Q: $\phi(x,y,z) = 3x^2y - y^3z^2$. Find $\nabla\phi$ (grad. of ϕ) at point $(1,-2,-1)$.

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(3x^2y - y^3z^2).$$

$$= 6xy\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - 2y^3z\hat{k}$$

Putting $(x,y,z) = (1,-2,-1)$

$$= 6(1)(-2)\hat{i} + \{3(1)^2 - 3(-2)^2(-1)^2\}\hat{j} - 2(-2)^3(-1)\hat{k}$$

$$\boxed{\nabla\phi = -12\hat{i} - 9\hat{j} - 16\hat{k}}$$ Ans

$\log a^b = b\log a \Rightarrow \ln(r)^{1/2} \Rightarrow \frac{1}{2}\ln r$

Q: Find $\nabla\phi$. (a) $\phi = \ln|r|$ (b) $\phi = \frac{1}{r}$

$|r| = \sqrt{x^2+y^2+z^2}$

a. $\phi = \frac{1}{2}\ln(x^2+y^2+z^2)$

$$\nabla\phi = \frac{1}{2}\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\{\ln(x^2+y^2+z^2)\}$$

$$\nabla\phi = \frac{1}{2}\left[\frac{2x}{x^2+y^2+z^2}\hat{i} + \frac{2y}{x^2+y^2+z^2}\hat{j} + \frac{2z}{x^2+y^2+z^2}\hat{k}\right]$$

$$\nabla\phi = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{x^2+y^2+z^2}\,(x\hat{i} + y\hat{j} + z\hat{k})$$

$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore r = \sqrt{x^2+y^2+z^2}$

$\therefore r^2 = x^2+y^2+z^2$

$$\nabla\phi = \frac{1}{r^2} \times \vec{r}$$

$$\boxed{\nabla\phi = \frac{\vec{r}}{r^2}}$$

b. $\phi = \frac{1}{r} \Rightarrow \phi = (x^2+y^2+z^2)^{-1/2}$

$$\nabla\phi = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x\hat{i} + 2y\hat{j} + 2z\hat{k})$$

$$= -\frac{1}{\cancel{2}} \times \cancel{2}\,(x^2+y^2+z^2)^{-3/2}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\nabla\phi = -(r)^{-3}(\vec{r})$$

$$\boxed{\nabla\phi = -\frac{\vec{r}}{r^3}}$$

— x —

Q: Find directional derivative of $\phi = x^2yz + 4xz^2$, at $(1,-2,-1)$ in the direction $2\hat{i}-\hat{j}-2\hat{k}$.

Directional derivative $\Rightarrow \nabla\phi \cdot \hat{a}$.

Unit Vector:

$$\hat{a} = \frac{\vec{a}}{|a|}$$

$$\hat{a} = \frac{2i-j-2k}{3}$$

$\nabla\phi$:

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)(x^2yz+4xz^2)$$

$$\nabla\phi = (2xyz+4z^2)\hat{i}+x^2z\hat{j}+(x^2y+8xz)\hat{k}.$$

Putting $(x,y,z)=(1,-2,-1)$

$$\boxed{\nabla\phi = 8\hat{i}-\hat{j}-10\hat{k}}$$

$$\nabla\phi \cdot \hat{a} = (8\hat{i} - \hat{j} - 10\hat{k})\left(\frac{2\hat{i} - \hat{j} - 2\hat{k}}{3}\right)$$

$$= \frac{16 + 1 + 20}{3}$$

$$\boxed{\nabla\phi \cdot \hat{a} = \frac{37}{3}}$$ Ans

Q: $\nabla\psi = (y^2 - 2xyz^3)\hat{i} + (3 + 2xy - x^2z^3)\hat{j} + (6z^3 - 3x^2yz^2)\hat{k}$. Find ψ.

