CSC343 Assignment 3 Part 2

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1 Question 1

(a) Check the closure for L, M, N, O:
$L^+ = LMNOPQRS$, so L is a super key, and $L \to NO$ does not violate BCNF
$M^+ = MP$, M is not a super key of R, thus $M \to P$ violates BCNF.
$N^+ = MNQRP$, N is not a super key of R, thus $N \to MQR$ violates BCNF.
$O^+ = OS$, O is not a super key of R, thus $O \to S$ violates BCNF.
Thus FDs $\underline{M \to P,\ N \to MQR \text{ and } O \to S}$ violate BCNF

(b) Decompose R using FD $N \to MQR$, we have $N^+ = MNQRP$, so this yields two relations:
R1 = MMNQRP, and R2 = LNOS
Project the FDs onto R1 = MNQRP,

M	N	Q	R	P	closure	FDs
✓					$M^+ = MP$	$M \to P$; violates BCNF; abort the projection

We must decompose R1 further,
So we decompose R1 using FD $M \to P$, this yield two relations:
R3 = MP, and R4 = MNQR
Project the FDs onto R3 = MP,

M	P	closure	FDs
✓		$M^+ = MP$	$M \to P$, M is a super key of R3
	✓	$P^+ = P$	Nothing

This relation satisfies BCNF.

Project the FDs onto R4 = MNQR

M	N	Q	R	closure	FDs
✓				$M^+ = MP$	nothing
	✓			$N^+ = MNQRP$	$N \rightarrow MQR$, N is a superkey of R4
		✓		$Q^+ = Q$	nothing
			✓	$R^+ = R$	nothing
Super set of N				irrelevant	can only generate weaker FDs than what we already have
✓		✓		$MQ^+ = MPQ$	nothing
✓			✓	$MR^+ = MPR$	nothing
		✓	✓	$QR^+ = QR$	nothing
✓		✓	✓	$MQR^+ = MQRP$	nothing

This relation satisfies BCNF

Return to R2 = LNOS and project FDs onto it.

L	N	O	S	closure	FDs
✓				$L^+ = LMNOPQRS$	$L \rightarrow NOS$, L is a superkey of R2
	✓			$N^+ = MNPQR$	nothing
		✓		$O^+ = OS$	$O \rightarrow S$ not a super key; violates BCNF; abort the projection

We must decompose R2 further.

Decompose R2 using FD $O \rightarrow S$, this yields two relations:

R5 = OS, and R6 = LMO

Project the FDs onto R5 = OS,

O	S	closure	FDs
✓		$O^+ = OS$	$O \rightarrow S$, O is a super key of R5
	✓	$S^+ = S$	Nothing

This relation satisfies BCNF.

Project the FDs onto R6.

L	N	O	closure	FDs
✓			$L^+ = LMNOPQRS$	$L \to NOS$, L is a superkey of R6
	✓		$N^+ = MNPQR$	nothing
		✓	$O^+ = OS$	nothing
Super set of L			irrelevant	can only generate weaker FDs than what we already have
	✓	✓	$N0^+ = MN0PQR$	nothing

This relation satisfies BCNF.

Final decomposition:

a: R3 = MP with FD $M \to P$

b: R4 = MNQR with FD $N \to MQR$

c: R5 = OS with FD $O \to S$

d: R6 = LNO with FD $L \to NO$

(c) Yes, we can check it using closure for L, M, N, O for our decomposition:

closure	FDs	preserve function dependency
$L^+ = LMNOPQRS$	$L \to NOS$	✓
$M^+ = MP$	$M \to P$	✓
$N^+ = MNQRP$	$N \to MQR$	✓
$O^+ = OS$	$O \to S$	✓

We can also see that all original FDs are in our decomposition, so my schema preserves dependencies.

(d) Throughout these solutions, I use boldface letters to show values whose presence we can infer from the structure of the decomposition, non-boldface letters to show values whose presence we can then infer from the FDs, and 3 for a value that could be anything.

a: show if a tuple ⟨l,m,n,o,p,q,r,s⟩. is in $R3 \bowtie R4 \bowtie R5 \bowtie R6$.

This is guaranteed because our decomposition included all attributes and a way to join them.

b: Show if a tuple ⟨l,m,n,o,p,q,r,s⟩. is in $R3 \bowtie R4 \bowtie R5 \bowtie R6$., then it is in R.

