Programmology

Proposition:

A proposition is a statement that is either true or false, but not both.

Statement
- Declarative: this is apple
- Imperative: eat this apple.
- Interrogative: Is this apple?

- Today is Friday
- Washington is the capital of America.

True, false
↓
Truth value of proposition

Notation: Proposition can be denoted by any letter especially p, q, r, s, t.

p: Today is Friday.

Propositional Logic:

The area of logic that deals with propositions.

Negation:

Opposite to proposition.

eg:

p: Today is Friday

~p: Today is not Friday

OR

It isn't the case that today is friday.

q: Today is not Friday

~q: Today is Friday.

It makes True → False & False → True.

let p be a proposition. The negation of p, denoted by ~p, is the statement "It is not the case that p".

→ the proposition ~p is read as "not p".

Truth Table: Method to show the relationship of propositions (negations)

p (input)	~p (output)
T	F
F	T

Example:

$20 < 11

a- Statement? yes

b- Proposition? yes

c- Truth value? false

$y > 5$

a- Statement? yes

b- proposition? no

c- Truth value? depends upon 'y' which isn't specified.

What time is it?

a- statement? no

b- proposition? no

A proposition has to be a statement.

Logical Operators (Connectives)

They are used to form new compound propositions from two or more existing propositions. The logical operators are also called connectives.

p: Today is Friday

q: It is raining today.

Today is Friday and it is raining today.

↳ logical operator / connective

Compound Proposition.

1. Conjunction (AND)
2. Disjunction (OR)
3. Exclusive or (XOR)
4. Implication (if-then)
5. Biconditional (if and only if)

Conjunction (AND Operator)

Let p & q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q" which is true when both p and q are true and is false otherwise.

p: Today is Friday.
q: It is raining today.

$p \wedge q$: Today is Friday and it is raining today.

The proposition is true when this condition happens.

Truth Table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR Operator):

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q" which is false when both p and q are false and is true otherwise.

Inclusive OR:

The disjunction is true when at least one of the two propositions is true.

"Students who have taken calculus or computer science can take this class".

Exclusive OR:

The disjunction is true only when one of the proposition is true.

"Ice-cream or pudding will be served after lunch".

Inclusive OR

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement / Implication:

Let p and q be propositions. The conditional statement $p \to q$, is the proposition "If p, then q" which is false when p is true and q is false, and true otherwise.

→ p is called hypothesis (or antecedent or premise) and q is called the conclusion (or consequence).

$$p \to q \quad (p \text{ implies } q)$$

Example: If I am elected, then I will lower taxes.

elected, lower taxes — T
elected, not lower taxes — F
not elected, lower taxes — T
not elected, not " " — T

Truth Table:

P	Q	P → q
T	T	T
T	F	F
F	T	T
F	F	T

Example: Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job". Express the statement $p \rightarrow q$ as a statement in English.

If Maria learns discrete mathematics, then she will find a good job.

Ex: Convert "If you are working hard, then you are a topper" into symbolic form.

$p \rightarrow q$

Conditional:

If you are working hard, then you are a topper. $p \rightarrow q$

Converse:

If you are a topper, then you are working hard. $p \rightarrow q : q \rightarrow p$

Inverse:

If you are not working hard, then you aren't a topper. $p \rightarrow q : \sim p \rightarrow \sim q$

Contrapositive:

If you are not topper, then you are not working hard. $p \rightarrow q : \sim q \rightarrow \sim p$

Biconditional Statement:

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$, is the proposition "p if and only if q" which is true when p and q have the same truth values, and is false otherwise.

⇒ $p \leftrightarrow q$ has the same truth values as $(p \rightarrow q) \wedge (q \rightarrow p)$

Example:

Let p be the statement "Shape is a triangle" and let q be the statement "It has exactly three sides".

$p \leftrightarrow q$ =

"Shape is a triangle if and only if it has exactly three sides".

$(p \rightarrow q) \wedge (q \rightarrow p)$

(Both should be True or False to get True value)

Truth Table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

① q

If ... you are hungry, then ... you are eating a burger. p

Conditional | Implication

$p \rightarrow q$

Truth Table:

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

② If you have a ticket, then only you can travel.

Biconditional

$p \leftrightarrow q$.

Truth table:

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

③ Automated reply can be send, if the file system is not full.

Conditional (Converse)

$q \rightarrow p$. $(p \rightarrow q)$

p	q	$q \rightarrow p$
T	T	T
T	F	F
F	F	T
F	F	T

Translating English into propositional logic.

Q: If you work hard, then you will succeed.

p: you work hard.
q: You will succeed.

$$p \rightarrow q$$

Q: You can access the Internet from campus if you are a computer science major or you are not a freshman.

p: You can access the Internet from campus
q: You are a comp. sci major
r: you are a freshman.

$$p \rightarrow (q \vee \neg r)$$

$$(q \vee \neg r) \rightarrow p$$

result he ye
end me aega

Q: You cannot ride a roller coaster if you are under 4 feet tall unless you are older than 16 years old.

$$(q \wedge \sim r) \rightarrow \sim p.$$

p: You can ride the r.c

q: You are under 4 feet tall.

r: You are older than 16 years old.