(more than one ind. variable)

Partial Differential Equation

1. Formation of PDE:

- Find partial differential eq. by eliminating a & b from $z = ax + by + a^2 + b^2$.

$$\frac{\partial z}{\partial x} = a, \quad \frac{\partial z}{\partial y} = b$$

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

$$\cancel{z = (x-a)^2 + (y-b)^2}$$

Q: $z = (x-a)^2 + (y-b)^2$

$$\frac{\partial z}{\partial x} = 2(x-a) \Rightarrow (x-a)^2 = \left(\frac{1}{2}\frac{\partial z}{\partial x}\right)^2$$

$$\frac{\partial z}{\partial y} = 2(y-b) \Rightarrow (y-b)^2 = \left(\frac{1}{2}\frac{\partial z}{\partial y}\right)^2$$

$$z = \frac{1}{4}\left(\frac{\partial z}{\partial x}\right)^2 + \frac{1}{4}\left(\frac{\partial z}{\partial y}\right)^2$$

$$4z = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

$e^{2y} \times 2$

Q3: $z = axe^{y} + \frac{1}{2} a^{2} e^{2y} + b$

$\frac{\partial z}{\partial x} = ae^{y}$

$\frac{\partial z}{\partial y} = axe^{y} + a^{2}e^{2y}$

$\frac{\partial z}{\partial y} = x\,(ae^{y}) + (ae^{y})^{2}$

$\frac{\partial z}{\partial y} = x\frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^{2}$

Q4: $z = (x+y)a + b$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} = a \quad , \quad \frac{\partial z}{\partial y} = a$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$$

Q: Find P.D.E and eliminate h and k from $(x-h)^2 + (y-k)^2 + z^2 = \lambda^2$.

$$\frac{\partial (\lambda)^2}{\partial x} = \frac{\partial}{\partial x}\left[(x-h)^2 + (y-k)^2 + z^2\right]$$

$$0 = 2(x-h) + 2z\frac{\partial z}{\partial x}$$

$$0 = 2(y-k) + 2z\frac{\partial z}{\partial y}$$

$$x - h = -z\frac{\partial z}{\partial x}$$

$$y - k = -z\frac{\partial z}{\partial y}$$

$$\left(-z\frac{\partial z}{\partial x}\right)^2 + \left(-z\frac{\partial z}{\partial y}\right)^2 + z^2 = \lambda^2$$

$$z^2\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1\right] = \lambda^2$$

Ans

Q: $z = (x^2+a)(y^2+b)$ —— eq (A)

$$z = x^2y^2 + bx^2 + ay^2 + ab$$

$$\frac{\partial z}{\partial x} = 2xy^2 + 2xb \quad \text{—— eq (i)}$$

$$\frac{\partial z}{\partial y} = 2x^2y + 2ay \quad \text{—— eq (ii)}$$

eq (i) => $\frac{\partial z}{\partial x} = 2x(b+y^2)$

$$y^2 + b = \frac{1}{2x}\frac{\partial z}{\partial x}$$

eq(ii) $\Rightarrow$ $$\frac{\partial z}{\partial y} = 2y(x^2+a)$$

$$x^2 + a = \frac{1}{2y}\frac{\partial z}{\partial y}.$$

$$z = \left(\frac{1}{2y}\frac{\partial z}{\partial y}\right)\left(\frac{1}{2x}\frac{\partial z}{\partial x}\right)$$

$$z = \frac{1}{2 \cdot 2}\left[\frac{\partial z}{\partial y} \times \frac{1}{y}\right]\left[\frac{1}{x}\frac{\partial z}{\partial x}\right]$$

$$4xyz = \left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial x}\right)$$

Ans

Eliminate A & P.

$z = (x,t)$

Q7. $z = Ae^{pt}\sin px$

$$\frac{\partial z}{\partial x} = pAe^{pt}\cos px \cdot p$$

$$\frac{\partial z}{\partial t} = pAe^{pt}\sin px$$

$$\cancel{z = \frac{A}{p}\,\frac{\partial z}{\partial t}}$$

$$\frac{\partial^2 z}{\partial x^2} = -p^2Ae^{pt}\sin px$$

$$\frac{\partial^2 z}{\partial t^2} = p^2Ae^{pt}\sin px$$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{\partial^2 z}{\partial t^2}$$

Solution of PDE with degree 1.

LAGRANGE'S Equation

$$P\frac{\partial z}{\partial x} + Q\frac{\partial z}{\partial y} = R$$

P, Q, R is a function of x, y, z.

$$xy\frac{\partial t}{\partial x} + yz\frac{\partial z}{\partial y} = zx$$

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

$$Pp + Qq = R$$

$$\frac{dx}{P} \neq \frac{dy}{Q} = \frac{dz}{R}$$

Q1:- $-x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$

$P = -x \,,\; Q = y \,,\; R = z$

$\frac{dx}{-x} = \frac{dy}{y} = \frac{dz}{z}$

take $\frac{dx}{-x} = \frac{dy}{y}$

$dx \, \frac{(-1)}{-x} \longrightarrow$ der. of $-x$

$-\ln x = \ln y - \ln c_1$

$-\ln x = \ln \frac{y}{c_1}$

$\frac{1}{x} = \frac{y}{c_1}$

$\boxed{c_1 = xy}$

take $\frac{dy}{y} = \frac{dz}{z}$

$\ln y = \ln z + \ln c$

$\ln y = \ln c_2 z$

$y = c_2 z$

$c_2 = \frac{y}{z}$

$f\left(xy, y/z\right) = 0$ Ans

Q2: $y^2 \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} = x(z - 2y)$.

$P = y^2, \; Q = -xy, \; R = xz - 2xy$

$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{xz - 2xy}$

take $\frac{dx}{y^2} = \frac{dy}{-xy}$

$-x\,dx = y\,dy$

$-\int x\,dx = \int y\,dy$

$$-\frac{x^2}{2} = \frac{y^2}{2} + c_1$$

$$c_1 = -\frac{x^2}{2} - \frac{y^2}{2}$$

~~take~~ $\cancel{\frac{dy}{-xy}} = \frac{dz}{xz-2xy}$

$$2c_1 = -(x^2+y^2)$$

$$\boxed{c_1 = x^2+y^2}$$

$$c_2 = y^2 - yz$$

take $\frac{dy}{-xy} = \frac{dz}{xz-2xy}$

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$$

$$\cancel{-\frac{1}{y}\,dy = dy}$$

$$-z\,dy + 2y\,dy = y\,dz$$

$$2y\,dy = y\,dz + z\,dy$$

$$\int 2y\,dy = \int d(yz)$$

$$y^2 = yz + c$$

$$c_2 = y^2 - yz$$

$$\xi = f(x^2 + y^2,\; y^2 - yz) = 0$$

Ans

Q3: $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

take $\cancel{\dfrac{dx}{x^2 - yz} = \dfrac{dy}{y^2 - zx}}$

$$\cancel{\frac{dx}{x^2 - yz} - \frac{dy}{y^2 - zx} = 0}$$

$$\cancel{\frac{y^2\,dx - xz\,dx -}{(x^2 - yz)(y^2 - xz)}}$$

$x^2 - y^2 - yz + zx$

$(x+y)(x-y) + z(-y+x)$

$(x-y)(x+y+z)$

$$\frac{dx - dy}{x^2 - yz - y^2 + zx} = \frac{dy - dz}{y^2 - zx - z^2 + xy} = \frac{dz - dx}{z^2 - xy - x^2 + yz}$$

