

June 19, 2017

Contents

Data Structures

1.1 Data Types

Primitive Types

Primitives: [Wiki Link](#)

These are the basic elements:

1. Bool
2. char
3. float
4. double
5. int
6. string
7. reference
8. enum

Composite Types

Composites: [Wiki Link](#)

Abstract Data Types

Abstract Types: [Wiki Link](#)

| | |
|------------------|---|
| Nodes | A vertex connected to other vertexes by edges, in a tree nodes are connected by edges |
| Edge | Connections to other nodes |
| Subtrees | A connected group of children nodes connected to a parent that's not the root |
| Digraph | |
| Root | Top node of a tree |
| Child | A node directly connected to another, away from the root |
| Parent | Directly connected node towards the root. Can only have 1 parent as this causes cycles. |
| Sibling | Node with same parent |
| Decendant | Node accessible by parent to child |
| Ancestor | Node accessible by traversing child to parent |
| Leaf | (or external node) A node with no children (degree 1) |
| Branch | (or internal node) A node with children (degree >1) |
| Degree | The number of edges on a nodes |
| Path | Sequence of nodes and edges to reach another node |
| Level | 1+connections to root of a node. (R)-()-()-(A) A has level 4. |
| Node Height | Number of edges on the longest path between that node and a leaf. |
| Depth | Number of edges from the root to a given node |
| Forest | A set of disjointed trees |
| Branching Factor | Maximum number of children per node |

Linear Data Structures

1.2 Abstract Data Types

Abstract data types (ADT) → it's only a data structure if you're talking about the implementation.

Trees

Tree: [Wiki Link](#)

A tree is a data structure made up of nodes connected by edges without having a cycle in it (a node cannot call itself in anyway). A linear list is a trivial tree.

Search Tree

Search Tree: [Wiki Link](#)

Tree data structure used for locating specific keys from within a set. Needs to be relatively balanced to be efficient.

Binary Trees

Binary Trees: [Allisons Link](#)

Binary Trees: [Wiki Link](#)

Hash Tree

Hash Array Tree: [Wiki Link](#)

Merkle Tree: [Wiki Link](#)

Trie

Trie: [Wiki Link](#)

Type of search tree. No node stores the key associated with its node, the position in the tree decides its key. All of the descendants of a node have the same prefix, the root is an empty string.

Commonly used for autocomplete and predictive text.

A trie can replace hash table:

- Worst case lookup is better
- No key collisions
- Buckets are only necessary if a key identifies multiple values
- Can provide alphabetical ordering

Drawbacks:

- Tends to be slower than a hash for lookups
- Floats can cause nasty long search chains
- Can require more memory as the keys are split up instead of contiguous

Heap

The Heap: [Wiki Link](#)

Tree data type, subtype of a priority queue. two types \rightarrow min and max. In a min/max heap the root is the lowest/highest value in the tree.

The binary heap was introduced for the heap sort algorithm. The heap is partially ordered.

- Heap Property: with $P \rightarrow$ parent node, $C \rightarrow$ child node, then the key of P is ordered with respect to C . This applies for every child and parent.
- The root is the lowest or highest value
- Items always go in the next free slot. If it isn't in the right place compare to its parent and swap if the parent is smaller/larger.

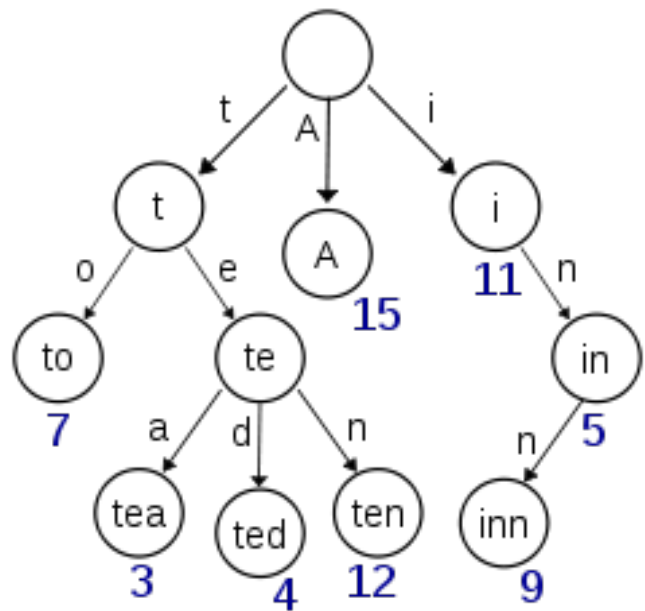


Figure 1.1: A trie data type visualization

Table 1.1: **Time Complexity**

| Algo | Ave | Worst |
|--------|--------|--------|
| Space | $O(n)$ | $O(n)$ |
| Search | $O(1)$ | $O(n)$ |
| Insert | $O(1)$ | $O(n)$ |
| Delete | $O(1)$ | $O(n)$ |

Sets

Sets: [Wiki Link](#)

1.3 Hashes

Hash function

Hash Table

Hash Table: [Wiki Link](#)

Hash tables tend to be faster than other table data structures, degrading to the same average lookup time of an unordered array only in the worst case senario. If the hash function is complex and the entry count small, this advantage can be lost.

