1 \text{ if } z \geq 0 \text{ else } 0$$

Normalizing Rain - 0-20mm
 Rent Cost Ground - 500 → 3000
 Home work -

Perceptions



Updates

$$w_0 = w_0 - \frac{1}{m} (\alpha \circ \partial w_0)$$

Learning Rate $[0.1 \rightarrow 0.001]$

$\alpha \leftarrow \# \text{ of Rows}$

$$w_1 = w_1 - \frac{1}{m} (\alpha \circ \partial w_1)$$

$$w_2 = w_2 - \frac{1}{m} (\alpha \circ \partial w_2)$$

Forward propagation

$$z = g(x) = w_0 + \sum_{i=1}^n (w_i * x_i)$$

$$\hat{y} = a = f(z)$$

$$\hat{y} = a = 1 \text{ if } z \geq 0 \text{ else } 0$$

Back propagation

$$\partial z = a - y$$

$$\partial w_0 = \partial z$$

$$\partial w_1 = x_1 \circ \partial z$$

$$\partial w_2 = x_2 \circ \partial z$$

$$z = w_0 + \sum x_i \cdot w_i$$

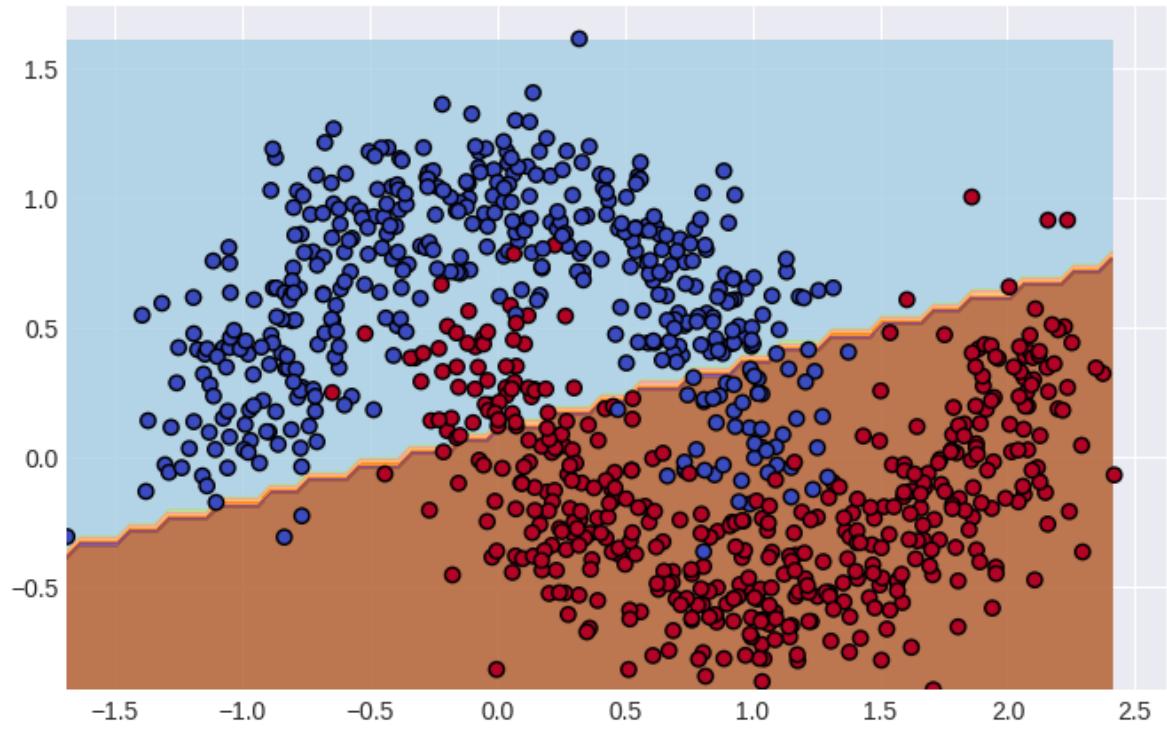
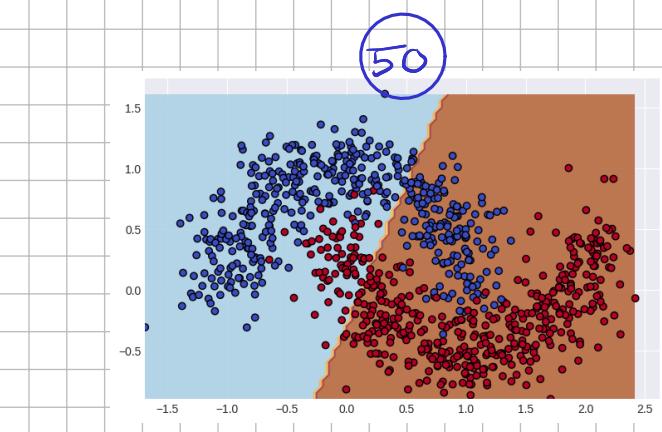
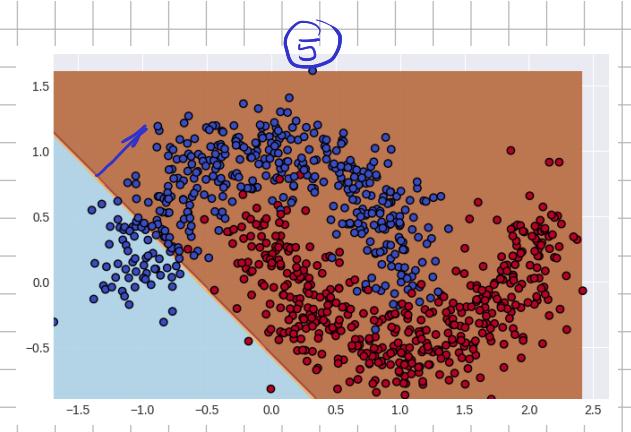
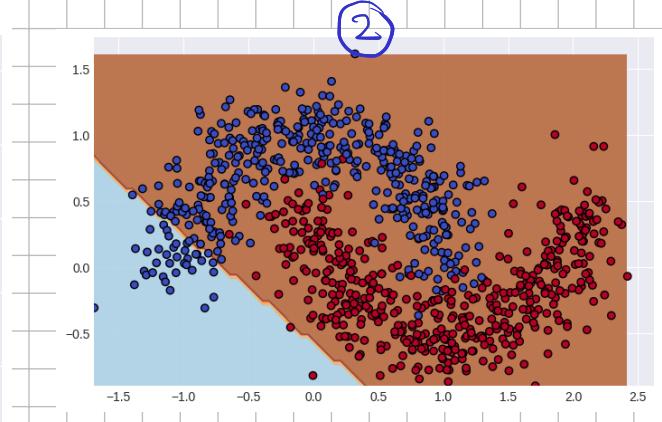
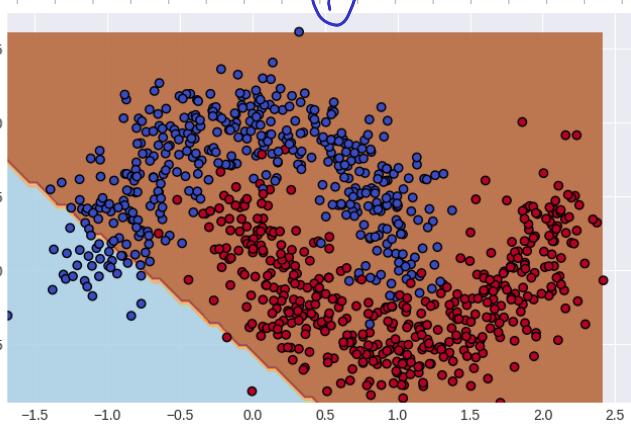
$$\text{Loss} = -[y \cdot \log(a) + (1-y) \cdot \log(1-a)]$$

$$\frac{-y}{a} \rightarrow \frac{(1-y)}{1-a} \rightarrow 1$$

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial a} \cdot \frac{\partial a}{\partial z}$$