We know that,

$$\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).$$

Comparing.

$$\frac{\partial\psi_1}{\partial x} = y^2 - 2xyz^3.$$

$$\frac{\partial\psi_2}{\partial y} = 3 + 2xy - x^2z^3.$$

$$\frac{\partial\psi_3}{\partial z} = 6z^3 - 3x^2yz^2.$$

$$\int \partial\psi_1 = \int (y^2 - 2xyz^3)\,dx$$

$$\psi_1 = xy^2 - x^2yz^3 + c.$$

$$\int \gamma\psi_2 = \int (3 + 2xy - x^2z^3)\, \partial y$$

$$\psi_2 = 3y + xy^2 + x^2yz^3 + c.$$

$$\int \gamma\psi_3 = \int (6z^3 - 3x^2yz^2)\, \partial x$$

$$\psi_3 = \frac{3}{2}z^4 - x^2yz^3 + c.$$

sare ψ me se repeated terms ko ek dafa likhenge

After ignoring the repeating terms, we get:

$$\boxed{\psi = xy^2 - x^2yz^3 + 3y + \frac{3}{2}z^4 + C}$$

Q4: Show that $\nabla r^n = n r^{n-2} r$.

$$\nabla r^n = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\sqrt{x^2+y^2+z^2}\right)^n$$

$$= \frac{\partial}{\partial x}(x^2+y^2+z^2)^{n/2}\hat{i} + \frac{\partial}{\partial y}(x^2+y^2+z^2)^{n/2}\hat{j} + \frac{\partial}{\partial z}(x^2+y^2+z^2)^{n/2}\hat{k}$$

$$= \frac{n}{\cancel{2}}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cancel{2}x\,\hat{i} + \frac{n}{\cancel{2}}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cancel{2}y\,\hat{j} + \frac{n}{\cancel{2}}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cancel{2}z\,\hat{k}$$

$$\nabla r^n = nx(x^2+y^2+z^2)^{\frac{n}{2}-1}\hat{i} + ny(x^2+y^2+z^2)^{\frac{n}{2}-1}\hat{j} + nz(x^2+y^2+z^2)^{\frac{n}{2}-1}$$

$$= nx(r^2)^{\frac{n}{2}-1}\hat{i} + ny(r^2)^{\frac{n}{2}-1}\hat{j} + nz(r^2)^{\frac{n}{2}-1}\hat{k}$$

$$= nxr^{n-2}\hat{i} + nyr^{n-2}\hat{j} + nzr^{n-2}\hat{k}$$

$$= nr^{n-2}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\boxed{\nabla r^n = nr^{n-2}\vec{r}}$$ **Proved!**

Q6: Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

Unit vector find krna ko mtlb $\frac{\text{vector}}{\text{magnitude}}$ lekin hme pehle vector find krna hoga jo hm ase krenge.

$$\nabla(x^2y + 2xz) = \frac{\partial}{\partial x}(x^2y+2xz)\hat{i} + \frac{\partial}{\partial y}(x^2y+2xz)\hat{j} + \frac{\partial}{\partial z}(x^2y+2xz)\hat{k}$$

$$\vec{n} = (2xy+2z)\hat{i} + (x^2)\hat{j} + 2x\hat{k}$$

At $(2, -2, 3)$

$$\vec{n} = \{2(2)(-2)+2(3)\}\hat{i} + (2)^2\hat{j} + 2(2)\hat{k}$$

$$\vec{n} = (-8+6)\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\boxed{\vec{n} = -2\hat{i} + 4\hat{j} + 4\hat{k}}$$

Now, we find unit normal:

$$\hat{n} = \frac{\vec{n}}{|n|}$$

$$\hat{n} = \frac{-2\hat{i} + 4\hat{j} + 4\hat{k}}{\sqrt{(-2)^2 + (4)^2 + (4)^2}}$$

$$\boxed{\hat{n} = -\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}}$$

Q11: a) In what direction from the point $(2,1,-1)$ is the directional derivative of $\phi = x^2yz^3$ a maximum?

b) What is the magnitude of this maximum?

a. $\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (x^2yz^3)$

$\nabla\phi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$

At $(2, 1, -1)$:

$\nabla\phi = 2(2)(1)(-1)^3\hat{i} + (2)^2(-1)^3\hat{j} + 3(2)^2(1)(-1)^2\hat{k}$

$\boxed{\nabla\phi = -4\hat{i} - 4\hat{j} + 12\hat{k}}$ Ans

The directional derivative is maximum in the direction of $-4\hat{i} - 4\hat{j} + 12\hat{k}$.

b.

Magnitude of maximum is $|\nabla\phi|$:

$|\nabla\phi| = \sqrt{(-4)^2 + (-4)^2 + (12)^2}$

$|\nabla\phi| = \sqrt{16 + 16 + 144}$

$\boxed{|\nabla\phi| = 4\sqrt{11}}$ Ans

Q12: Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2, -1, 2)$.

