| Section VI | Big O  |   |   |                                       |                           |  |  |  |  |
|------------|--|---|---|---------------------------------------|---------------------------|--|--|--|--|
|            | Asymptotic notations                             |   |   |                                       |                           |  |  |  |  |
|            | O(Big O)   | Describes an upper bound on time  | for example: algo that prints all the values in an array: could have Big O time as O (n), O(n^2), O(n^3) or O(2^n) and many other Big O's | on the runtime;<br>similar to a less- | then we also say that X<= | In industry, O<br>and theta have<br>been put<br>together and we<br>have to give the<br>tightest<br>description of<br>runtime |  |  |  |
|            | Omega(n)   | Describes the lower bound   | for example: printing the<br>values in an array is Omega<br>(n) as well as Omega(logn)<br>as well as Omega(1)                             |                                       |                           |  |  |  |  |
|            | Theta(n)   | Describes the tight bound on runtime  | Theta here means both O and Omega; in this example, it would be Theta(n)  |                                       |                           |  |  |  |  |
|            | Best Case,<br>Worst Case<br>and Expected<br>Case |   |   |                                       |                           |  |  |  |  |
|            | Best Case:                                       | For example, in Quick Sort, if all the elements are equal, then quick sort will, on average, just traverse through the array once - O(N) time   | elements greater than pivot -<br>this gives partial sort. then it<br>recursively sorts the left and                                       |                                       |                           |  |  |  |  |
|            | Worst Case:                                      | The pivot could be repeatedly the biggest element in the array. If pivot is the first element in a reversely sorted array. In this cae, our recursion does not divide the array in half and recurse on other half. Instead, it justs shrinks the subarray by 1 element. | Time taken would O(N^2)   |                                       |                           |  |  |  |  |
|            | Expected Case:                                   | both the above best and<br>worst conditions would rarely<br>happen; thus we can expect<br>a runtime of O(nlogn)   |   |                                       |                           |  |  |  |  |
|            |  | etween Asymptotic notations<br>, Worst Case and Expected  |   |                                       |                           |  |  |  |  |

| There is no particular relationship between the two concepts   |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
|--|--|---------------------------------|---|---------------------|------------------|-------------------|------------------|--------------------|---------------------|--------|
|  | 10 15 110  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Best Case, Wor   | st Case and Expected Case act  | ually describe the big O or big | g Theta time for part                                     | ticular scenarios w | hereas these asy | mptotic notations | describe the upp | er, lower and tigh | t bounds for the ru | ıntime |
|  |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Space complexity   |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Memory or space required by an algorithm   | to create an array - if it is<br>unidimensional, O(N) space<br>complexity; for a 2-D array, O<br>(N^2)                     |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Stack space in recursive calls counts too. Each call adds a level tot he stack and takes up actual memory. | However, just because you have N calls does not mean it will take O(N) time: check the example on Page 41 for more details |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Drop the constants   |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| O(2N) is actually O(N)   |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Drop the non-<br>dominant<br>terms   |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| O(N^2 + N)<br>becomes O<br>(N^2)   |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| O(N + logN)<br>becomes O(N)  |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| O(5*2^N +<br>1000N^100)<br>becomes O<br>(2^N)  |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| $O(x!) > O(2^x) > 0$   | $O(x^2) > O(x\log x) > O(x)$   |                                 |   |                     |                  |                   |                  |                    |                     |        |
|  |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Multi-Parts<br>algorithms:<br>add versus<br>mutiply  |  |                                 |   |                     |                  |                   |                  |                    |                     |        |
| Add:   | Non-nested chunk of work A and B   | O(A + B)                        | "DO THIS<br>THEN WHEN<br>YOU ARE ALL<br>DONE, DO<br>THAT" |                     |                  |                   |                  |                    |                     |        |

| Multiply   | Nested A and B   | O(AB)  | "DO THIS FOR<br>EACH TIME<br>YOU DO THAT"                |   |  |  |  |
|--|--|--|--|---|--|--|--|
| Multiply   | Nesteu A anu B   | O(AB)  | TOO DO THAT  |   |  |  |  |
| Amortized time   |  |  |  |   |  |  |  |
|  | That copying might take additional O(N) time after accounting for initial O(N) time of adding the elements | into a new array does not  | that the worst ca<br>every once in a<br>happens it won't |   |  |  |  |
| Adding X more space to an array takes additional O(X) time; thus the amortized time for each adding is O(1)  | X + X/2 + X/4 + X/8 = 2X   |  |  |   |  |  |  |
| logN runtimes  |  |  |  |   |  |  |  |
| Example:   |  |  |  |   |  |  |  |
| Binary search. We are looking for an element x in a sorted array. We first compare to the midpoint. If x == middle, then we return else if x < middle, we search on the left side of array |  | The total runtime is then a matter of how many steps we can take before it becomes 1 | $2^k = N \Rightarrow k = \log N$ with base 2             | Basically, when you see a problem with logN runtime, the problem space gets halved in each step |  |  |  |
|  |  |  |  |   |  |  |  |
| Recursive runtimes   |  |  |  |   |  |  |  |
| Program:   | int f(int n){  |  |  |   |  |  |  |
|  | if(n <= 1){  |  |  |   |  |  |  |
|  | return 1}  |  |  |   |  |  |  |
|  | return f(n -1) + f(n -1);}   |  |  |   |  |  |  |
|  |  |  |  |   |  |  |  |
| How many calls in the tree?  |  |  |  |   |  |  |  |
| Do not count and say 2   |  |  |  |   |  |  |  |