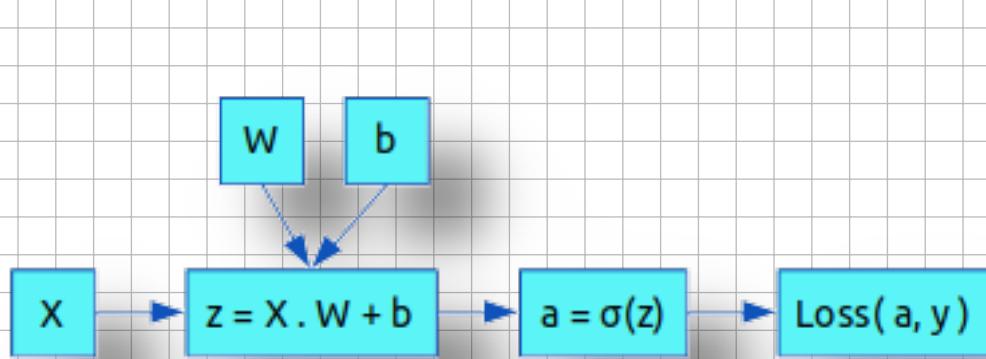
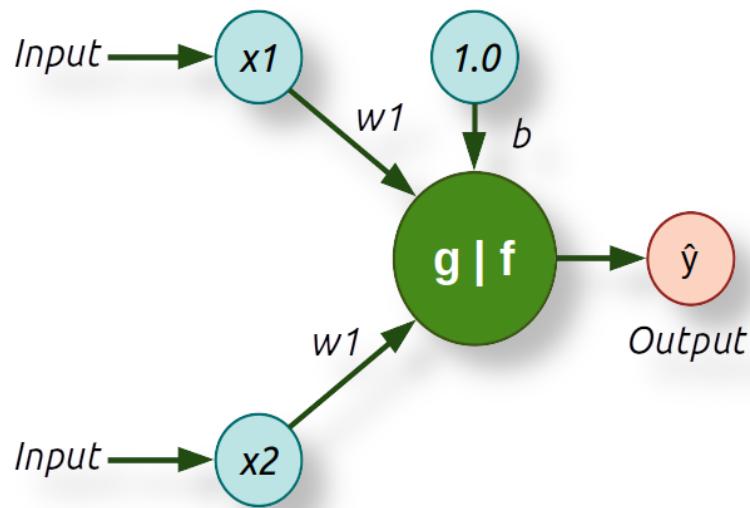
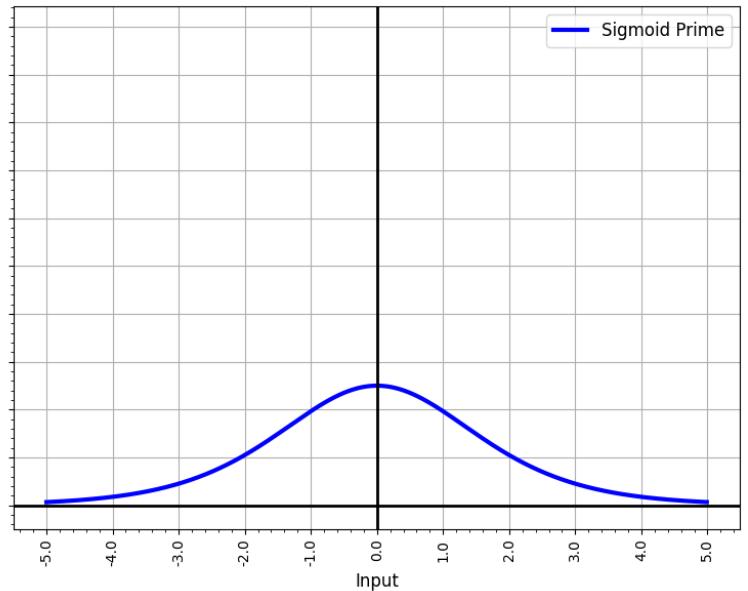
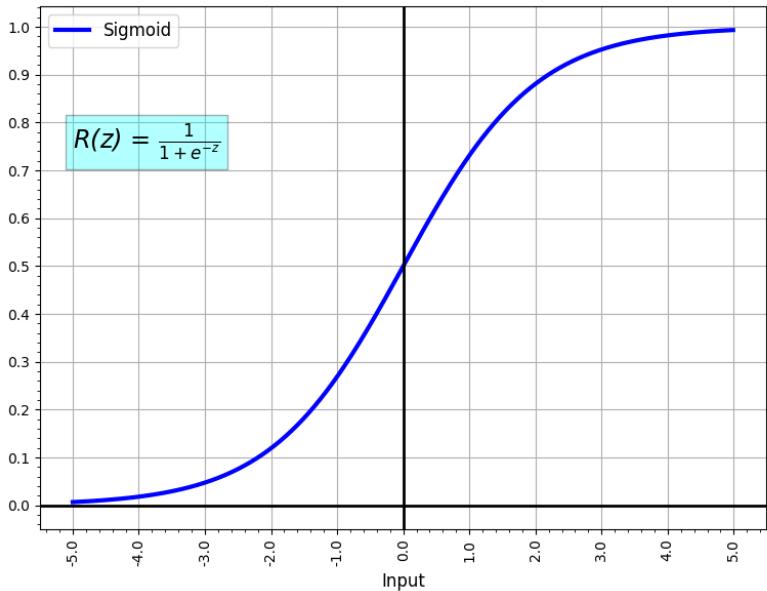
\downarrow