$$\frac{dx - dy}{(x-y)(x+y+z)} = \frac{dy - dz}{(y-z)(x+y+z)} = \frac{dz - dx}{(z-x)(x+y+z)}$$

First two Quantities,

$$\frac{dx - dy}{(x-y)\cancel{(x+y+z)}} = \frac{dy - dz}{(y-z)\cancel{(x+y+z)}}$$

$$\int \frac{dx - dy}{x-y} = \int \frac{dy - dz}{y-z}$$

$$\ln(x-y) = \ln(y-z) + \ln c_1$$

$$\ln(x-y) - \ln(y-z) = \ln c_1$$

$$\cancel{\ln} \frac{x-y}{y-z} = \cancel{\ln} c_1$$

$$c_1 = \frac{x-y}{y-z}$$

last two Quantities,

$$\frac{dy - dz}{(y-z)\cancel{(x+y+z)}} = \frac{dz - dx}{(z-x)\cancel{(x+y+z)}}$$

$$\frac{dy - dz}{y - z} = \frac{dz - dx}{z - x}$$

$$\ln(y-z) = \ln(z-x) + \ln C_2$$

$$\ln(y-z) - \ln(z-x) = \ln C_2$$

$$\cancel{\ln} \frac{y-z}{z-x} = \cancel{\ln} C_2$$

$$\boxed{C_2 = \frac{y-z}{z-x}}$$

Q4: $\frac{y^2z}{x}\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = y^2$

$P = \frac{y^2 z}{x}$, $Q = xz$, $R = y^2$

$$\frac{dx}{\frac{y^2z}{x}} = \frac{dy}{xz} = \frac{dz}{y^2}$$

~~da~~ First two

$$dx \times \frac{x}{y^2\cancel{z}} = \frac{dy}{x\cancel{z}}$$

$$\int x^2\,dx = \int y^2\,dy$$

$$\frac{x^3}{3} = \frac{y^3}{3} + C_1$$

$$\boxed{C_1 = x^3 - y^3}$$

Last two c

$$dx \times \frac{x}{\cancel{y^2 z}} = \frac{dz}{\cancel{yz}}$$

$$x\,dx = z\,dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c_2$$

$$\boxed{x^2 - z^2 = c_2}$$

$$f(x^3 - y^3,\ x^2 - z^2) = 0$$

Q5: $a\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = z.$

$P = a, \quad Q = a, \quad R = z$

$$\frac{dx}{a} = \frac{dy}{a} = \frac{dz}{z}$$

<u>First two</u>:

$$\frac{dx}{\cancel{a}} = \frac{dy}{\cancel{a}}$$

$$\int du = \int dy$$

$$x = y + c$$

$$\boxed{C_1 = x - y}$$

last two:

$$\frac{dy}{a} = \frac{dz}{z}$$

$$\frac{1}{a}\int dy = \int \frac{1}{z}\, dz$$

$$\frac{y}{a} = \ln z + C_2$$

$$y = a\ln z + a\ln c_2$$

$$y = \ln z^a + \ln c_2^a$$

$$y = a \ln z^a c_2$$

$\ln(\sin x)$

$$e^y = z^a c_2$$

$$\frac{e^y}{z^a} = c_2$$

$$\boxed{e^y z^{-a} = C_2}$$

$$f(x-y,\ e^y z^{-a}) = 0.$$

Q6: $\tan x \frac{\partial z}{\partial x} + \tan y \frac{\partial z}{\partial y} = \tan z$

$P = \tan x,\ Q = \tan y,\ R = \tan z$

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

<u>first two</u>:

$$\int \frac{dx}{\tan x} = \int \frac{dy}{\tan y}$$

$$\ln \sin x = \ln \sin y + \ln c_1$$

$$\ln \sin x - \ln \sin y = \ln C_1$$

$$\ln \frac{\sin x}{\sin y} = \ln C_1$$

$$\boxed{C_1 = \frac{\sin x}{\sin y}}$$

Last two:

$$\int \frac{dy}{\tan y} = \int \frac{dz}{\tan z}$$

$$\ln \sin y = \ln \sin z + \ln C_2$$

$$\ln \frac{\sin y}{\sin z} = \ln C_2$$

$$\boxed{C_2 = \frac{\sin y}{\sin z}}$$

$$f\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$$

Q7: $z\frac{\partial z}{\partial x} = -x$

$P = z,\ Q = 0,\ R = -x$

$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$.

~~first~~ take $\frac{\partial x}{z} = \frac{dz}{-x}$

$-x\,dx = z\,dz$.

$-\frac{x^2}{2} = \frac{z^2}{2} + c_1$

$-c_1 = \cancel{\frac{1}{2}}(x^2 + z^2)$

$\boxed{c_1 = x^2 + z^2}$

take $\frac{dy}{0} = \frac{dz}{-x}$

$dy = 0$

$\int dy = 0$

$$\boxed{y = c_2}$$

$$F(x^2+z^2, y) = 0$$

Q8: $(x^2+2y^2)p - xyq = xz$

$P = x^2+2y^2$, $Q = -xy$, $R = xz$

$$\frac{dx}{x^2+2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}$$

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}z}$$

$$-\ln y = \ln z + \ln c_1$$

$$-\ln y = \ln z c_1$$

$$\frac{1}{y} = z c_1$$

$$c_1 = \frac{1}{yz} \Rightarrow \boxed{c_1 = yz}$$

take $\frac{dx}{x^2+2y^2} = \frac{-1}{xy} dy$

$$\frac{dy}{dx} = -\frac{xy}{x^2+2y^2}$$

let $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{-vx^2}{x^2+2v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{x^2(-v)}{x^2(1+2v^2)}$$

$$x\frac{dv}{dx} = \frac{-v}{1+2v^2} - v$$

$$x\frac{dv}{dx} = -\left(\frac{v+v+2v^3}{1+2v^2}\right)$$

$$x\frac{dv}{dx} = -\left(\frac{2v^3+2v}{1+2v^2}\right)$$

$$-\frac{1}{x}dx = \frac{1+2v^2}{2v^3+2v}$$

$$y^2x^2 + y^4 = C_2$$

$$-\frac{1}{x}\,dx = \frac{2v^2+1}{2v^3+2v}$$

$$-\frac{3}{x}\,dx = \frac{6v^2+2+1}{2v^3+2v}$$

$$-\frac{3}{x}\,dx = \frac{6v^2+2}{2v^3+2v} + \frac{1}{2v^3+2v}$$

$$-\frac{3}{x}\,dx = \frac{6v^2+2}{2v^3+2v} + \frac{1}{2} \times \left[\frac{1}{v^3+v}\right]$$

ye incomplete hai, kuch milena complete...

