Pg 1

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Course Code : PH-122.

Batch : 2020.

Q1

(a)

That there is net force on the object directed north.

(b)

Work

(c)

1

(d)

0

Q1 (e):

If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy-plane from (0,0) to (1,4) along a curve, $y = 4x^2$. Calculate the work done ....

Solution:

$$\boxed{y = 4x^2}$$

$$\frac{dy}{dx} = 4\frac{d}{dx}(x^2)$$

$$\boxed{dy = 8x\,dx}$$

$$W = \int F \cdot dr$$

$$W = \int (2x^2y\hat{i} + 3xy\hat{j})\,dr$$

$$W = \int 2x^2y\,dx + 3xy\,dy$$

$$= \int 2x^2(4x^2)\,dx + 3x(4x^2)(8x\,dx)$$

$$= \int 8x^4\,dx + 96x^4\,dx$$

$$W = 8\int x^4\, dx + 96\int x^4\, dx$$

$$W = \left[\frac{8x^5}{5} + \frac{96x^5}{5}\right]_0^1$$

$$W = \left(\frac{8(1)^5}{5} + \frac{96(1)^5}{5}\right) - \left(\frac{8(0)}{5} + \frac{96(0)}{5}\right)$$

$$W = \frac{8}{5} + \frac{96}{5}$$

$$\boxed{W = \frac{104}{5}\,J}$$

Ans

Q1 (f):

Electric Field:

An electric field is a physical field that surrounds each electric charge and exerts force on all other charges in the field, either attracting or repelling them.

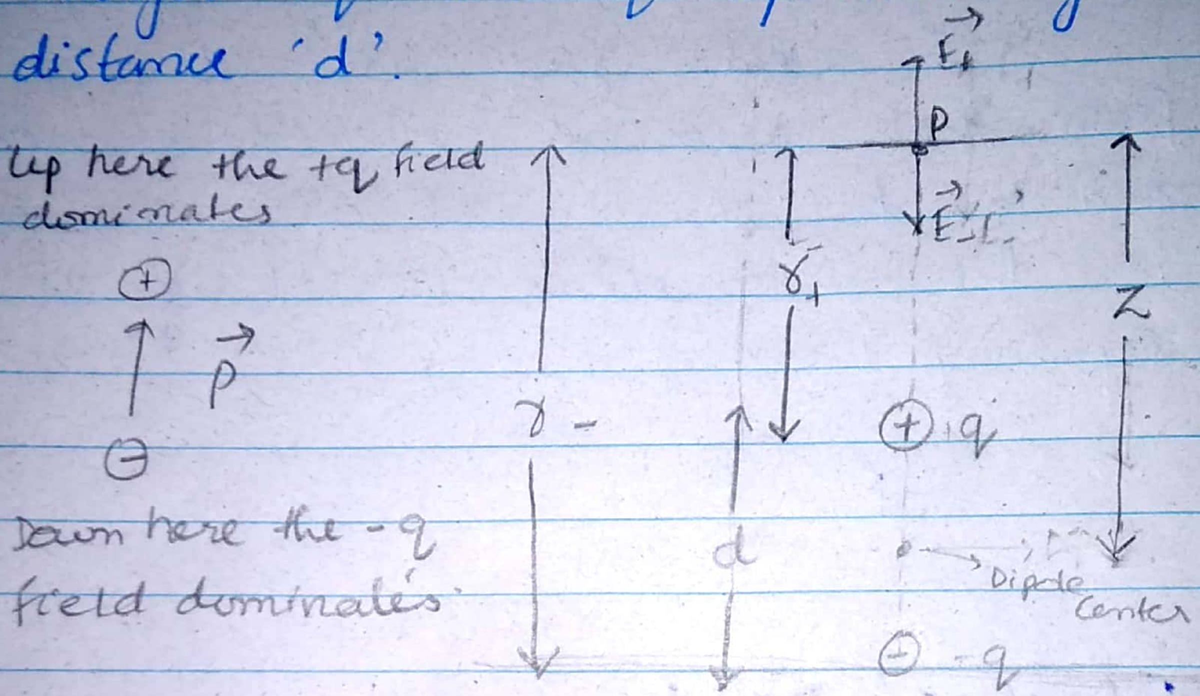
$$E = \frac{F}{q_0}$$

Electric Dipole:

When positive and negative charge is placed at a distance 'd' so we can say that it is creating an electric dipole. It is denoted by $\vec{P}$.

Electric field of a Dipole:

Consider an electric dipole with charges $+q$ and $-q$ separated by a distance 'd'.



$$E = E_+ - E_-$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r)_{(-)}^2}$$

$$E = \frac{q}{4\pi\epsilon_0\left(z - \frac{1}{2}d\right)^2} - \frac{q}{4\pi\epsilon_0\left(z + \frac{1}{2}d\right)^2}$$

$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right]$$

$$E = \frac{q}{4\pi\epsilon_0 z^2} \times \frac{2d/z}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}$$

$$E = \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{\left(\left(1-\frac{d}{2z}\right)^2\right)^2}$$

$$\boxed{E = \frac{1}{2\pi\epsilon_0} \times \frac{qd}{z^3}}$$

or

$$\boxed{E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}}$$

Does Electric field lines intersect?

Electric lines of force can be defined as a way or a direction, it can be straight or curved so that at any point the tangent gives the path of the electric field's strength.