$a \cdot (1-a)$

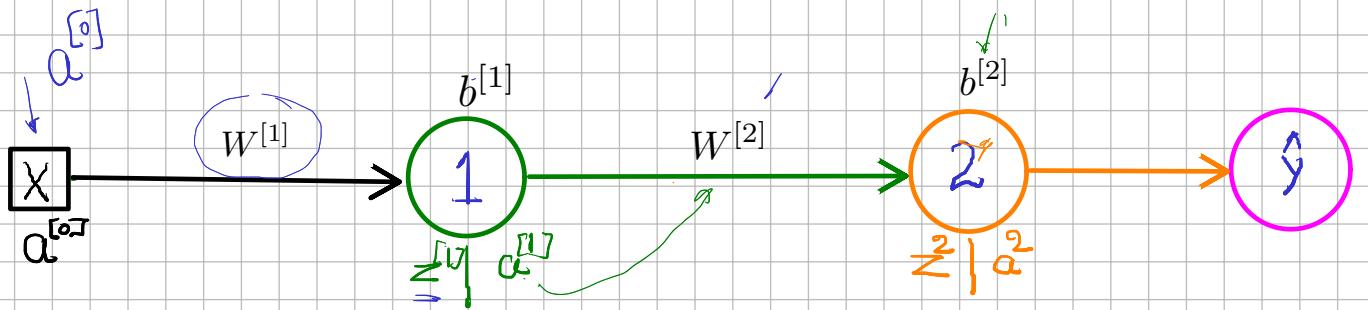


Assignment A02

Changes : - Sigmoid Activation function
 :- Loss = $\text{row}[-1] \cdot \text{Log}(a)$