LAGRANGE'S MULTIPLIER

$$\frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR}$$

l, m, n are multipliers in such a way that $lP + mQ + nR = 0$.

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{0}$$

Equating fourth quantity with any other quantity

$$l\,dx + m\,dy + n\,dz = 0$$

$$u_1 = c_1$$

Take l_1, m_1 & n_1

$$u_2 = c_2$$

$$c_1 = f(c_2)$$

Q1: $(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} = ly-mx$

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$

Take l, m, n as multipliers

$$\frac{l\,dx + m\,dy + n\,dz}{l(mz-ny) + m(nx-lz) + n(ly-mx)}$$

$$\frac{l\,dx + m\,dy + n\,dz}{0}$$

Equating above quantity with $\frac{dz}{ly-mx}$

$$\frac{l\,dx + m\,dy + n\,dz}{0} = \frac{dz}{ly-mx}$$

$$l\int dx + m\int dy + n\int dz = 0$$

$$lx + my + nz = c_1$$

Take again multiplier x, y, z

$$\frac{x\,dx + y\,dy + z\,dz}{x(mz - ny) + y(nx - lz) + z(ly - mx)}$$

$$\frac{x\,dx + y\,dy + z\,dz}{m\cancel{x}z - n\cancel{x}y + n\cancel{x}y - l\cancel{y}z + ly\cancel{z} - m\cancel{x}z}$$

$$\frac{x\,dx + y\,dy + z\,dz}{0}$$

Equating above quantity by

$$\frac{dx}{mz - ny}$$

$$\int x\,dx + y\,dy + z\,dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_2$$

$$x^2 + y^2 + z^2 = c_2$$

$$x^2 + y^2 + z^2 = f(lx + my + nz)$$

Q2: $x(z^2 - y^2)\dfrac{\partial z}{\partial x} + y(x^2 - z^2)\dfrac{\partial z}{\partial y} =$

$z(y^2 - x^2)$

$P = x(z^2 - y^2),\ Q = y(x^2 - z^2),\ R = z(y^2 - x^2)$

$$\frac{dx}{xz^2 - xy^2} = \frac{dy}{x^2y - yz^2} = \frac{dz}{y^2z - x^2z}$$

Taking x, y, z multiplier

$$\frac{x\,dx + y\,dy + z\,dz}{\cancel{x^2z^2} - \cancel{x^2y^2} + \cancel{x^2y^2} - \cancel{y^2z^2} + \cancel{y^2z^2} - \cancel{x^2z^2}}$$

$$\frac{x\,dx + y\,dy + z\,dz}{0}$$

Equating with $\dfrac{dx}{xz^2 - xy^2}$

$$\frac{dx}{xz^2 - xy^2} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$\int x\,dx + \int y\,dy + \int z\,dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_1$$

$$\boxed{x^2 + y^2 + z^2 = c_1}$$

Taking multiplier $1/x$, $1/y$, $1/z$.

$$\frac{1/x\,dx + 1/y\,dy + 1/z\,dz}{\frac{\cancel{x}z^2}{\cancel{x}} - \frac{\cancel{x}y^2}{\cancel{x}} + \frac{x^2\cancel{y}}{\cancel{y}} - \frac{\cancel{y}z^2}{\cancel{y}} + \frac{y^2\cancel{z}}{\cancel{z}} - \frac{x^2\cancel{z}}{\cancel{z}}}$$

$$\frac{1/x\,dx + 1/y\,dy + 1/z\,dz}{\cancel{z^2} - \cancel{y^2} + \cancel{x^2} - \cancel{z^2} + \cancel{y^2} - \cancel{x^2}}$$

$$\int \frac{1}{x}\,dx + \int \frac{1}{y}\,dy + \int \frac{1}{z}\,dz = 0$$

$$\log x + \log y + \log z = c_2$$

$$\cancel{\ln} xyz = \cancel{\ln} c_2$$

$$\boxed{c_2 = xyz}$$

$xyz = f(x^2 + y^2 + z^2)$.

Q3: $x(y-z)p + y(z-x)q = z(x-y)$.

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

Taking $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as first multiplier.

$$\frac{dx/x + dy/y + dz/z}{\frac{\cancel{xy}}{x} - \frac{\cancel{xz}}{x} + \frac{\cancel{yz}}{y} - \frac{\cancel{xy}}{y} + \frac{\cancel{xz}}{z} - \frac{\cancel{yz}}{z}}$$

$$\frac{dx/x + dy/y + dz/z}{0}$$

$$\int \frac{1}{x}\,dx + \int \frac{1}{y}\,dy + \int \frac{1}{z}\,dz = 0$$

$$\ln x + \ln y + \ln z = \ln c_1$$

$$\boxed{xyz = c_1}$$

Taking 2^{nd} multiplier 1.

$$\frac{dx + dy + dz}{\cancel{xy} - \cancel{xz} + \cancel{yz} - \cancel{xy} + \cancel{xz} - \cancel{yz}}$$

$$\frac{dx + dy + dz}{0}$$

$$\int dx + \int dy + \int dz = 0$$

$$x + y + z = c_2$$

$$x + y + z = f(xyz)$$

Q4: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

last two

$$\frac{dy}{\cancel{2x}y} = \frac{dz}{\cancel{2x}z}$$

$\ln y = \ln z + \ln c_1$

$\ln y = \ln z c_1$

$$\boxed{c_1 = \frac{y}{z}}$$

Taking multiplier x, y, z.

$$\frac{x\,dx + y\,dy + z\,dz}{x(x^2-y^2-z^2)+2xy^2+2xz^2}$$

$$\frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)}$$

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} = \frac{x\,dx+y\,dy+z\,dz}{x(x^2+y^2+z^2)}$$

$$\frac{x\,dx + y\,dy + z\,dz}{\cancel{x}(x^2+y^2+z^2)} = \frac{dz}{2\cancel{x}z}$$

$$\int \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2} = \int \frac{dz}{z}$$

$$\ln(x^2+y^2+z^2) = \ln z + \ln c_2$$

$$\frac{x^2+y^2+z^2}{z} = c_2$$

$$x^2+y^2+z^2 = zf\left(y/z\right)$$ Ans

Q5. $x^2p + y^2q = (x+y)z$.