$\vec{A}\cdot\vec{B} = AB\cos\theta$ (Is formula se angle find krenge lekin jahle do vectors chahiye honge hmen) Or vectors find krne ke lie $\nabla\phi$ use krenge hm.

The formula would be:

$$\vec{\nabla\phi_1} \cdot \vec{\nabla\phi_2} = |\nabla\phi_1| \cdot |\nabla\phi_2| \cos\theta \quad \text{— eq (i)}$$

Here,

$$\phi_1 = x^2+y^2+z^2 = 9$$
$$\phi_2 = x^2+y^2-z = 3$$

$$\nabla\phi_1 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(x^2+y^2+z^2)$$

$$\nabla\phi_1 = 2x\,\hat{i} + 2y\,\hat{j} + 2z\,\hat{k}$$

At $(x, y, z) = (2, -1, 2)$

$$\nabla\phi_1 = 2(2)\hat{i} + 2(-1)\hat{j} + 2(2)\hat{k}$$

$$\boxed{\nabla\phi_1 = 4\hat{i} - 2\hat{j} + 4\hat{k}}$$

$$\nabla\phi_2 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(x^2 + y^2 - z).$$

$$\nabla\phi_2 = 2x\,\hat{i} + 2y\,\hat{j} - \hat{k}$$

At $(x, y, z) = (2, -1, 2)$

$$\nabla\phi_2 = 2(2)\,\hat{i} + 2(-1)\hat{j} - \hat{k}$$

$$\boxed{\nabla\phi_2 = 4\hat{i} - 2\hat{j} - \hat{k}}$$

Now, consider eq (1).

$$\text{eq (1)} \Rightarrow \quad \vec{\nabla\phi_1} \cdot \vec{\nabla\phi_2} = |\nabla\phi_1||\nabla\phi_2| \cos\theta$$

$$(4i - 2j + 4k)(4i - 2j - k) = \left(\sqrt{(4)^2 + (-2)^2 + (4)^2}\right)\left(\sqrt{(4)^2 + (-2)^2 + (-1)^2}\right) \cos\theta$$

$$16 + 4 - 4 = (6)\sqrt{21}\, \cos\theta$$

$$\frac{\cancel{16}^{\,8}}{\cancel{6}_{\,3}\sqrt{21}} = \cos\theta \quad \Rightarrow \theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$$

$$\Rightarrow \boxed{\theta = 54.41^\circ}$$

Q42 : If $\phi = 2xz^4 - x^2 y$, find $\nabla\phi$ and $|\nabla\phi|$ at the point $(2, -2, -1)$

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(2xz^4 - x^2 y)$$

$$\nabla\phi = (2z^4 - 2xy)\hat{i} + (-x^2)\hat{j} + (8xz^3)\hat{k}$$

$$\nabla\phi = (2z^4 - 2xy)\hat{i} - x^2\hat{j} + 8xz^3\hat{k}$$

At $(x, y, z) = (2, -2, -1)$

$$\nabla\phi = \{2(-1)^4 - 2(2)(-2)\}\hat{i} - (2)^2\hat{j} + 8(2)(-1)^3\hat{k}$$

$$\boxed{\nabla\phi = 10\hat{i} - 4\hat{j} - 16\hat{k}}$$

$$|\nabla\phi| = \sqrt{(10)^2 + (-4)^2 + (-16)^2}$$

$$|\nabla\phi| = \sqrt{100 + 16 + 256}$$

$$\boxed{|\nabla\phi| = 2\sqrt{93}}$$

Q43: If $A = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$ and $\phi = 2z - x^3y$, find $A \cdot \nabla\phi$ and $A \times \nabla\phi$ at the point $(1, -1, 1)$.

$A \cdot \nabla\phi$:

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(2z - x^3y)$$

$$\boxed{\nabla\phi = -3x^2y\,\hat{i} - x^3\hat{j} + 2\hat{k}}$$

$$A \cdot \nabla\phi = (2x^2)(-3x^2y) + (-3yz)(-x^3) + (xz^2)(2)$$

$$A \cdot \nabla\phi = -6x^4y + 3x^3yz + 2xz^2$$

At point $(1, -1, 1)$

$$A \cdot \nabla\phi = -6(1)^4(-1) + 3(1)^3(-1)(1) + 2(1)(1)^2$$

$$= +6 - 3 + 2$$

$$\boxed{A \cdot \nabla\phi = +5}$$

$\underline{A \times \nabla\phi}$.

