2.4 证明为奇异矩阵

$$A = \begin{bmatrix} I & X \\ Z & Y \end{bmatrix}$$

由于 I 为单位矩阵

$D(I) = 1$

Z 为全0矩阵

$D(Z) = 0$

$D(A) = 1 \times Y_{11} \times Y_{22} \times \cdots \times Y_{nn} - 0$

因为 $Y_{11} = 0$

$D(A) = 0$

A为奇异矩阵.

线性回归证明

3.3 $E = \frac{1}{2}\sum_{i=1}^{n}(y_i - m x_i - b)^2$

3.3.1 证明1 $E = \frac{1}{2}\sum_{i=1}^{n}[(y_i - b)^2 + m^2 x_i^2 - 2m x_i(y_i - b)]$

$$\frac{\partial E}{\partial m} = \frac{1}{2}\sum_{i=1}^{n}[2m x_i^2 - 2x_i(y_i - b)]$$

$$= \sum_{i=1}^{n} -x_i(y_i - b - m x_i)$$

证明2 $E = \frac{1}{2}\sum_{i=1}^{n}[(y_i - m x_i)^2 + b^2 - 2(y_i - m x_i)b]$

$$\frac{\partial E}{\partial b} = \frac{1}{2}\sum_{i=1}^{n}[2b - 2(y_i - m x_i)]$$

$$= \sum_{i=1}^{n} -(y_i - m x_i - b)$$

3.3.2 $E = \frac{1}{2}\sum_{i=1}^{n}(y_i - m x_i - b)^2$ 将(1,1),(2,2),(3,2)代入

目标函数 $E = \frac{1}{2}(9 + 3b^2 + 14m^2 - 22m - 10b + 6mb)$

二元一次方程 $14 + 3b = 11$

$6m + 3b = 5$

求解 $m = 3/4$, $b = 1/6$

3.3.3. 证明 $\begin{bmatrix} \frac{\partial E}{\partial m} \\ \frac{\partial E}{\partial b} \end{bmatrix} = X^T X h - X^T Y$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \quad h = \begin{bmatrix} m \\ b \end{bmatrix}$$

解：$X^T X = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} 1 \end{bmatrix}$

$$X^T X h = \begin{bmatrix} \sum_{i=1}^{n} (m x_i^2 + b x_i) \\ \sum_{i=1}^{n} (m x_i + b) \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{bmatrix}$$

$$X^T X h - X^T Y = \begin{bmatrix} \sum_{i=1}^{n} -x_i (y_i - b - m x_i) \\ \sum_{i=1}^{n} -(y_i - m x_i - b) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial E}{\partial m} \\ \frac{\partial E}{\partial b} \end{bmatrix}$$