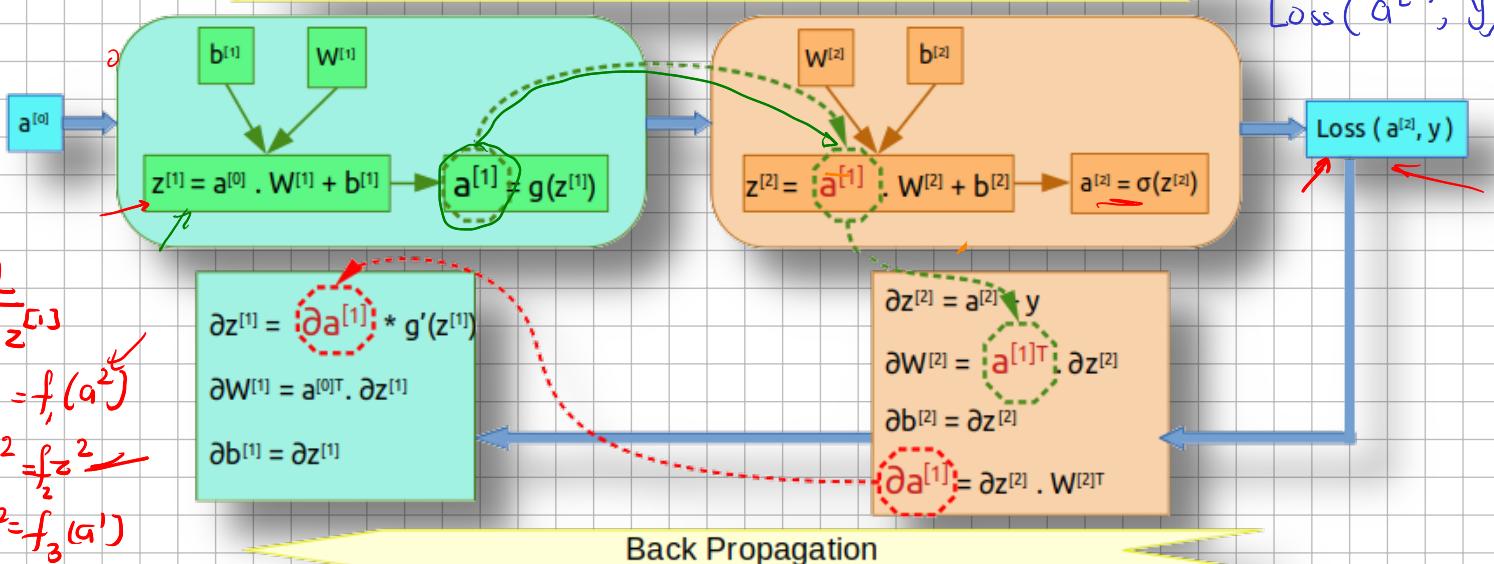


One Hidden Layer



$$z_i = \tilde{x}_i \cdot W^{[1]} + b^{[1]} \quad a^{[1]} = \tanh(z^{[1]}) \quad z^{[2]} = a^{[1]} \cdot W^{[2]} + b^{[2]} \quad a^{[2]} = \sigma(z^{[2]})$$

Forward Propagation



$$\begin{aligned} \frac{\partial L}{\partial z^{[1]}} &= \frac{\partial L}{\partial a^{[2]}} * g'(z^{[1]}) \\ L &= f_1(a^{[2]}) \\ a^2 &= f_2(z) \\ z^2 &= f_3(a') \\ a' &= f_4(z') \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial z^{[1]}} &= \frac{\partial L}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \cdot \frac{\partial a^{[1]}}{\partial z^{[1]}} \\ &= \frac{\partial L}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \cdot \frac{\partial a^{[1]}}{\partial z^{[1]}} \end{aligned}$$

$$\begin{aligned} \frac{\partial z^{[2]}}{\partial a^{[1]}} &= q^{[2]} - y \\ \frac{\partial a^{[1]}}{\partial z^{[1]}} &= a^{[1]T} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \\ \frac{\partial a^{[1]}}{\partial z^{[1]}} &= a^{[1]T} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \\ \frac{\partial z^{[2]}}{\partial z^{[1]}} &= a^{[1]T} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \end{aligned}$$

For single perceptron:

$$z = x_1 \cdot w_1 + x_2 \cdot w_2 + b \quad \textcircled{1}$$

$$a = \sigma(z) \quad \textcircled{2}$$

$$a = \sigma(x_1 \cdot w_1 + x_2 \cdot w_2 + b)$$

$$a = \sigma([x_1, x_2] \odot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + b)$$

For multiple Rows of X:

$$a = \sigma(\begin{bmatrix} x_1^{(1)} & x_2^{(1)} \\ x_1^{(2)} & x_2^{(2)} \\ \dots & \dots \\ x_1^{(m)} & x_2^{(m)} \end{bmatrix} \odot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + b)$$

In matrix form it can be represented as:

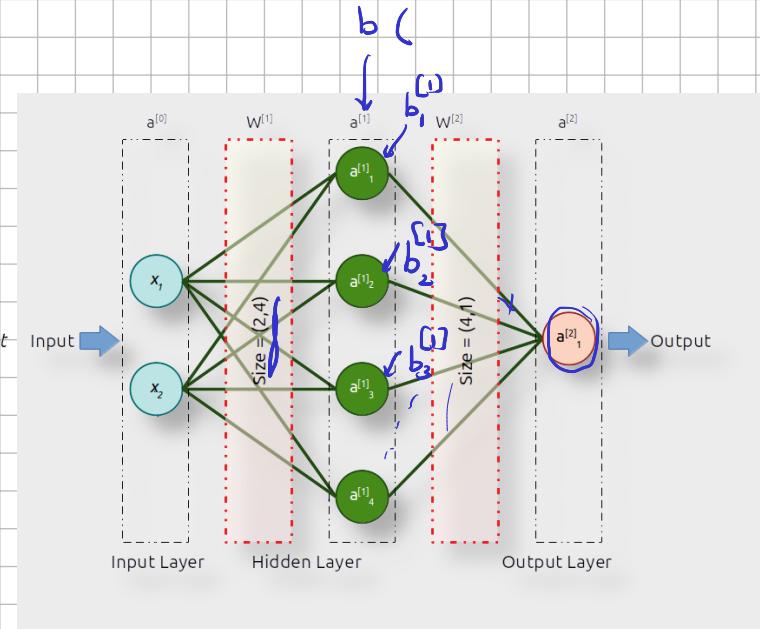
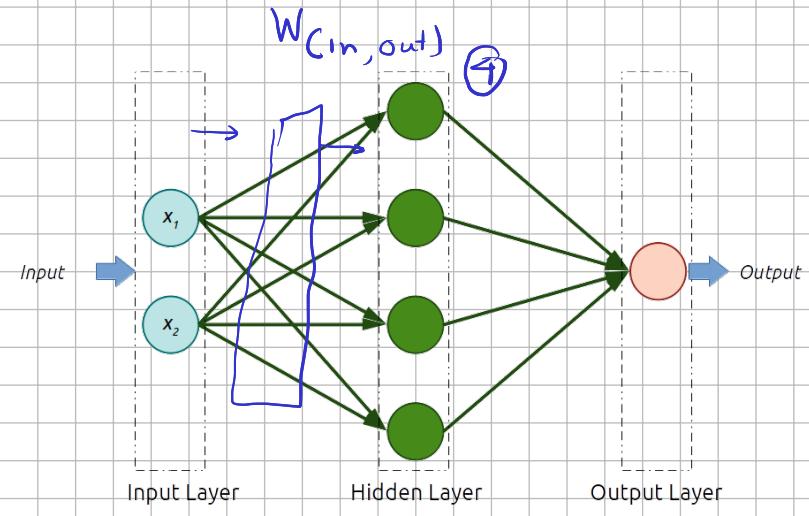
$$a = \sigma(X_{shape=(m,2)} \odot W_{shape=(2,1)}^{[1]} + b_{shape=(1,1)})$$

Scalar

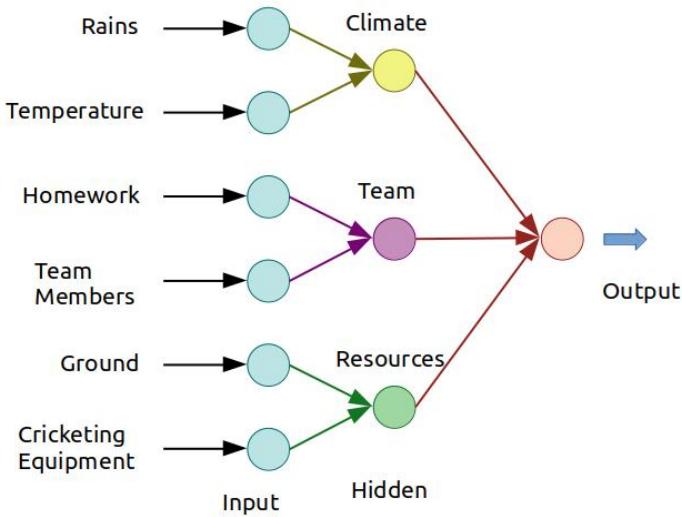
For single row of perceptrons:

$$\begin{aligned} a^{[1]} &= g(X_{shape=(m,2)} \odot W_{shape=(2,1)}^{[1]} + b_{shape=(1,1)}^{[1]}) \quad \textcircled{1} \\ a^{[2]} &= \sigma(a^{[1]}_{shape=(m,1)} \odot W_{shape=(1,1)}^{[2]} + b_{shape=(1,1)}^{[2]}) \quad \textcircled{2} \end{aligned}$$

Neural Network



What is this hidden layer?