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{xz+yz}$$

First two

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

$$-\frac{1}{x} = -\frac{1}{y} + c_1$$

$$-\frac{1}{x} + \frac{1}{y} = c_1$$

$$-\left(\frac{y-x}{xy}\right) = c_1$$

$$\boxed{C_1 = \frac{x \pm y}{xy}}$$

taking $1/x$, $1/y$ & $-1/z$ as multipliers

$$\frac{1/x\,dx}{1/x(x^2)} = \frac{1/y\,dy}{1/y\,y^2} = \frac{-1/z\,dz}{-1/z(xz+yz)}$$

$$\frac{1/x\,dx}{x} = \frac{1/y\,dy}{y} = \frac{-1/z\,dz}{-x-y}$$

$$\frac{1/x\,dx + 1/y\,dy - 1/z\,dz}{\cancel{x} + \cancel{y} - \cancel{x} - \cancel{y}}$$

$$\int \frac{1}{x}\,dx + \int \frac{1}{y}\,dy - \int \frac{1}{z}\,dz = 0$$

$$\ln x + \ln y - \ln z = \ln C_2$$

$$\cancel{\ln}\, xy = \cancel{\ln}\, z C_2$$

$$\boxed{\frac{xy}{z} = C_2}$$

Q6: $(y+z)p-(x+z)q = x-y$.

$$\frac{dx}{y+z} = \frac{dy}{-x-z} = \frac{dz}{x-y}$$

$xy+xz \quad -xy-yz \quad -xz+yz$

Taking 1 as multiplier.

$$\frac{dx+dy+dz}{\cancel{y}+\cancel{z}-\cancel{x}-\cancel{z}+\cancel{x}-\cancel{y}}$$

$$\int dx + \int dy + \int dz = 0$$

$$x+y+z = c_1$$

$$c_1 = x+y+z$$

Taking x, y and $-z$ as multipliers

$$\frac{x\,dx + y\,dy - z\,dz}{\cancel{xy}+\cancel{xz}-\cancel{xy}-\cancel{yz}-\cancel{xz}+\cancel{yz}}$$

$$\frac{x^2}{2} + \frac{y^2}{2} - \frac{z^2}{2} = c_2$$

$$\boxed{x^2+y^2-z^2 = c_2}$$

$$x+y+z = f(x^2+y^2-z^2)$$

Q7: $z\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x$

$$\frac{dx}{z} = \frac{dy}{y} = \frac{dz}{x}$$

Taking $\frac{dx}{z} = \frac{dz}{x}$

$$x\,dx = z\,dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c_1$$

$$\frac{x^2}{2} - \frac{z^2}{2} = c_1$$

$$c_1 = x^2 - z^2$$

$$z = \sqrt{x^2 - c_1}$$

$\ln\{x + \sqrt{}$

$$\frac{dy}{y} = \frac{dx}{z}$$

$$\frac{dy}{y} = \frac{dx}{\sqrt{x^2 - c^2}}$$

$$\int \frac{dy}{y} = \int \frac{dx}{\sqrt{x^2 + c^2}}$$

$$\ln y = \sinh^{-1} \frac{x}{\sqrt{c_1}} + c_2$$

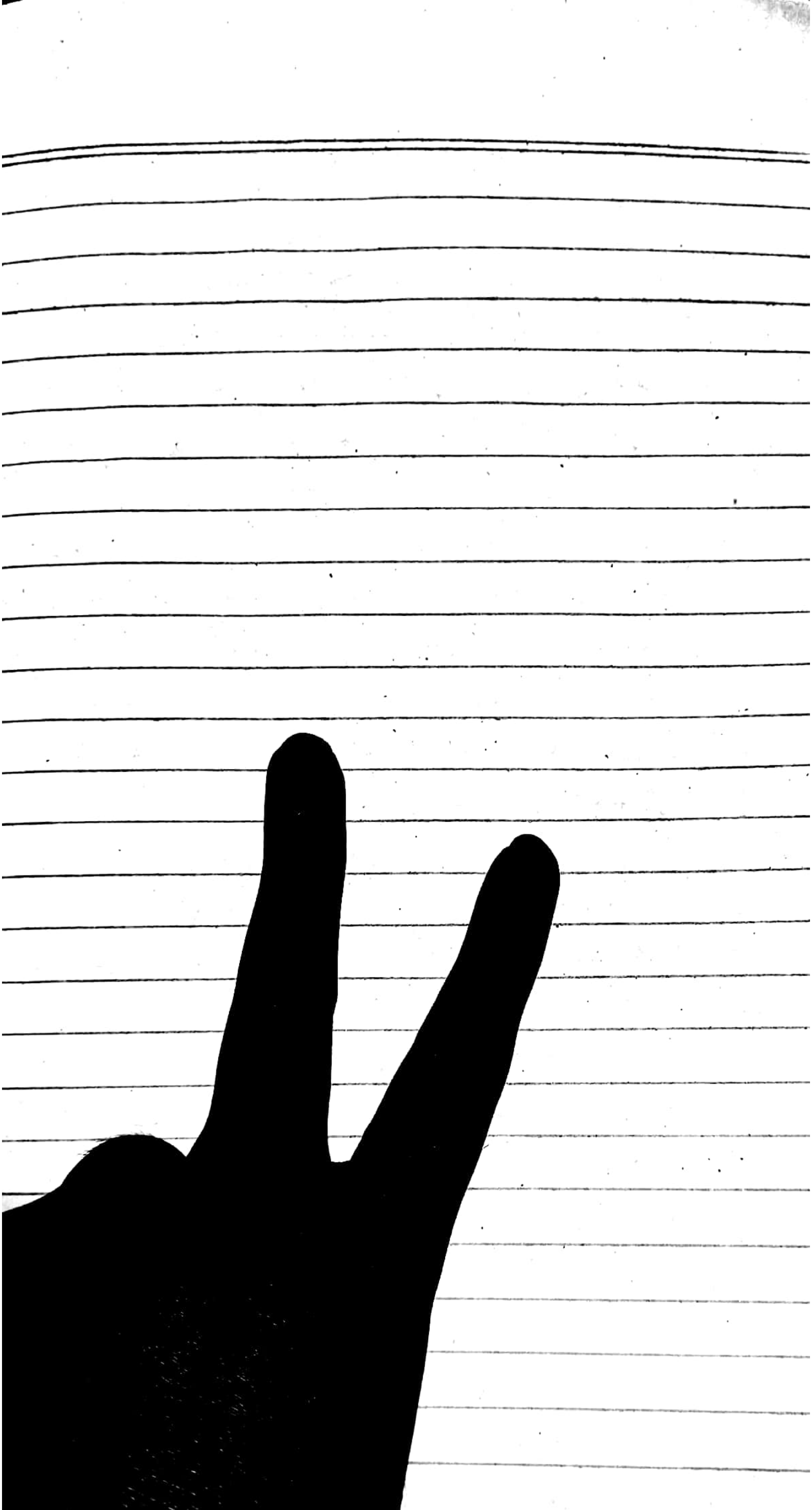
$$\sinh^{-1} \frac{x}{\sqrt{c_1}} - \ln y = c_2$$