We start with:

L	M	N	O	P	Q	R	S
3	**m**	3	3	**p**	3	3	3
3	**m**	**n**	3	3	**q**	**r**	3
3	3	3	**o**	3	3	3	**s**
l	3	**n**	**o**	3	3	3	3

Then, because $M \to P$, $N \to MQR$, $O \to S$, we make these changes:

L	M	N	O	P	Q	R	S
3	**m**	3	3	**p**	3	3	3
3	**m**	**n**	3	p	**q**	**r**	3
3	3	3	**o**	3	3	3	**s**
l	m	**n**	**o**	p	q	r	s

We observe that the tuple ⟨l,m,n,o,p,q,r,s⟩. does occur. The chase test succeeded, so it is a lossless join decomposition.

2 Question 2

(a) Step 1: Split RHS: Let S1 = {a) $ACD \to E$, b) $B \to C$, c) $B \to D$, d) $BE \to A$, e) $BE \to C$, f)$BE \to F$, g) $D \to A$, h) $D \to B$, i) $E \to A$, j) $E \to C$}

Step 2: for each FD, try to reduce the LHS.
a) $A^+ = A$, $C^+ = C$, $D^+ = ABCD$, $AC^+ = AC$, $AD^+ = ABCD$, $CD^+ = ABCD$, no E in it, so can't be reduced
b) $B^+ = ABD$, no C in it, can't be reduced
c) $B^+ = BC$,no D in it, can't be reduced
d) $B^+ = ABCD$, A is in the closure, so it can be reduced to $B \to A$, which is equivalent to b)
e) $B^+ = ABCD$, C is the closure, so it can be reduced to $B \to C$
f) $B^+ = ABCD$, $E^+ = ACE$, $BE^+ = ABCE$, no F in it, can't be reduced
g)h)i)j): only one attribute on LHS, can't be reduced more.
So S2 = {a) $ACD \to E$, b) $B \to C$, c) $B \to D$, d) $B \to A$, e)$BE \to F$, f) $D \to A$, g) $D \to B$, h) $E \to A$, i) $E \to C$}

Step 3: Try to eliminate each FD.

FD	Exclude these from $S2$ when computing closure	Closure	Decision
a	a	$CDA^+ = ABCD$	keep
b	b	$B^+ = ABD$	keep
c	c	$B^+ = ABC$	keep
d	d	$B^+ = ABCDEF$	discard
e	d, e	$BE^+ = ABCDE$	keep
f	d, f	$D^+ = BCD$	keep
g	d, g	$D^+ = AD$	keep
h	d, h	$E^+ = CE$	keep
i	d, i	$E^+ = AE$	keep

Our final set of FDs is: S2 = {a) $ACD \to E$, b) $B \to C$, c) $B \to D$, d)$BE \to F$, e) $D \to A$, f) $D \to B$, g) $E \to A$, h) $E \to C$}

(b)

Attribute	Appears on LHS	Appears on RHS	Conclusion
G, H	–	–	must be in every key
$\emptyset$	✓	–	must be in every key
F	–	✓	is not in any key
A, B, C, D, E	✓	✓	must check

check: $GH^+ = GH$,
Now consider all combinations of A,B,C,D,E. For each, we must add in G and H, since they are in every key.

A	B	C	D	E	closure	conclusion
✓					$AGH^+ = AGH$	not a key
	✓				$BGH^+ = ABCDEFGH$	It's a key
		✓			$CGH^+ = CGH$	not a key
			✓		$DGH^+ = ABCDEFGH$	It's a key
				✓	$EGH^+ = ACEGH$	not a key

Since BGH, DGH is minimal and it's a key, so we don't have to further consider any super set of BGH and DGH.
From the minimal basis for T we can see that only D can determine B, so without D or B itself, we can never get the closure of a set that can give us everything.

Thus BGH and DGH are keys for P.

(C) The set of relations that would result would have those attributes:
R1(A,C,D,E), R2(B,C), R3(B,D), R4(B,E,F), R5(A,D), R6(B,D), R7(A,E), R8(C,E)
Since R5,R7,R8 are all subset of R1, and R6 is a subset of R3, so we left with

R1(A,C,D,E), R2(B,C), R3(B,D), R4(B,E,F).

Since there's no G and no H in either LHF or RHS of FDs, and G, H do not appear in R1,R2,R3,R4, so none of them is a super key.
So we have to add another relation that is a key, from (b), we know that BGH is a key, so we have R5(B,G,H).

Thus the final set of relationns is:
R1(A,C,D,E), R2(B,C), R3(B,D), R4(B,E,F), R5(B,G,H)

(d) Clearly we can see that $B \to C$, $B \to D$ would project onto R5, and $B^{+} = BCD$, so B is not a super key of R5, so $B \to C$ and $B \to D$ violate BCNF, which means that our relation would allow redundency.