$$A \times \nabla\phi = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2x^2 & -3yz & xz^2 \\ -3x^2y & -x^3 & 2 \end{vmatrix}$$

$$A \times \nabla\phi = \hat{i}\begin{vmatrix} -3yz & xz^2 \\ -x^3 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2x^2 & xz^2 \\ -3x^2y & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2x^2 & -3yz \\ -3x^2y & -x^3 \end{vmatrix}$$

$$A \times \nabla\phi = \hat{i}\,(-6yz + x^4z^2) - \hat{j}\,(4x^2 + 3x^3yz^2) + k\,(-2x^5 - 9x^2y^2z)$$

$$A \times \nabla\phi = \hat{i}\,(x^4z^2 - 6yz) + \hat{j}\,(-4x^2 - 3x^3yz^2) + k\,(-2x^5 - 9x^2y^2z)$$

At $(1, -1, 1)$:

$$A \times \nabla\phi = \hat{i}\,\{(1)^4(1)^2 - 6(-1)(1)\} + \hat{j}\,\{(-4)(1)^2 - 3(1)^3(-1)(1)^2\} + k\,\{-2(1)^5 - 9(1)^2(-1)^2(1)\}$$

$$\boxed{A \times \nabla\phi = 7\hat{i} - \hat{j} - 11\hat{k}}$$

<u>dr</u>

Q44. If $F = x^2z + e^{y/x}$ and $G = 2z^2y - xy^2$, find (a) $\nabla(F+G)$ and (b) $\nabla(FG)$ at the point $(1,0,-2)$.

(a)

$$F+G = x^2z + e^{y/x} + 2z^2y - xy^2$$

$$\nabla(F+G) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(x^2z + e^{y/x} + 2z^2y - xy^2)$$

$$\nabla(F+G) = \left[2xz + e^{y/x}\left(\frac{-y}{x^2}\right) - y^2\right]\hat{i} + \left[e^{y/x}\left(\frac{1}{x}\right) + 2z^2 - 2xy\right]\hat{j} + \left[x^2 + 4zy\right]\hat{k}$$

$$\nabla(F+G) = \left[2(1)(-2) + 0 - 0\right]\hat{i} + \left[e^0\left(\frac{1}{1}\right) + 2(-2)^2 - 0\right]\hat{j} + \left\{(1)^2 + 4(-2)(0)\right\}\hat{k}$$

$$\boxed{\nabla(F+G) = -4\hat{i} + 9\hat{j} + \hat{k}}$$

(b)

$$FG = (x^2z + e^{y/x})(2z^2y - xy^2)$$

$$FG = 2x^2yz^3 - x^3y^2z + 2z^2ye^{y/x} - xy^2e^{y/x}$$

$$\nabla(FG) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(2x^2yz^3 - x^3y^2z + 2z^2ye^{y/x} - xy^2e^{y/x}\right)$$

$$\nabla(FG) = \hat{i}\left[4xyz^3 - 3x^2y^2z + 2z^2ye^{y/x}\left(\frac{-y}{x^2}\right) - y^2e^{y/x}\left(\frac{1}{x}\right)\right]$$
$$+\hat{j}\left[2x^2z^3 - 2x^3yz + 2z^2e^{y/x}\left(\frac{1}{x}\right) - 2xye^{y/x}\frac{1}{x}\right]$$
$$+\hat{k}\left[6x^2yz^2 - x^3y^2 + 4zye^{y/x} - 0\right]$$

at $(1, 0, -2)$

$$\nabla(FG) = (0-0+0-0)\hat{i} + \hat{j}\left[2(1)^2(-2)^3 - 0 + 2(-2)^2e^0 - 0\right] + \hat{k}\,(0-0+0-0)$$

$$= (-16+8)\hat{j}$$

$$\boxed{\nabla(FG) = -8\hat{j}}$$ Ans

Q45. Find $\nabla|r|^3$.