Ans

Q8: $x(z - 2y^2)p + y(z - y^2 - 2x^3)q = z(z - y^2 - 2x^3).$

$$\frac{dx}{xz - 2xy^2} = \frac{dy}{yz - y^3 - 2yx^3} = \frac{dz}{z^2 - y^2 z - 2x^3 z}$$

complete on your own....



Laplace Transformation

Basic formula:

$$\mathcal{L}\, f(t) = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

$$f(t) \longrightarrow F(S)$$

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{t}\, dt$$

Q: $f(t) = 1$

$$\mathcal{L}\,[f(t)] = \mathcal{L}\,(1) = \int_0^\infty e^{-st}(1)\, dt$$

$$\mathcal{L}(1) = \lim_{b \to \infty} \int_0^b e^{-st}\, dt$$

$$\mathcal{L}(1) = \lim_{b \to \infty} \left[-\tfrac{1}{s} e^{-st} \right]_0^b$$

$$\mathcal{L}(1) = \lim_{b \to \infty} \left[\frac{-1}{se^{sb}} + \frac{1}{s(1)} \right]$$

Applying limit.

$$\mathcal{L}(1) = \frac{1}{s}$$

$\frac{\infty}{\infty}, \frac{0}{0}$ $u\int v - \int u' \int v$ $\frac{}{\infty}$

$1/s$

$\mathcal{L}(1) = \frac{1}{s}$ Ans.

Q2: $f(t) = t$.

$$\mathcal{L}[F(t)] = \mathcal{L}(t) = \int_0^\infty e^{-st}(t)\,dt$$

$$\mathcal{L}(t) = \left[t\left(\frac{e^{-st}}{-s}\right) - \int 1 \times \frac{e^{-st}}{-s} \right]_0^\infty$$

$$= \left[\frac{-t}{s} e^{-st} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right) \right]_0^\infty$$

$$= \left[\frac{-t}{s} e^{-st} - \frac{1}{s^2} e^{-st} \right]_0^\infty$$

$$= \lim_{b\to\infty} \left[\left(\frac{-b}{se^{sb}} - \frac{1}{s^2 e^{sb}} \right) - \left(\frac{-1}{s^2} \right) \right]$$

$$= \lim_{b\to\infty} \left[\frac{-b}{se^{sb}} - \frac{1}{s^2 e^{sb}} + \frac{1}{s^2} \right]$$

$$= \lim_{b\to\infty} \left[\frac{-1}{s^2 e^{sb}} - \frac{1}{s^2 e^{sb}} + \frac{1}{s^2} \right]$$

~~obtain~~ Applying limit

$$= \frac{1}{\infty} - \frac{1}{\infty} + \frac{1}{s^2}$$

$$\mathcal{L}(t) = \frac{1}{s^2}$$

Q3 $f(t) = e^{at}$

$$\mathcal{L}[f(t)] = \mathcal{L}(e^{at})$$

$$= \int_0^\infty e^{-st} e^{at} dt = \int_0^\infty e^{-st+at}$$

$$= \int_0^\infty e^{-t(s-a)} dt = -\frac{e^{-t(s-a)}}{s-a}\Bigg|_0^b$$

$$= \frac{1}{s-a}$$

(1) $\mathcal{L}(1) = \frac{1}{s}$

(2) $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$

(3) $\mathcal{L}(e^{at}) = \frac{1}{s-a}$ $\quad \mathcal{L}(e^{-at}) = \frac{1}{s+a}$

(4) $\mathcal{L}(\sin kt) = \frac{k}{s^2+k^2}$

(5) $\mathcal{L}(\cos kt) = \frac{s}{s^2+k^2}$

(6) $\mathcal{L}(\sinh kt) = \frac{k}{s^2-k^2}$

(7) $\mathcal{L}(\cosh kt) = \frac{s}{s^2-k^2}$

(8) $\mathcal{L}(e^{at}\cos bt) = \frac{s-a}{(s-a)^2+b^2}$

(9) $\mathcal{L}(e^{at}\sin bt) = \frac{b}{(s-a)^2+b^2}$

Practice Ques

Q1: $f(t) = 2t^4$

2: $f(t) = 4t - 10$

3: $f(t) = t^2 + 6t - 3$

4. $f(t) = (t+1)^3$

5. $f(t) = 1 + e^{4t}$

6: $f(t) = (1 + e^{2t})^2$

7. $f(t) = (e^t - e^{-t})^2$

8. $f(t) = \cos 5t + \sin 2t$

9. $f(t) = e^t \sinh t$

10. $f(t) = e^{-t} \cosh t$

11. $f(t) = \sin 2t \cos 2t$

12. $f(t) = \cos^2 t$

13. $f(t) = \sin(4t+5)$

14. $F(t) = 10 \cos(t - \pi/6)$

Gamma Properties

$$\Gamma(n) = \int_0^\infty t^{n+1} e^{-t}\, dt$$

1. $\Gamma(1) = 1$

2. $\Gamma(n+1) = n\Gamma(n)$

3. $\Gamma(n+1) = n!$

4. $\Gamma(1/2) = \sqrt{\pi}$

$$\mathcal{L}(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$$

Q: $\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$.