| It will have recursive calls with a depth N and 2^N nodes at the bottom most level | More genrically, 2^0 + 2^1 + 2^2 +2^n-1 = 2^n - 1 nodes  |  | The space complexity would still be O (n) - even though we have O(2^n) nodes in tree total, only O(n) exists at a time |  |  |  |  |
|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |
| Examples and I   |  |  |  |  |  |  |  |
| Example 1  | O(n) time as we iterate<br>through array once in each<br>loop which are non-nested   |  |  |  |  |  |  |
| Example 2  | O(N^2) time as we have two nested loops  |  |  |  |  |  |  |
| Example 3  | j basically runs for N-1 steps<br>in first iteration, N-2 steps in<br>second iteration and so on.  | 1 + 2 + 3 ++ N-1 = N(N -<br>1)/2 ~ N^2   | O(N^2)   |  |  |  |  |
| Example 4  | For each element of array A, the inner loop goes through b iterations where b is the length of array B. Thus, time complexity is O(ab)   |  |  |  |  |  |  |
| Example 5  | Similar to example 4, 100,000 units of work is still constant; so the run time is O(ab)  |  |  |  |  |  |  |
| Example 6  | O(N) time as the array is<br>iterated even if half of it<br>(constant 1/2 can be ignored)  |  |  |  |  |  |  |
| Example 7  | There is no established relationship between N and M , thus all but the last one are equivalent to O(N)  |  |  |  |  |  |  |
| Example 8  | s = length of the longest string<br>Sorting each string would take<br>elements of the array; thus O(a<br>array would take O(s * aloga) a<br>would take O(s) time in additio<br>take O(aloga); thus adding the<br>aloga) = O(a *s (log a + log s) | O(slogs) and we do this for a a * slogs) . Now, sorting the as each string comparison in to array sorting that would |  |  |  |  |  |
| Example 9  | Approach 1: for summing up<br>the nodes in a BST, each<br>node is exactly traversed<br>once, thus O(N) time<br>complexity  |  |  |  |  |  |  |
|  | Approach 2: The number of recursive calls is 2 and the depth is logN in a BST> O (branches ^ depth) = O(2 ^ logN) where the base of logN is also 2 => time complexity = O(N) after simpllifying  |  |  |  |  |  |  |

|  | ) as the for loop<br>stant time and runs<br>N)) time  |   |  |  |
|--|---|---|--|--|
| process c  | ne recursive<br>alls from N to N-1 to<br>o on until 1   |   |  |  |
| that we br<br>time. this<br>string. So<br>path with  | h 1: We make a tree for an example string say 'abc<br>anch 4 times at the root , then 3 times, then 2 time<br>gives us 4*3*2*1 leaf nodes. We could say n! leaf r<br>, total nodes would be n * n! as each leaf node is a<br>n nodes. Also, string concatenation will also take C<br>me complexity in worst case would be O(n * n * n!)   | es, and then 1<br>nodes for n length<br>attached to a<br>D(n) time. Thus,     |  |  |
| nodes; at<br>total node<br>1/n!). Nov<br>n! * e who<br>Thus, the                             | n 2: At level 6, we have 6! / 0! nodes; at level 5 we level 4 we have 6!/2! nodesat level 0, we have 6! s in the tree in terms of n can be: n!(1/0! + 1/1 | !/6! nodes. so, the<br>1/2! + 1/3!+<br>of Euler's number:<br>Iropped further. |  |  |
| pattern for  | We can also get tighter runtime as O(1.6^N) if we consider that there might be just one call instead of 2 at the bottom of call stack of the bottom of call stack sometimes.  |   |  |  |
| time. And,<br>2^2 + 2^3  | vious example, we deduce that fib(n) taken $2^n$ we have fib(1) + fib(2) + fib(3) +fib(n) = $2^1$ ++ $2^n$ = $2^n$ (n+1) - 2, thus we can say that the s approx. $2^n$  |   |  |  |
| are doing<br>which the<br>reduces to<br>and fib(i -<br>array at ea<br>we are do<br>amount of | e in this program we memoization due to amount of work b looking up fib(i - 1) 2) values in memo ach call fib(i). Thus, ing a constant f work n times in n ce time complexity:  |   |  |  |
| times we until we guntil we gunte case 1. As number of                                       | ne is the number of can divide n by 2 et down to the base s, we know the firmes we can ntil we get 1 is O   |   |  |  |
|  |   |   |  |  |
| Additional Problems  |   |   |  |  |
|  | op iterates through<br>ne complexity is O   |   |  |  |
| through b  | sive call iterates calls as it subtracts iteration, thus time y is O(b)   |   |  |  |