$$\begin{aligned} z_1^{[1]} &= X \cdot W_1^{[1]} + b_1^{[1]} \\ a_1^{[1]} &= \tanh(z_1^{[1]}) \end{aligned} \quad \boxed{1}$$

$$\begin{aligned} z_2^{[1]} &= X \cdot W_2^{[1]} + b_2^{[1]} \\ a_2^{[1]} &= \tanh(z_2^{[1]}) \end{aligned} \quad \boxed{2}$$

$$\begin{aligned} z_3^{[1]} &= X \cdot W_3^{[1]} + b_3^{[1]} \\ a_3^{[1]} &= \tanh(z_3^{[1]}) \end{aligned} \quad \boxed{3}$$

$$\begin{aligned} z_4^{[1]} &= X \cdot W_4^{[1]} + b_4^{[1]} \\ a_4^{[1]} &= \tanh(z_4^{[1]}) \end{aligned} \quad \boxed{4}$$

Or

$$a^{[1]} = \tanh(X \circ [W_1^{[1]}, W_2^{[1]}, W_3^{[1]}, W_4^{[1]}] + [b_1^{[1]}, b_2^{[1]}, b_3^{[1]}, b_4^{[1]}])$$

$$a^{[1]} = \tanh(X \circ W^{[1]} + b^{[1]})$$

Where:

$$W^{[1]} = [W_1^{[1]}, W_2^{[1]}, W_3^{[1]}, W_4^{[1]}]$$

$$b^{[1]} = [b_1^{[1]}, b_2^{[1]}, b_3^{[1]}, b_4^{[1]}]$$

Hence:

$$z_{shape=(m,4)}^{[1]} = X_{shape=(m,2)} \circ W_{shape=(2,4)}^{[1]} + b_{shape=(1,4)}^{[1]}$$
$$a_{shape=(m,4)}^{[1]} = \tanh(z^{[1]})$$

Similarly for second layer.

$$z_{shape=(m,1)}^{[2]} = a_{shape=(m,4)}^{[1]} \circ W_{shape=(4,1)}^{[2]} + b_{shape=(1,1)}^{[2]}$$
$$a_{shape=(m,1)}^{[2]} = \hat{y} = \text{sigmoid}(z^{[2]})$$

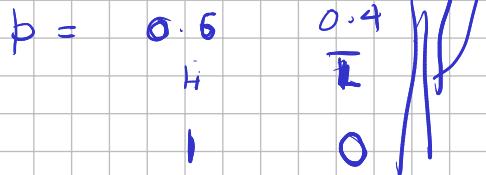
Where:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Loss Function

$$L(a_i, y_i) = - [y_i \log(a_i) + (1 - y_i) \log(1 - a_i)]$$

$$\underline{J(a, y)} = - \frac{1}{m} \sum_{i=1}^m [y_i \log(a_i) + (1 - y_i) \log(1 - a_i)]$$



In case of Binary Classification:

$$L(a_i, y_i) = -y_i \log(a_i)$$

$$J(a, y) = - \frac{1}{m} \sum_{i=1}^m y_i \log(a_i)$$

Backpropagation

$$Loss = f_1(a^{[2]})$$

$$a^{[2]} = f_2(z^{[2]})$$

$$z^{[2]} = f_3(a^{[1]})$$

$$a^{[1]} = f_4(z^{[1]})$$

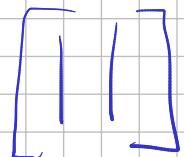
$$z^{[1]} = f_5(X)$$

.

Therefore:

$$\frac{\partial Loss}{\partial z^{[1]}} = \frac{\partial Loss}{\partial z^{[2]}} \circ \frac{\partial z^{[2]}}{\partial a^{[1]}} \circ \frac{\partial a^{[1]}}{\partial z^{[1]}}$$

4, 2

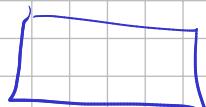


Back-propagation for all Rows

$$\partial z^{[2]} = a^{[2]} - y$$

$$\partial W^{[2]} = \frac{1}{m} a^{[1]T} \circ \partial z^{[2]}$$

(2,



(1, 2)

$$\partial b^{[2]} = \frac{1}{m} \text{np.sum}(\partial z^{[2]}, axis=0, keepdims=True)$$

$$\partial a^{[1]} = \partial z^{[2]} \circ W^{[2]T}$$

$$\partial z^{[1]} = \partial a^{[1]} * (1 - \tanh(z^{[1]})^2)$$

$$\partial W^{[1]} = \frac{1}{m} X^T \circ \partial z^{[1]}$$

$$\partial b^{[1]} = \frac{1}{m} \text{np.sum}(\partial z^{[1]}, axis=0, keepdims=True)$$