Q: $\mathcal{L}(t^n) = ?$

$$\mathcal{L}(t^n) = \frac{\Gamma(n+1)}{s^{n+1}} \quad \text{(Formula)}.$$

$$\Rightarrow \mathcal{L}(t^{1/2}) = \frac{\Gamma(1/2+1)}{s^{1/2+1}}$$

$$= \frac{1/2\, \Gamma(1/2)}{s^{3/2}}$$

$$\mathcal{L}(t^{1/2}) = \frac{1/2 \sqrt{\pi}}{s^{3/2}} \quad \underline{\underline{\text{Ans}}}$$

Q: $f(t) = t^{3/2}$

$$\mathcal{L}\, f(t) = \mathcal{L}(t^{3/2})$$

$$\mathcal{L}(t^{3/2}) = \frac{\Gamma(n+1)}{s^{n}+1}$$

$$= \frac{\Gamma\left(\frac{3}{2}+1\right)}{s^{\frac{3}{2}+1}}$$

$$= \frac{\frac{3}{2}\,\Gamma\left(\frac{3}{2}\right)}{s^{5/2}}$$

$$= \frac{\frac{3}{2}\,\Gamma\left(\frac{1}{2}+1\right)}{s^{5/2}}$$

$$= \frac{\frac{3}{2} \times \frac{1}{2}\,\Gamma\left(\frac{1}{2}\right)}{s^{5/2}}$$

$$= \frac{\frac{3}{4}\sqrt{\pi}}{s^{5/2}}$$

$$= \frac{3\sqrt{\pi}}{4s^{5/2}}$$

Q: $f(t) = 2t^{1/2} + 8t^{5/2}$

$$\mathcal{L}[f(t)] = 2\mathcal{L}(t^{1/2}) + 8\mathcal{L}(t^{5/2})$$

$$= 2\frac{\Gamma(1/2+1)}{s^{1/2+1}} + 8\frac{\Gamma(5/2+1)}{s^{5/2+1}}$$

$$= \frac{\cancel{2}\,\frac{1}{\cancel{2}}\Gamma(1/2)}{s^{3/2}} + \frac{\overset{4}{\cancel{8}}\,\frac{5}{\cancel{2}}\Gamma(5/2)}{s^{7/2}}$$

$$= \frac{\sqrt{\pi}}{s^{3/2}} + \frac{20}{s^{7/2}}\Gamma(3/2+1)$$

$$= \frac{\sqrt{\pi}}{s^{3/2}} + \frac{\overset{10}{\cancel{20}}}{s^{7/2}}\frac{3}{\cancel{2}}\Gamma(3/2)$$

$$= \frac{\sqrt{\pi}}{s^{3/2}} + \frac{30}{s^{7/2}}\,\frac{1}{2}\Gamma(1/2+1)$$

$$= \frac{\sqrt{\pi}}{s^{3/2}} + \frac{\overset{15}{\cancel{30}}}{s^{7/2}} \times \frac{1}{\cancel{2}}\Gamma(1/2)$$

$$= \frac{\sqrt{\pi}}{s^{3/2}} + \frac{15\sqrt{\pi}}{s^{7/2}}$$ Ans

Practice Questions.

Q1: $f(t) = 2t^4$

$$\mathcal{L}[f(t)] = 2\mathcal{L}(t^4)$$

$$\because \mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$\therefore$

$$\mathcal{L}[f(t)] = 2\frac{4!}{s^{4+1}}$$

$$\mathcal{L}[f(t)] = \frac{2 \times 4!}{s^5}$$

Ans.

Q3: $f(t) = t^2 + 6t - 3$

$\mathcal{L}[f(t)] = \mathcal{L}(t^2) + 6\mathcal{L}(t) - 3$

$= \frac{2!}{s^{2+1}} + \frac{6}{s^2} - \frac{3}{s}$

$= \frac{2}{s^3} + \frac{6}{s^2} - \frac{3}{s}$ Ans

Q4: $f(t) = (t+1)^3$

$\because (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$f(t) = t^3 + 3t^2 + 3t + 1$

$\mathcal{L} f(t) = \mathcal{L}(t^3) + 3\mathcal{L}(t^2) + 3\mathcal{L}(t) + \mathcal{L}(1)$

$\mathcal{L}[f(t)] = \frac{3!}{s^{3+1}} + \frac{3 \times 2!}{s^{2+1}} + \frac{3}{s^2} + \frac{1}{s}$

$= \frac{3!}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}$

$= \frac{6}{s^4} + \frac{6}{s^3} + 3/s^2 + 1/s$ Ans

Q5: $f(t) = 1 + e^{4t}$

$$\mathcal{L}[f(t)] = \mathcal{L}(1) + \mathcal{L}(e^{4t})$$

$$\mathcal{L} f(t) = \frac{1}{s} + \frac{1}{s-4}$$ Ans

Q6: $f(t) = (1 + e^{2t})^2$

$$f(t) = 1 + 2e^{2t} + e^{4t}$$

$$\mathcal{L} f(t) = \mathcal{L}(1) + 2\mathcal{L}(e^{2t}) + \mathcal{L}(e^{4t})$$

$$= \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$$ Ans

Q2: $f(t) = 4t - 10$

$\mathcal{L}\, f(t) = 4\,\mathcal{L}(t) - \mathcal{L}(10)$

$\mathcal{L}[f(t)] = \frac{4}{s^2} - \frac{10}{s}$ Ans

Q7: $f(t) = (e^{t} - e^{-t})^2$

$f(t) = e^{2t} - 2\frac{e^t}{e^t} + e^{-2t}$

$f(t) = e^{2t} - 2 + e^{-2t}$

$\mathcal{L}[f(t)] = \mathcal{L}(e^{2t}) - \mathcal{L}(2) + \mathcal{L}(e^{-2t})$

$= \frac{1}{s-2} - \frac{2}{s} + \frac{1}{s+2}$ Ans

Q8: $f(t) = \cos 5t + \sin 2t$.

$\mathcal{L}[f(t)] = \mathcal{L}(\cos 5t) + \mathcal{L}(\sin 2t)$

$= \frac{s}{s^2+25} + \frac{2}{s^2+4}$ Ans

Q9: $f(t) = e^{t} \sinh t.$

$$f(t) = e^{t}\left(\frac{e^{t} - e^{-t}}{2}\right)$$

$$f(t) = \frac{e^{2t}}{2} - \frac{e^{t-t}}{2}$$

$$f(t) = \frac{e^{2t}}{2} - \frac{1}{2}$$

$$f(t) = \frac{1}{2}\mathcal{L}(e^{2t}) - \mathcal{L}(1/2)$$

$$f(t) = \frac{1}{2}\left(\frac{1}{s-2}\right) - \frac{1/2}{s}$$

$$= \frac{1}{2s-4} - \frac{1}{2s}$$

$\cos 2\theta = 1 - 2\cos^2 t$

$\cos^2 t = \frac{1 - \cos 2\theta}{2}$

$$f(t) = \frac{1}{2}\left[\sin(2t+2t) + \sin(2t-2t)\right]$$

$$f(t) = \frac{1}{2}(\sin 4t)$$

$$\mathcal{L}[f(t)] = \frac{1}{2}\mathcal{L}(\sin 4t)$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{4}^{\,2}}{s^2+16}\right)$$

