- Original Heuristic:

Instead of simply calculating *Manhattan distance (MD)*, which is the sum of the absolute differences between the two vectors, we can find a closer cost added to MD.

$$H'(n)=\begin{cases} MD & ,if\ MD <= 1 \\ MD+3 & ,o.w. \end{cases}$$

Relax the problem by simplifying the 1*2 pieces to two 1*1 pieces, there are only one 2*2 pieces, and all rest pieces are 1*1 (labeled 'x').

- Say MD=0, we are at goal state, actual cost is 0.
- Say MD =1, the best case, we can move to goal state at cost of 1.
  - Ex.

```
xxxx                 xxxx
xxxx                 xxxx
x11x   ----down-→    x00x
x11x     cost:1      x11x
x00x                 x11x
```

- Say MD=2, the best case, we can move to goal state in 5 moves
  - Ex1.

```
xxxx                xxxx                      xxxx                  xxxx
xxxx                xxxx                      xxxx                  xxxx
110x.  ---right -→  011x  -move empty →       x11x     --down->     x00x
110x.   cost: 1     011x     cost: 3          x11x      cost: 1     x11x
xxxx                xxxx                      x00x                  x11x
```

  - Ex2.

```
xxxx                xxxx                      xxxx                  xxxx
x11x                x00x                      xxxx                  xxxx
x11x.  ---right -→  x11x  -move empty →       x11x     --down->     x00x
x00x.   cost: 1     x11x     cost: 6          x11x      cost: 1     x11x
xxxx                xxxx                      x00x                  x11x
```

- In best case for MD>= 2, 2*2 piece need **at least cost of 3** to swap the empty pieces with 1*1 pieces and prepare the empty pieces for the next move.

- Why admissible?

  H'(n) is admissible as it never overestimates the cheapest cost from state n to a goal state.

  In the relaxed rules, for MD>=2, the 2*2 piece needs to move at least **MD** moves, and between **(MD-1)** pairs of consecutive moves we need to move the empty pieces, with a **min cost of 3** each.

- Why does it dominate *Manhattan distance heuristic*?
  - For all possible number of MD numbers, H'(n) >= H(n); and for MD>=2, H'(n) >H(n).
- Implementation:
  - get_original_heuristic(board)
  - A* with original heuristic for the classic HRD config still runs at costs of 116.