A hash table is an associative array (i.e. a dictionary) that maps keys to values. Puts a key through a hash function to find/retrieve an index to an array of buckets

Table 1.2: Hash table terms

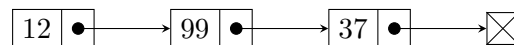
| | |
|---------------|---|
| Key | The name of a value/ attribute |
| Bucket | The array elements. Typically a dynamic array |
| Slot | Synonym for bucket |
| Hash function | computes the index from a key |
| Load Factor | $\frac{\text{entries}}{\text{buckets}}$ The higher the load factor the slower the search, the lower the load factor the more memory wasted. |

Collision Resolution

| | |
|-------------------|---|
| Separate Chaining | Buckets have a list of their own entries to a single has, if the hash matches and the key doesn't do a linear search of the list |
| Open Addressing | On a collision, the new entry takes the next open slot. Useful if memory is an issue and the entries are smaller that $\sim 4 \times \text{sizeof}(*)$ |

Linked Lists

Singly linked lists



Doubly linked lists

Queue

Queue: [Wiki Link](#)

Fifo, any added item are added to the end/tail of the structure (inqueued). Items are only removed from the front/head (dequeued).

- Simulate waiting lines
- Buffers for I/O

Priority Queue

Stack

The Stack: [Wiki Link](#)

Opposite of a queue \rightarrow LIFO

Ring Buffer

Ring Buffer: [Wiki Link](#)

Union

Tagged Union

Heuristics → efficient algos that get a good although not necessarily perfect solution

Algorithms

2.1 Asymptotics

How the algorithm grows as $N \rightarrow \infty$.

Algorithms by David Sedgewick: Page 67

David Mount Notes: [Link](#)

Big-O Notation: [Wiki Link](#)

Time Complexity: [Wiki Link](#)

Notation

Θ : The asymptotic class of an algo.

$$\Theta(g(n)) \equiv \left\{ f(n) \mid 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \mid c_1, c_2, n_0 \in |\mathbb{R}| \text{ and } n_0 \leq n \right\} \quad (2.1)$$

For a algo to be $\Theta(g(n))$ it needs to be both $O(g(n))$ and $\Omega(g(n))$. A $\Theta(g(n))$ grows at exactly $g(n)$.

O: Upper asymptotic bound for an algo. An algo that has $O(g(n))$ grows at or slower than that rate.

$$O(g(n)) \equiv \left\{ f(n) \mid 0 \leq f(n) \leq c g(n) \mid c, n_0 \in |\mathbb{R}| \text{ and } n_0 \leq n \right\} \quad (2.2)$$

Ω : The lower bound on the growth. An algo w/ $\Omega(g(n))$ grows at or faster than $g(n)$.

$$\Omega(g(n)) \equiv \left\{ f(n) \mid 0 \leq c g(n) \leq f(n) \mid c_1, c_2, n_0 \in |\mathbb{R}| \text{ and } n_0 \leq n \right\} \quad (2.3)$$

Table 2.1: Asymptotic growth types

| Notation | Name | Example |
|----------------------------|--------------------------|--|
| $O(1)$ | Constant | Seeing if a binary number is even or odd |
| $O(\log \log n)$ | Double Logarithmic | |
| $O(\log n)$ | Logarithmic | Finding an item in a sorted array with binary search |
| $O((\log n)^c)$ w/ $c > 1$ | Polylogarithmic | |
| $O(n^c)$ w/ $0 < c < 1$ | Fractional power | Searching in a kd-tree |
| $O(n)$ | Linear | Find an item in an unsorted list |
| $O(n \log^* n)$ | n log-star n | Union-find |
| $O(n \log n)$ | quasilinear/linearithmic | FFT |
| $O(n^2)$ | Quadratic | Common limit on sorting |
| $O(n^c)$ | Polynomial | LU decomposition |

Performace**Worst Case****Best Case****Average Case****Asymptotic Analysis****(Strong) Induction****Iteration****Recurrence****Master's Theorem****2.2 Algorithm Types****Divide & Conquer****Selection****2.3 Searching****Binary Search**Binary Search: [Wiki Link](#)**Linear Search**Linear Search: [Wiki Link](#)

2.4 Selection

Sieve Technique

2.5 Sorting

Merge Sort

Heap Sort

Quick Sort

Bubble Sort

Insertion Sort

Selection Sort

Count Sort

Radix Sort

Math

3.1 Series

Arithmetic:

$$\sum_{i=1}^n i = 1 + 2 + \dots + n = \frac{n(n+1)}{2} \quad (3.1)$$

Geometric:

$$\sum_{i=0}^n x^i = 1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1} \quad (3.2)$$

Harmonic:

$$H_n \equiv \sum_{i=1}^n \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \approx \ln(n) \quad (3.3)$$