$$= \frac{2}{s^2+16} \quad \text{Ans}$$

Q12: $f(t) = \cos^2 t$

$$= \frac{1 - \cos 2t}{2}$$

$$f(t) = \frac{1 - \cos 2t}{2S}$$

$$\mathcal{L}[f(t)] = \mathcal{L}(1/2) - \frac{1}{2}\mathcal{L}(\cos 2t)$$

Q10 : $f(t) = e^{-t} \cosh t$

$$f(t) = e^{-t}\left(\frac{e^{t} + e^{-t}}{2}\right)$$

$$f(t) = \frac{e^{-t+t}}{2} + \frac{e^{-t-t}}{2}$$

$$f(t) = \frac{1}{2} + \frac{e^{-2t}}{2}$$

$$\mathcal{L}[f(t)] = \mathcal{L}(1/2) + \frac{1}{2}\mathcal{L}(e^{-2t})$$

$$= \frac{1/2}{s} + \frac{1}{2}\left(\frac{1}{s+2}\right)$$

$$= \frac{1}{2s} + \frac{1}{2s+4}$$ Ans

Q11. $f(t) = \sin 2t \cos 2t$

$$\because \sin\alpha \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\sin(at+b) = \frac{a\cos(b) + s(\sin(b))}{a^2+s^2}$$

$$\cancel{\cos(at+b) = \frac{s\cos b - a\sin b}{a^2+s^2}}$$

$$= \frac{1/2}{s} - \frac{1}{2}\left(\frac{s}{s^2+4}\right)$$

$$= \frac{1}{2s} - \frac{s}{2s^2+8} \quad \text{Ans}$$

Q13: $f(t) = \sin(4t+5)$

$$\because \sin(at+b) = \frac{a\cos(b) + s(\sin(b))}{a^2+s^2}$$

$$\mathcal{L}\, f(t) = \mathcal{L}\, \sin(4t+5)$$

$$= \frac{4\cos 5 + s\sin 5}{16+s^2}$$

Q14: $f(t) = 10\cos(t - \pi/6)$

$$\mathcal{L}[f(t)] = 10\,\mathcal{L}\,\cos(t-\pi/6)$$

$$\because \cos(a-b) = \cos a\cos b + \sin a \sin b$$

$$= 10\,\mathcal{L}\left[\cos t\cos \pi/6 + \sin t \sin \pi/6\right]$$

$$\mathcal{L}\{f(t)\} = 10\,\mathcal{L}\left[\frac{\sqrt{3}}{2}\cos t + \frac{1}{2}\sin t\right]$$

$$= 5\sqrt{3}\,\mathcal{L}(\cos t) + 5\,\mathcal{L}(\sin t)$$

$$= 5\sqrt{3}\left(\frac{s}{s^2+1}\right) + 5\left(\frac{1}{s^2+1}\right)$$

$$= \frac{5\sqrt{3}\times s}{s^2+1} + \frac{5}{s^2+1}$$

Inverse Laplace Trans.

$f(t) = \mathcal{L}^{-1}[F(s)]$

$1 = \mathcal{L}^{-1}[1/s]$

$e^{at} = \mathcal{L}^{-1}[1/s-a]$

$\sin kt = \mathcal{L}^{-1}[k/s^2+k^2]$

$\cos kt = \mathcal{L}^{-1}[s/s^2+k^2]$

$\sinh kt = \mathcal{L}^{-1}[k/s^2-k^2]$

$\cosh kt = \mathcal{L}^{-1}[s/s^2-k^2]$

$t^n = \mathcal{L}^{-1}\left[\frac{n!}{s^{n+1}}\right]$

Q1: $\mathcal{L}^{-1}\left[{}^{1}/{s^5}\right]$

$n+1 = 5 \Rightarrow \boxed{n=4}$

$$\mathcal{L}^{-1}\left(\frac{1}{s^5}\right) = \frac{1}{4!} \quad \frac{4!}{s^{4+1}}$$

$$= \frac{1}{4!} t^4$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^5}\right) = \frac{t^4}{24}$$ Ans

Q2: $\mathcal{L}^{-1}\left[{}^{1}/{s^2+7}\right]$

$K^2 = 7 \Rightarrow K = \sqrt{7}$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2+7}\right] = \frac{1}{\sqrt{7}} \mathcal{L}^{-1}\left[\frac{\sqrt{7}}{s^2+7}\right]$$

$$= \frac{1}{\sqrt{7}} \sin\sqrt{7}\,t$$ Ans

Q3: $\mathcal{L}^{-1}\left[\dfrac{1}{4s+1}\right]$

~~aaa~~/4

$\dfrac{1}{4}\mathcal{L}^{-1}\left[\dfrac{1}{s+1/4}\right]$

$= \dfrac{1}{4}e^{-\frac{1}{4}t}$

Ans

Q4: $\mathcal{L}^{-1}\left[\dfrac{1}{s^2}+\dfrac{48}{s^5}\right]$

$\mathcal{L}^{-1}\left[\dfrac{1}{s^2}+\dfrac{48}{s^5}\right] = \left(\dfrac{1}{s^2} + \dfrac{1}{s^5}\times\dfrac{48}{\cancel{6}}\right)$

$= n = 1 \quad , \quad n = 4$

$= \left(\dfrac{1!}{1!}\times\dfrac{1}{s^2} + \dfrac{4!}{4!}\,\dfrac{48}{s^5}\right)$

$= t + 2t^4 \quad \dfrac{48}{24} = 2$

Ans

Q5: $\mathcal{L}^{-1}\left[\frac{-2s+6}{s^2+4}\right]$.

$= \mathcal{L}^{-1}\left[\frac{-2s}{s^2+4} + \frac{6}{s^2+4}\right]$

$= -2\,\mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] + \frac{6}{2}\,\mathcal{L}^{-1}\left[\frac{\kappa 2}{s^2+4}\right]$

$\Rightarrow \kappa^2 = 4 \Rightarrow \kappa = 2 \;,\; \kappa^2 = 4 \Rightarrow \kappa = 2$

$= -2\cos 2t + 3\sin 2t$. Ans

Q6: $\mathcal{L}^{-1}\left[\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right]$

$\frac{s^2+6s+9}{(s-1)(s-2)(s+4)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s+4}$ — eq(i)

$s^2+6s+9 = A(s-2)(s+4) + B(s-1)(s+4) +$
$C(s-1)(s-2)$.