$$|r|^3 = \left(\sqrt{x^2+y^2+z^2}\right)^3$$

$$|r|^3 = (x^2+y^2+z^2)^{3/2}$$

$$\nabla|r|^3 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(x^2+y^2+z^2)^{3/2}$$

$$\nabla |r|^3 = \left[\frac{3}{\cancel{2}}\sqrt{x^2+y^2+z^2}\,(\cancel{2}x)\right]\hat{i} + \left[\frac{3}{\cancel{2}}\sqrt{x^2+y^2+z^2}\,(\cancel{2}y)\right]\hat{j}$$

$$+ \left[\frac{3}{\cancel{2}}\sqrt{x^2+y^2+z^2}\,(\cancel{2}z)\right]\hat{k}$$

$$\nabla |r|^3 = 3rx\,\hat{i} + 3ry\,\hat{j} + 3rz\,\hat{k}$$

$$\nabla |r|^3 = 3r\,(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\boxed{\nabla |r|^3 = 3r\,.\,\vec{r}}$$

Q46: Prove $\nabla f(r) = \frac{f'(r)\,\vec{r}}{r}$.

$$f(r) = \sqrt{x^2+y^2+z^2}$$

$$\nabla f(r) = \frac{\partial}{\partial x} f(r)\,\hat{i} + \frac{\partial}{\partial y} f(r)\,\hat{j} + \frac{\partial}{\partial z} f(r)\,\hat{k}$$

× and ÷ by '∂r'

$$\nabla f(r) = \frac{\partial f(r)}{\partial r} \times \frac{\partial r}{\partial x}\,\hat{i} + \frac{\partial f(r)}{\partial r} \times \frac{\partial r}{\partial y}\,\hat{j} + \frac{\partial f(r)}{\partial r} \times \frac{\partial r}{\partial z}\,\hat{k}$$

$$\nabla f(r) = f'(r)\,.\,\frac{\partial r}{\partial x}\,\hat{i} + f'(r)\,.\,\frac{\partial r}{\partial y}\,\hat{j} + f'(r)\,.\,\frac{\partial}{\partial z}\,\hat{k}$$

$$\nabla f(r) = f'(r)\left[\frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}\right]$$

For $\frac{\partial r}{\partial x}$:

$$r = \sqrt{x^2+y^2+z^2}$$

$$\frac{\partial r}{\partial x} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$\boxed{\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}}$$

For $\frac{\partial r}{\partial y}$:

$$\frac{\partial r}{\partial y} = \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2+z^2}} \Rightarrow \boxed{\frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}}$$

For $\frac{\partial r}{\partial z}$:

$$\frac{\partial r}{\partial z} = \frac{\cancel{2}z}{\cancel{2}\sqrt{x^2+y^2+z^2}} \Rightarrow \boxed{\frac{\partial r}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}}$$

So,

$$\nabla f(r) = \overset{f'(r)}{\uparrow}\left[\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}\right]$$

$$\nabla f(r) = \overset{f'(r)}{\uparrow}\frac{1}{\sqrt{x^2+y^2+z^2}}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\nabla f(r) = f'(r) \cdot \frac{1}{r} \cdot \vec{r}$$

$$\boxed{\nabla f(r) = \frac{f'(r) \cdot \vec{r}}{r}}$$

Proved!

Q47: Evaluate $\nabla\left(3r^2 - 4\sqrt{r} + \frac{6}{\sqrt[3]{r}}\right)$

For $3r^2$:

$$3r^2 = 3\left(\cancel{\sqrt{}}\,x^2+y^2+z^2\right)^{\cancel{2}}$$

$$\boxed{3r^2 = 3x^2 + 3y^2 + 3z^2}$$

For $4\sqrt{r}$:

$$4\sqrt{r} = 4\left\{(x^2+y^2+z^2)^{1/2}\right\}^{1/2}$$

$$\boxed{4\sqrt{r} = 4(x^2+y^2+z^2)^{1/4}}$$

For $\frac{6}{\sqrt[3]{r}}$:

$$\frac{6}{\sqrt[3]{r}} = \frac{6}{\{(x^2+y^2+z^2)^{1/2}\}^{1/3}}$$

$$\boxed{\frac{6}{\sqrt[3]{r}} = \frac{6}{(x^2+y^2+z^2)^{1/6}}}$$

Now,

$$\nabla\left(3r^2-4\sqrt{r}+\frac{6}{\sqrt[3]{r}}\right) = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\left[(3x^2+3y^2+3z^2)-4(x^2+y^2+z^2)^{1/4} + \frac{6}{(x^2+y^2+z^2)^{1/6}}\right]$$

$$= \left[6x - \frac{1}{\cancel{4}}\times\cancel{4}(x^2+y^2+z^2)^{-3/4}\,2x - \cancel{6}\times\frac{1}{\cancel{6}}(x^2+y^2+z^2)^{-7/6}\,2x\right]\hat{i}$$