| 3  | It does constant amount of work, thus time complexity is O(1)  |  |  |  |
|----|--|--|--|--|
| 4  | The variable count will eventually equal a/b. The while loop iterates through count times. Thus time complexity is O(a/b)  |  |  |  |
| 5  | The algorithm is actually doing a binary search to find the square root. Thus the runtime is O(logN)   |  |  |  |
| 6  | This is straightforward loop<br>that stops when guess*guess<br>> n or guess> sqrt(n); hence<br>time complexity is O(sqrt(n))   |  |  |  |
| 7  | Ifa binary tree is not balanced, the max time to find an element would be the depth of the tree. The tree since it is imbalanced could be a straight list downwards and ave depth n. Thus, runtime is O(n)   |  |  |  |
| 8  | Without any ordering or sorting in a binary tree, we might need to traverse through all the nodes in the tree , thus the time complexity is O(n) where n is the number of nodes in the tree.   |  |  |  |
| 9  | The first call to appendToNew takes 1 copy. The second call takes 2 copies. The third call takes 3 copies. And so on. The total time will be sum through 1 to n, which is O(n^2)   |  |  |  |
| 10 | The runtime would be O(d) where d is the number of digits in the given number. A number with d digits can have a value upto 10^d. If n = 10^d, then d = log n. Thus, time complexity is O(log n) where the log is with base 10.  |  |  |  |
| 11 | If the length of the string is k, then to check if the string is inOrder or sorted , takes $O(k)$ time. Also, suppose the length of the string is c characters. Now, to get strings of c characters and k length would be $O(c^{\Lambda}k)$ . for example, you wish to construct astring length 3 with just two characters a and b, thus the number of strings possible would be $2^{\Lambda}3$ . Similarly here , that runtime would be $O(c^{\Lambda}k)$ . Thus, overall runtime to get all the strings of k length with c characters and check if they are sorted would be $O(kc^{\Lambda}k)$ . |  |  |  |

|            | 12   | First of all the runtime for merg<br>then , for each element in a , w<br>b - runtime would be O(a * log<br>2 ( b log b + a log b).                  | ve are doing binary search of   |  |  |  |  |  |
|------------|--|---|---|--|--|--|--|--|
| Section IX | Interview Ques   | tions   |   |  |  |  |  |  |
| Chapter 1  | Arrays and Stri  |   |   |  |  |  |  |  |
|            | Hash Table   | 90  |   |  |  |  |  |  |
|            |  | a data structure that maps keys cient lookup.   |   |  |  |  |  |  |
|            | Hash Table Imp   | olementation  |   |  |  |  |  |  |
|            | Approach 1   | We use an array of Linked List  | s and a hash code function.   |  |  |  |  |  |
|            |  | To insert a key ( a string or any we follow the following steps:  | other datatype) and value   |  |  |  |  |  |
|            |  | Compute the key's hashcod<br>or long. Two different keys cou<br>as there may be numerous key  | ld have the same hashcode,  |  |  |  |  |  |
|            |  | 2. Then, we map the hash cod<br>This could be done with somet<br>array_length. Two different has<br>to the same index.                              | hing like hash (key) %  |  |  |  |  |  |
|            |  | 3. At this index, there is a linke<br>Store the key and value in the<br>List to tackle collisions: you co<br>with same hashcode or two dif<br>index | index. We must use a Linked uld have two different keys   |  |  |  |  |  |
|            |  |   | If it is the worst case, collisions are very high, runtime would be O(N) where N is the number of keys. And, if it is the best case, collisions are minimum, look up time would be O(1) |  |  |  |  |  |
|            | Approach 2   | We can implement a look up s search tree. This gives us O(lo advantage of this approach is we no longer allocate a large a through keys in order.   | ogN) lookup time. The potentially less space, since   |  |  |  |  |  |
|            | ArrayList & Res  | sizable Arrays  |   |  |  |  |  |  |
|            | When you need<br>an array-like<br>datastrure<br>with dynamic<br>resizing, you<br>should use an<br>ArrayList. | the size might instead  | many elements we copied at a increase to get an array of size N/8 ++ 2+ 1 = N. Therefore  | each capacity e N: N/2 + N/4 + e, inserting N I. Thus, each O(1), even |  |  |  |  |
|            | StringBuilder  |   |   |  |  |  |  |  |