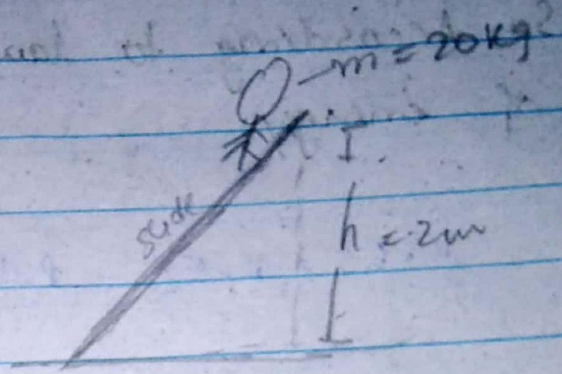
Electric lines of force never intersect each other because, at the point of intersection, two tangents can be drawn to the two lines of force. this means two direction of the electric field at the point of intersection, which is not possible.

Why like charges repel & unlike attract?

Since the electron is much smaller and lighter than a proton, when they are attracted to each other due to their unlike charges, the electron usually does ~~not~~ most of the moving. This is because the protons have more mass and are harder to get moving.

Q2:

(a)



Data:

$m = 20kg$

$h = 2m$

$v = ?$

$\Delta E = ?$

Solution:

a(i):

First we calculate energy at Top:

$\boxed{K.E_1 = 0}$

$P.E_1 = mgh$

$P.E_1 = 20 \times 9.8 \times 2$

$\boxed{P.E_1 = 392J}$

Now, at bottom:

$P.E_2 = 0$

$K.E_2 = \frac{1}{2} mv^2$

So, According to law of conservation of energy:

$$K.E_1 + K.E_2 = P.E_1 + P.E_2$$

$$0 + \frac{1}{2}mv^2 = 392 + 0$$

$$\frac{1}{2}mv^2 = 392$$

$$mv^2 = 784$$

$$v^2 = \frac{784}{m}$$

$$v^2 = \frac{784}{20}$$

$$v^2 = 39.2$$

$$\boxed{v = 6.26\,m/s}$$

Ans of part (ai).

a(ii):

In the presence of friction:

$\because$ $V = 3m/s$.

So,

$$K.E = \frac{1}{2}mv^2$$

$$K.E = \frac{1}{2}(20)(3)^2$$

$$\boxed{K.E = 90J}$$

The mechanical energy lost is given by:

$$\Delta E = 392 - 90$$

$\boxed{\Delta E = 302J}$ ans of a(ii)

Q2 (b):

Solution:

$x = 2\cos t \Rightarrow dx = -2\sin t\,dt$

$y = 3\sin t \Rightarrow dy = 3\cos t\,dt$

~~$F\cdot dr = (2\cos t - 9\sin t)($~~ -

$\oint F\cdot dr = (x-3y)\,dx + (y-2x)\,dy$

$= (2\cos t - 9\sin t)(-2\sin t\,dt) + (3\sin t - 4\cos t)(3\cos t\,dt)$

$\oint F\cdot dr = -4\cos t\sin t\,dt + 18\sin^2 t\,dt + 9\sin t\cos t\,dt - 12\cos^2 t\,dt$

$\oint F\cdot dr = 4\int \cos t(-\sin t)\,dt + \frac{\cancel{18}^{\,9}}{\cancel{2}}\int (1-\cos 2t)\,dt + 9\int \sin t(\cos t)\,dt - \frac{\cancel{12}^{\,6}}{\cancel{2}}\int (1+\cos 2t)\,dt$

$\oint F\cdot dr = \left[\frac{\cancel{4}^{\,2}\cos^2 t}{\cancel{2}} + 9t + \frac{9}{2}\sin 2t - 9\sin^2 t - 6t - \frac{\cancel{6}^{\,3}}{\cancel{2}}\sin 2t\right]_0^{2\pi}$

$\oint F\cdot dr = \left[2\cos^2 t + 9t + \frac{9}{2}\sin 2t - 9\sin^2 t - 6t - 3\sin 2t\right]_0^{2\pi}$

$$\oint_c F \cdot dr = (2 - 0 + 6\pi + 0 - 0) - (2 - 0 + 0 + 0 - 0)$$

$$\oint_c F \cdot dr = \cancel{2} + 6\pi - \cancel{2}$$

$$\boxed{\oint F \cdot dr = 6\pi}$$ Ans

Q2 (c):

Evaluate $|(\text{grad } U).(\text{grad } V)|$ if $U = 3x^2y$ and $V = xz^2 - 2y$.

For grad U (∇U):

We know that,

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

So,

$$\nabla U = \frac{\partial}{\partial x}(3x^2y)\hat{i} + \frac{\partial}{\partial y}(3x^2y)\hat{j} + \frac{\partial}{\partial z}(3x^2y)\hat{k}.$$

$$\boxed{\nabla U = 6xy\hat{i} + 3x^2\hat{j}}$$

For grad V (∇V):

$$\nabla V = \frac{\partial}{\partial x}(xz^2 - 2y)\hat{i} + \frac{\partial}{\partial y}(xz^2 - 2y)\hat{j} + \frac{\partial}{\partial z}(xz^2 - 2y)\hat{k}$$

$$\nabla V = z^2\hat{i} - 2\hat{j} + 2xz\hat{k}$$

Now,

$$\nabla U \cdot \nabla V = (6xy\hat{i} + 3x^2\hat{j}) \cdot (z^2\hat{i} - 2\hat{j} + 2xz\hat{k})$$

$$\nabla U \cdot \nabla V = (6xy)(z^2) + (3x^2)(-2)$$

$$\boxed{\nabla U \cdot \nabla V = 6xyz^2 - 6x^2}$$ Ans