$$+\left[6y - \cancel{4}\times\frac{1}{\cancel{4}}(x^2+y^2+z^2)^{-3/4}\,2y - \cancel{6}\times\frac{1}{\cancel{6}}(x^2+y^2+z^2)^{-7/6}\,2y\right]\hat{j}$$

$$+\left[6z - \cancel{4}\times\frac{1}{\cancel{4}}(x^2+y^2+z^2)^{-3/4}\,2z - \cancel{6}\times\frac{1}{\cancel{6}}(x^2+y^2+z^2)^{-7/6}\,2x\right]\hat{k}.$$

$$= \left[6x - 2r^{-3/2}x - 2xr^{-7/3}\right]\hat{i} + \left[6y - 2r^{-3/2}y - 2yr^{-7/3}\right]\hat{j}$$
$$+ \left[6z - 2r^{-3/2}z - 2zr^{-7/3}\right]\hat{k}.$$

$$\nabla\left(3r^2-4\sqrt{r}+\frac{6}{\sqrt[3]{r}}\right)= x i\left(6-2r^{-3/2}-2r^{-7/3}\right)+yj\left(6-2r^{-3/2}-2r^{-7/3}\right)+z\hat{k}\left(6-2r^{-3/2}-2r^{-7/3}\right)$$

$$=\left(6-2r^{-3/2}-2r^{-7/3}\right)(xi+yj+zk)$$

$$\boxed{\nabla\left(3r^2-4\sqrt{r}+\frac{6}{\sqrt[3]{r}}\right)=\left(6-2r^{-3/2}-2r^{-7/3}\right).\vec{r}}$$ Ans

Q50: Find $\nabla\psi$ where $\psi=(x^2+y^2+z^2)e^{-\sqrt{x^2+y^2+z^2}}$.

$$\nabla\psi=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\left[(x^2+y^2+z^2)\left(e^{-\sqrt{x^2+y^2+z^2}}\right)\right]$$

For $\frac{\partial\psi}{\partial x}$:

$$\frac{\partial\psi}{\partial x}=\left(e^{-\sqrt{x^2+y^2+z^2}}(2x)+(x^2+y^2+z^2)e^{-\sqrt{x^2+y^2+z^2}}\left[-\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}\cancel{2}x\right]\right)i$$

$$\frac{\partial\psi}{\partial x}=2xe^{-r}+r^{\cancel{2}}e^{-r}\left(-\frac{x}{\cancel{r}}\right)$$

$$\boxed{\frac{\partial\psi}{\partial x}=xe^{-r}(2-r)}$$

For $\frac{\partial \psi}{\partial y}$:

$$\boxed{\frac{\partial \psi}{\partial y} = y e^{-r}(2-r)}$$

For $\frac{\partial \psi}{\partial z}$:

$$\boxed{\frac{\partial \psi}{\partial z} = z e^{-r}(2-r)}$$

Now,

$$\nabla.\psi = x e^{-r}(2-r)\hat{i} + y e^{-r}(2-r)\hat{j} + z e^{-r}(2-r)\hat{k}$$

$$= e^{-r}(2-r)(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\boxed{\nabla \psi = e^{-r}(2-r)\vec{r}}$$ Ans

Q51: If $\nabla \phi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$, find $\phi(x,y,z)$ if $\phi(1,-2,2) = 4$.

$$\nabla \phi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$$

As we know that,

$$\nabla \phi = \left(\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}\right)$$

$$\frac{\partial \phi}{\partial x} = 2xyz^3$$

$$\int \partial \phi = \int 2xyz^3 \, \partial x$$

$$\phi = \frac{\cancel{2}x^2yz^3}{\cancel{2}} + c \Rightarrow \boxed{\phi = x^2yz^3 + c}$$

$$\frac{\partial \phi}{\partial y} = x^2z^3$$

$$\int \partial \phi = \int x^2z^3 \, \partial y$$

$$\boxed{\phi = x^2yz^3 + c}$$

$$\frac{\partial \phi}{\partial z} = 3x^2yz^2$$

$$\int \partial \phi = \int 3x^2yz^2 \, \partial z$$

$$\phi = \frac{\cancel{3}x^2yz^3}{\cancel{3}} + c \Rightarrow \boxed{\phi = x^2yz^3 + c}$$

so,

$$\boxed{\phi = x^2yz^3 + c}$$

Now, we find the value of c,

$$\phi = x^2yz^3 + c$$

$$4 = (1)^2(-2)(2)^3 + c$$

$$-c = -16 - 4$$

$$+c = +20$$

$$\boxed{c = 20}$$

So,

$$\boxed{\phi = x^2yz^3 + 20}$$

Q52: If $\nabla\psi = (y^2 - 2xyz^3)\hat{i} + (3 + 2xy - x^2z^3)\hat{j} + (6z^3 - 3x^2yz^2)\hat{k}$. Find ψ.