|                        | Normally,   |  |  |  |   |  |  |  |
|------------------------|---|--|--|--|---|--|--|--|
|                        | concatenating n strings of x  |  |  |  |   |  |  |  |
|                        | characters each   |  |  |  |   |  |  |  |
|                        | would take O  | O(x + 2x + 3x + nx) = O  |  |  |   |  |  |  |
|                        | (xn^2).   | (xn^2)   |  |  |   |  |  |  |
|                        |   | n reduce this complexity as it cr  |  |  |   |  |  |  |
|                        |   |  |  |  |   |  |  |  |
| Chapter 2              | Linked Lists  |  |  |  |   |  |  |  |
| Oliupici 2             | Linked Lists  LinkedList is a   |  |  |  |   |  |  |  |
|                        | datastructure representing a sequence of nodes.                               | Singly Linked List> there is a pointer to the next node  | Doubly Linked List> there is a pointer to the next and previous nodes  |  |   |  |  |  |
|                        |   | LinkedList does not provide coess to any element of the list.  | Benefit of a Linked List is that one can add or remove   |  |   |  |  |  |
|                        | It takes iterating<br>Kth element   | through K elements to get the  | items from the beginning of the list in constant time  |  |   |  |  |  |
| Chapter 3              | Stacks and Que  | eues   |  |  |   |  |  |  |
|                        |   |  | A stack does not offer   |  |   |  |  |  |
|                        | Stack uses  | Operations of a stack: pop(), push(item), peek(), isEmpty()  | constant-time access to the ith item. However, it allows constant time adds and removes as it does not require shifting elements around. | most useful case<br>algorithms - one<br>temporary data one recurses, bu<br>them as one bac | needs to push<br>oto a stack as<br>ut then remove |  |  |  |
|                        | Queue<br>implements   | Operations of a queue: add (item), remove(), peek(),   | most useful case: breadth-<br>first search and   |  |   |  |  |  |
|                        | FIFO  | isEmpty()  | implementing a cache   |  |   |  |  |  |
| Chapter 4              | Trees and Grap  | ohs  |  |  |   |  |  |  |
|                        | Searching a   |  |  |  |   |  |  |  |
|                        | tree is more<br>complicated<br>than searching<br>any linear data<br>structure | The worst case and the average case time may vary wildly and we must evaluate both the aspects of any problem                  | Tree is actually a type of graph in which cycles / loops are not possible  |  |   |  |  |  |
| Trees                  | In programming. The root node h   | structure composed of nodes.<br>, each tree has a root node.<br>as zero or more children. Each<br>tero or more children and so | The nodes may be in any order and may have any data types as values and may or may not have links back to their parent nodes.            | A node is called<br>a leaf node if it<br>has no children.                                  |   |  |  |  |
| Trees vs. binary trees | A binary tree is<br>a tree in which<br>each node has                          |  |  |  |   |  |  |  |
|                        | upto two<br>children. Not all<br>trees are binary<br>trees.                   | For example, a 10-ary tree representing a bunch of phone numbers is not a binary tree.   |  |  |   |  |  |  |
|                        | evry node fits a left descendants   | tree is a binary tree in which specific ordering property: all s <= n < all right descendants. e for each node n               | This inequality condition must be true for all of a node's descendants, not just its immediate children.                                 |  |   |  |  |  |

| Balanced vs. unbalanced tree | not neccessarily<br>perfectly<br>balanced but<br>ensures O(log<br>n) times for<br>insert and find | Two common types of balanced trees: Red-black trees and AVL trees               |  |  |  |  |  |
|------------------------------|---|---|--|--|--|--|--|
| Complete binary trees        |   | ry tree is a binary tree in which the last level. To the extent the             |  |  |  |  |  |
| Full binary tree             | each node has e<br>is no node havin   | either 2 or zero children. There g only one child                               |  |  |  |  |  |
| Perfect binary tree          | All interior nodes have two children and all leaf nodes are at the same level.                    |   | It must have exactly 2 <sup>k</sup> - 1 nodes where k is the number of levels. |  |  |  |  |
| In-order<br>traversal        | "visit" the left<br>branch, then the<br>current node,<br>and finally the<br>right node            | when performed on a binary search tree, it visits the nodes in ascending order. |  |  |  |  |  |
| Pre-order<br>traversal       | "visits" the current node before its child nodes  | The root is always the first node visited                                       |  |  |  |  |  |
| Post-order<br>traversal      | "visits" the current node after its child nodes   |   |  |  |  |  |  |