If $s = 1$,

$1+6+9 = A(-1)(5)$.

$$\boxed{\frac{16}{-5} = A}$$

For B, putting $s = 2$

$$4 + 12 + 9 = B(1)(6)$$

$$\boxed{\frac{25}{6} = B}$$

For C, putting $s = -4$

$$16 - 24 + 9 = C(-5)(-6)$$

$$\boxed{\frac{1}{30} = C}$$

eq(i) becomes,

$$= \mathcal{L}^{-1}\left[-\frac{16}{5}\left(\frac{1}{s-1}\right)\right] + \mathcal{L}^{-1}\left[\frac{25}{6}\left(\frac{1}{s-2}\right)\right] + \mathcal{L}^{-1}\left[\frac{1}{30}\left(\frac{1}{s+4}\right)\right]$$

$$= -\frac{16}{5}e^{t} + \frac{25}{6}e^{2t} + \frac{1}{30}e^{-4t}$$

Ans

Q7: $\mathcal{L}^{-1}\left[\dfrac{s}{s^2+2s-3}\right]$

$$= \mathcal{L}^{-1}\left[\frac{s}{s^2+3s-s-3}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{s}{(s+3)(s-1)}\right]$$

$$\Rightarrow \frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$$

$$s = A(s-1) + B(s+3)$$

For A, putting $s = -3$.

$$+3 = A(-4)$$

$$\boxed{A = \frac{3}{4}}$$

For B, putting $s = 1$.

$$1 = B(4)$$

$$\boxed{B = 1/4}$$

$$C = \mathcal{L}^{-1}\left[\frac{3}{4}\left(\frac{1}{s+3}\right)\right] + \mathcal{L}^{-1}\left[\frac{1}{4}\left(\frac{1}{s-1}\right)\right]$$

$$= \frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$$ Ans

Q8: $\mathcal{L}^{-1}\left[\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right]$

$$= \frac{s^2+1}{s(s-1)(s+1)(s-2)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+1} + \frac{D}{s-2}$$

$$s^2+1 = A(s-1)(s+1)(s-2) + B(s)(s+1)(s-2) + C(s)(s-1)(s-2) + D(s)(s-1)(s+1)$$

For A, putting $s = 0$.

$$1 = A(-1)(1)(-2)$$

$$\boxed{A = 1/2}$$

For B, putting $s = 1$.

$$2 = B(1)(2)(-1)$$

$$\boxed{B = -1}$$

For C, putting $s = -1$.

$2 = C(-1)(-2)(-3)$

$2 = -C6$

$C = -\frac{\cancel{2}}{\cancel{6}3} \Rightarrow \boxed{C = -\frac{1}{3}}$

For D, putting $s = 2$:

$4+1 = D(2)(1)(3)$

$5 = D6$

$\boxed{D = \frac{5}{6}}$

$$\frac{s^2+1}{s(s-1)(s+1)(s-2)} = \frac{1}{2}\left(\frac{1}{s}\right) - \frac{1}{s-1} - \frac{1}{3}\left(\frac{1}{s+1}\right) + \frac{5}{6}\left(\frac{1}{s-2}\right)$$

Applying $\mathcal{L}^{-1}$

$$= \frac{1}{2}(1) - e^{at} - \frac{1}{3}e^{-t} + \frac{5}{6}e^{2t}$$

Transform. Of derivative.

$\mathcal{L}[f'(t)] = sF(s) - f(0)$

$\mathcal{L}[f''(t)] = s^2F(s) - sf(0) - (1)f'(0)$

$\mathcal{L}[f'''(t)] = s^3F(s) - s^2f(0) - sf'(0) - (1)f''(0)$

$\mathcal{L}[f''''(t)] = s^4F(s) - s^3f(0) - s^2f'(0) - sf''(0)$
$- (1)f'''(0)$

Paper wale question started!

Q:- $\frac{dy}{dt} + 3y = 13\sin 2t$; $y(0) = 6$.

Phle hm $Y(s)$ ki form me le aege question ko.

⇒ Applying laplace,

$\mathcal{L}\left(\frac{dy}{dt}\right) + 3\mathcal{L}(y) = 13\mathcal{L}(\sin 2t)$

↳ This is $\mathcal{L}f'(t)$, $= sF(s) - f(0)$

$sY(s) - f(0) + 3Y(s) = \frac{26}{s^2+4}$

$sY(s) - 6 + 3Y(s) = \frac{26}{s^2+4}$

$Y(s)[s+3] - 6 = \frac{26}{s^2+4}$

$Y(s) = \frac{26}{(s^2+4)(s+3)} + \frac{6}{(s+3)}$ —— eq(i)

$\frac{18}{3}$

$26-8$

Taking, $\frac{26}{(s^2+4)(s+3)}$

$$\frac{26}{(s^2+4)(s+3)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4}.$$

$$26 = A(s^2+4) + (Bs+C)(s+3)$$

if $s=-3$

$$26 = A(9+4) \Rightarrow \boxed{A=2}$$

for B, consider,

$$26 = A(s^2+4) + (Bs+C)(s+3)$$

$$26 = As^2 + 4A + 3Bs + 3C + Bs^2 + Cs.$$

Putting $A=2$ & $s=0$

$$26 = 8 + 3C$$

$$\frac{26-8}{3} = C \Rightarrow \boxed{C=6}$$

Now, finding B.

Considers

$$As^2 + 4A + 3Bs + 3C + Bs^2 + Cs = 26$$

Ek square wali term bnegi, ek linear or ek constant,

$$s^2(A+B) + s(3B+C) + 4A + 3C = 26$$

2 ki or 1 ki koi term nhi he R.H.S pe to wo equal to 0 hoga.

$A + B = 0$

$3B + C = 0$

$4A + 3C = 26$

As we have to find B. Taking,

$3B + C = 0$

Putting $C = 6$.

$3B + 6 = 0$

$$3(B+2)=0$$

$$\boxed{B=-2}$$

$$\frac{26}{(s^2+4)(s+3)} = \frac{2}{s+3} + \frac{(-2s)+6}{s^2+4}$$

so,

$$Y(s) = \frac{6}{(s+3)} + \frac{2}{s+3} - \frac{2s-6}{s^2+4}$$

$$Y(s) = \frac{8}{s+3} - \frac{2s}{s^2+4} + \frac{6}{s^2+4}$$

$$\mathcal{L}^{-1}Y(s) = 8\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) - 2\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right)$$

$$+ 6\mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right)$$

$$y(t) = 8e^{-3t} - 2\cos 2t + 3\sin 2t.$$