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Q58: Find a unit vector which is perpendicular to the surface of the paraboloid of revolution $z = x^2 + y^2$ at the point $(1, 2, 5)$.

$$x^2 + y^2 - z = 0$$

$$\nabla z = 2x\hat{i} + 2y\hat{j} - \hat{k}$$

$$\nabla z = 2(1)\hat{i} + 2(2)\hat{j} - \hat{k}$$

$$\nabla z = 2\hat{i} + 4\hat{j} - \hat{k}.$$

$$\hat{z} = \frac{\text{Vector}}{\text{Magnitude}} = \frac{\nabla z}{|\nabla z|}$$

$$\hat{z} = \frac{2\hat{i} + 4\hat{j} - \hat{k}}{\sqrt{(2)^2 + (4)^2 + (1)^2}}$$

$$\boxed{\hat{z} = \frac{2i + 4j - k}{\sqrt{21}}}$$ Ans

Q59: Find the unit outward drawn normal to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at the point $(3, 1, -4)$.

$$(x-1)^2 + y^2 + (z+2)^2 = 9$$

$$x^2 - 2x + 1 + y^2 + z^2 + 4z + 4 = 9$$

$$x^2 - 2x + y^2 + 4z + z^2 = 9 - 4 - 1$$

$$(2x - 2)\hat{i} + (2y)\hat{j} + (2z + 4)\hat{k}$$

$$(6 - 2)\hat{i} + 2j + [2(-4) + 4]\hat{k}$$

$$\phi = 4\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\hat{\phi} = \frac{\phi}{|\phi|}$$

$$\hat{\phi} = \frac{4i + 2j - 4\hat{k}}{\sqrt{(4)^2 + (2)^2 + (4)^2}}$$

$$\hat{\phi} = \frac{2 4\hat{i} + 2j - 4\hat{k}}{6}$$

$$\boxed{\hat{\phi} = \frac{2i + j - 2K}{3}}$$ Ans

Q60: Find an equation for the tangent plane to the surface $xz^2 + x^2 y = z - 1$ at the point $(1, -3, 2)$.

$$xz^2 + x^2y - z = -1$$

$$\nabla(xz^2 + x^2y - z) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(xz^2 + x^2y - z)$$

$$N = (z^2 + 2xy)\hat{i} + x^2\hat{j} + (2xz - 1)\hat{k}.$$

At $(1, -3, 2)$

$$N = [(2)^2 + 2(1)(-3)]\hat{i} + (1)^2\hat{j} + [2(1)(2) - 1]\hat{k}.$$

$$N = (4 - 6)\hat{i} + \hat{j} + 3\hat{k}$$

$$\boxed{N = -2\hat{i} + \hat{j} + 3\hat{k}}$$

Equation for the tangent plane:

$$\because (r - r_0) \cdot N = 0$$

$$r = x\hat{i} + y\hat{j} + z\hat{k}$$

$$r_0 = \hat{i} - 3\hat{j} + 2\hat{k} \qquad (\text{At } 1, -3, 2)$$

$$\{(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 3\hat{j} + 2\hat{k})\} \cdot N = 0$$

$$[(x-1)\hat{i} + (y+3)\hat{j} + (z-2)\hat{k}] \cdot (-2\hat{i} + \hat{j} + 3\hat{k}) = 0$$

$(x-1)(-2)+(y+3)(1)+(z-2)(3)=0$

$-2x+2+y+3+3z-6=0$

$-2x-1+y+3z=0$

$\boxed{2x-y-3z+1=0}$ Ans

Q62: Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ in the direction $(2i-3j+6k)$.

