NED UNIVERSITY OF ENGINEERING & TECHNOLOGY

{SSIC- SE (PP)} BATCH-2021-TE (BCIT) FALL SEMESTER BATCH-2020

Handout #2 P&S (MT-331)

RANDOM VARIABLE

1. The probability distribution of the discrete random variable X is Find the mean of X.

$$f(x) = \binom{3}{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{3-x}, \quad x = 0, 1, 2, 3.$$

2. Roulette wheel is divided in to 6 sectors of unequal area marked with the no's 1, 2, 3, 4, 5, & 6. The wheel is spun & X is the random variable the number on which the wheel stops. The probability of X is as follows:

x	1	2	3	4	5	6
P(X=x)	1/16	3/16	k	1/4	3/16	1/16

Find the value of 'k' and find (i) $E(3x-5)$ (ii) $E(6x^2 + 6x - 10)$

3. Suppose that an antique jewelry dealer is interested in purchasing a gold necklace for which the probabilities are 0.22, 0.36, 0.28, and 0.14, respectively, that she will be able to sell it for a profit of $250, sell it for a profit of $150, break even, or sell it for a loss of $150.What is her expected profit?

4. An attendant at a car wash is paid according to the number of cars that pass through. Suppose the probabilities are 1/12, 1/12, 1/4, 1/4, 1/6, and 1/6, respectively, that the attendant receives $7, $9, $11, $13, $15, or $17 between 4:00 P.M. and 5:00 P.M. on any sunny Friday. Find the attendant' s expected earnings for this particular period.

5. The random variable X, representing the number of errors per 100 lines of software code, has the

x	2	3	4	5	6
$f(x)$	0.01	0.25	0.4	0.3	0.04

Following probability distribution: find the variance of X.

6. Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1, respectively, that 0, 1, 2, or 3 power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable X representing the number of power failures striking this subdivision.

7. For a laboratory assignment, if the equipment is working, the density function of the observed outcome X is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{otherwise}. \end{cases}$$

Find the variance and standard deviation of X.

8. The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable Y = 3X −2, where X has the density function

$$f(x) = \begin{cases} \frac{1}{4}e^{-x/4}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean and variance of the random variable Y .

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9. The total time, measured in units of 100 hours, that a teenager runs her hair dryer over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Evaluate the mean of the random variable $Y = 60X^2 + 39X$, where Y is equal to the number of kilowatt hours expended annually.

10. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a Period of one year is a continuous random variable X that has the density function. Find the probability that over a period of one year, a family runs their vacuum cleaner

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

I. Less than 120 hours.
II. Between 50 and 100 hours.

11. The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has the density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

I. Show that $P(0 < X < 1) = 1$.
II. Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

12. A continuous random variable X that can assume values between x = 2 and x = 5 has a density function given by $f(x) = 2(1 + x)/27$. Find

I. $P(X < 4)$
II. $P(3 \le X < 4)$.
III. Find F(x), and use it to evaluate $P(3 \le X < 4)$.

13. Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

I. Evaluate k.
II. (b) Find F(x) and use it to evaluate $P(0.3 < X < 0.6)$.

14. Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

I. Verify that f(x) is a density function.

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II. Find $P(0 < X \le 1)$.

III. For the density function, find $F(x)$, and use it to evaluate $P(0 < X \le 1)$.

15. Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$, where X has the density function

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

16. The number of messages sent per hour over a computer network has the following distribution:

x = number of messages	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages sent per hour.

17. The monthly demand for transistors is known to have the following probability distribution.

Demand (n) :	1	2	3	4	5	6
Probability (P) :	0.10	0.15	0.20	0.25	0.18	0.12

Determine the expected demand for transistors. Also obtain the variance?

18. Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is

$$f(x) = \begin{cases} \frac{20{,}000}{x^3}, & x > 100, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected life of this type of device.

19. The density function of the time Z in minutes between calls to an electrical supply store is given by

$$f(z) = \frac{1}{10}\exp(-z/10) \quad ; z > 0$$

i. What is the mean time between calls?

ii. What is the variance in the time between calls?

iii. What is the probability that the time between calls exceeds the mean?

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