Directional derivative $= \nabla\phi \cdot \hat{a}$

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(4xz^3 - 3x^2y^2z)$$

$$\nabla\phi = (4z^3 - 6xy^2z)\hat{i} + (-6x^2yz)\hat{j} + (12xz^2 - 3x^2y^2)\hat{k}$$

$$\nabla\phi = (4z^3 - 6xy^2z)\hat{i} - 6x^2yz\hat{j} + (12xz^2 - 3x^2y^2)\hat{k}$$

$$\boxed{\nabla\phi = 8\hat{i} + 48\hat{j} + 84\hat{k}}$$

$$\hat{a} = \frac{\vec{a}}{|a|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (-3)^2 + (6)^2}}$$

$$\boxed{\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}}$$

$$\nabla\phi \cdot \hat{a} = (8\hat{i} + 48\hat{j} + 84\hat{k})\left(\frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}\right)$$

$$= \frac{16 - 144 + 504}{7}$$

$$\boxed{\nabla\phi \cdot \hat{a} = \frac{376}{7}}$$ Ans

Q63: Find the directional derivative of $P = 4e^{2x-y+z}$ at the point $(1, 1, -1)$ in a direction toward the point $(-3, 5, 6)$.

$$\nabla P = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(4e^{2x-y+z}\right)$$

$$\nabla P = \left[4e^{2x-y+z}(2)\right]\hat{i} + \left[4e^{2x-y+z}(-1)\right]\hat{j} + \left(4e^{2x-y+z}(1)\right)\hat{k}$$

At $(1, 1, -1)$

$$\nabla P = \left[4e^{2-1-1}(2)\right]\hat{i} + \left[4e^{2-1-1}(-1)\right]\hat{j} + \left[4e^{2-1-1}(1)\hat{k}\right]$$

$$\boxed{\nabla P = 8\hat{i} - 4\hat{j} + 4\hat{k}}$$

$\because \quad r = r_2 - r_1$

$$\vec{r} = (-3-1)\hat{i} + (5-1)\hat{j} + (6+1)\hat{k}$$

$$\vec{r} = -4\hat{i} + 4\hat{j} + 7\hat{k}$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{-4\hat{i} + 4\hat{j} + 7\hat{k}}{\sqrt{(-4)^2 + (4)^2 + (7)^2}}$$

$$\boxed{\hat{r} = \frac{-4\hat{i} + 4\hat{j} + 7\hat{k}}{9}}$$

Directional derivative $= \nabla P \cdot \vec{r}$

$$= (8\hat{i} - 4\hat{j} + 4\hat{k}) \cdot \left(\frac{-4\hat{i} + 4\hat{j} + 7\hat{k}}{9}\right)$$

$$= \frac{(-32) + (-16) + (28)}{9}$$

$$\boxed{= \frac{-20}{9}}$$

square wali terms ek side pe lagi +ve side pe.

Q66: find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, -2, 1)$

$$\phi_1 = 3x + z^2 - xy^2z$$

$$\phi_2 = 3x^2 - y^2 + 2z - 1$$

$$\nabla\phi_1 = (3 - y^2z)\hat{i} - 2xyz\hat{j} + (2z - xy^2)\hat{k}$$

$$\boxed{\nabla\phi_1 = -\hat{i} + 4\hat{j} - 2\hat{k}}$$

$$\boxed{\nabla\phi_2 = 6x\hat{i} - 2y\hat{j} + 2\hat{k}}$$

After putting points.

$$\nabla\phi_2 = 6(1)\hat{i} - 2(-2)\hat{j} + 2\hat{k}$$

$$\boxed{\nabla\phi_2 = 6\hat{i} + 4\hat{j} + 2\hat{k}}$$

$$|\phi_1| = \sqrt{(-1)^2 + (4)^2 + (-2)^2}$$

$$\boxed{|\phi_1| = \sqrt{21}}$$

$$|\phi_2| = \sqrt{(6)^2 + (4)^2 + (2)^2}$$

$$\boxed{|\overleftarrow{\phi_2}| = 2\sqrt{14}}$$

$\because \quad \nabla\phi_1 \cdot \nabla\phi_2 = |\nabla\phi_1|\,|\nabla\phi_2| \cos\theta.$

$(-i + 4j - 2k) \cdot (6i + 4j + 2k) = \sqrt{21} \cdot 2\sqrt{14} \cos\theta$

$$\frac{-6 + 16 - 4}{\sqrt{21} \cdot 2\sqrt{14}} = \cos\theta.$$

$$\cos\theta = \frac{\sqrt{6}}{14}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{6}}{14}\right)$$

$$\boxed{\theta = 79.92^\circ}$$ Ans