S.No.			
1	**Algorithm**	**Program**	
	Design time	Implementation time	
	Domain knowledge	Programmer	
	Any language even English and Maths	Programming language	
	Hardware and software independent	Hardware and operating system dependent	
	Analyze an algorithm	Testing of programs	
2	**Priori Analysis**	**Posterior Testing**	
	Algorithm	Program	
	Independent of language	Language dependent	
	Hardware independent	Hardware dependent	
	Time and space function	watch time and bytes	
3	**Characteristics of algorithm**		
	Zero or more inputs		
	Must generate atleast one output		
	Definiteness		
	Finiteness		
	Effectiveness		
4	**How to analyze an algorithm**		
	Time		
	Space		
	Network consumtion : Data transfer amount		
	Power consumption		
	CPU registers		
5	**Frequency Count Method**	Used for time snalysis of an algorithm	
	Assign 1 unit of time for each statement		

	For any repitition, calculate the frequency of repetition		
	for(i = 0; i < n; i++) --> condition is checked for n+1 times	2n + 2 units of time ~ n+1 as we see condition i < n only for now	
	any statement within the loop will execute for n times		
	Space complexity depends upon number and kind of variables used		
6	**Algorithm : sum(A, n)**		
	Single for loop -		
	Time complexity: O(N)		
	Space complexity: O(N)		
7	**Algorithm : Add(A, B, n)**	Sum of two square matrices of dimesions nXn	
	Two nested for loops -		
	Time complexity: O(N^2)		
	Outer for loop executes for N+1 times		
	Inner for loop xecutes for N *(N+1) times		
	Any statement within inner for loop executes for (N + 1) * (N + 1) times		
	Space complexity: O(N^2)		
8	**Algorithm : Multiply(A, B, n)**		
	Three nested for loops -		
	Time complexity: O(N^3)		
	Space complexity: O(N^2)		
9	**Different algorithm conditions**		

	For loops		
	for(i = n; i > 0; i--)	n+1 times	
	for(i = 0; i < n; i = i + 2)	n/2 times	
	2 nested for loops where both i and j range from 0 to n	n^2 times	
	2 nested for loops where j ranges from 0 to i	when i = 0; j loop repeats 0 times; when i = 1; j loop repeates 1 times; and so on...total number of repetitions: 0 + 1+ 2+3+4....+n = O(n^2)	
	p = 0; for(i = 1; p<= n; i++){ p = p + i; }	p = k(k+1)/2 --> assuming that the loop exits when p is greater than n --> k(k+1) / 2 > n	~ k^2 > n --> O(root(n))
	for(i = 1; i < n; i = i *2)	will execute for 2^k times	O(logn)
		Assume i >= n ; i = 2^k >= n	
		k = logn with base 2	
	for(i = n; i >= 1; i = i/2)	i	
		n	
		n/2	
		n/2^2	
		n/2^3	
		.....	
		n/2^k	
		Assume i < 1 => n / 2^k < 1	~ O(logn) with base 2
	for(i = 0; i * i < n; i++)	i*i < n	
		i*i > -n	
		i^2 = n --> i = root(n)	~O(root(n))
	for(i = 0; i < n; i++) {.....}for(j = 0; j < n ; j++) {....}	O(n)	
	p = 0; for(i = 1; i < n; i*2){.....} for(j = 1; j < p; j*2){.....}	log n times for upper loop; lop p times for lower loop	~ O(log(logn))
	for(i = 0; i < n; i++) {.....for(j = 0; j < n ; j*2) {....}}	Outer loop repeats n times; inner loop repeats logn times	~O(nlogn)
	for(i = 1; i < n; i = i*3)		~O(logn) with base 3

	While loops		
	while vs. do while	do while will execute for minimum one time	
	for and while are almost similar	do while will execute as long as the condition is true; for loop will execute until the condition is false	
	a = 1;		
	while(a < b){ ..... a = a *2;}	1, 2, 2^2, 2^3.....2^k repetitions	~O(logb) with base 2
		assume a > b; 2^k > b ==> k = logb with base 2	
	i = n; while(i > 1) {....i = i/2;}		~O(logn) with base 2
	i = 1; k = 1; while(k <n){....k = k + i; i++;}		
	i	k	
	1	1	
	2	1 + 1	
	3	2 + 2	
	4	2 + 2 + 3	
	5	2 + 2 + 3 + 4	
	.....		
	m	m(m + 1) /2	
	Assume, k >= n	m(m + 1)/ 2 >= n	~O(root(n))
	while(m != n) { if(m > n) m = m - n; else n = n - m;}		~O(n)
10	**Types of time functions**		
	O(1) --- constant		
	O(logn) --- logarithmic		
	O(n) --- linear		
	O(n^2) --- quadratic		
	O(n^3) --- cubic		

	O(2^n) --- exponential		
11	**Order of complexity**		
	1 < logn < root(n) < n < nlogn < n^2 < n^3 <.....< 2^n < 3^n .....< n^n		
12	**Asymptotic Notations**		
	Representation of time omplexity in simple form which is understandable		
	Big O Notation - works as an upper bound	The function f(n) = O(g(n)) iff for all positive constants c and n_0, such that f(n) <= c * g (n) for all n >= n_0; here, f(n) = O(n)	e.g. 2n + 3 <= 10n; All those functions in time order complexity above n become upper bound; below n become lower bound and n is the average bound
	Big Omega Notation - works as a lower bound	The function f(n) = Omega(g(n)) iff for all positive constants c and n_0, such that f(n) >= c * g(n) for all n >= n_0; here, f(n) = Omega(n)	e.g. 2n + 3 >= 1n
	Theta Notation - works as an average bound	The function f(n) = theta(g(n)) iff for all positive constants c1, c2 and n0 such that c1 * g(n) <= f(n) <= c2 * g(n)	e.g. f(n) 2n + 3; 1n <= 2n + 3 <= 5n
	Most useful is theta notation, then why do we need the other two?	In case we are not able to get the average bound, then we point to its upper or lower bound	
13	**Examples for asymptotic notations**		
a	**f(n) = 2n^2 + 3n + 4**		
	2n^2 + 3n + 4 <= 2n^2 + 3n^2 + 4n^2 i.e. 9n^2	O(n^2)	
	2n^2 + 3n + 4 >= 1n^2	Omega(n^2)	
	1n^2 <= 2n^2 + 3n + 4 <= 9n^2	Theta(n^2)	
b	**f(n) = n^2logn + n**		
	n^2logn <= n^2logn + n <= 10n^2logn	O(n^2logn)	
		Omega(n^2logn)	

		Theta(n^2logn)	
c	**f(n) = n!**		
	1 <= 1*2*3*4....*n-1*n <= n*n*n*n*.....*n	O(n^n)	
		Omega(1)	
		Cannot find theta for n!	
d	**f(n) = logn!**		
	1 <= log(1*2*3.....*n) <= log(n*n*n*n....*n)	O(logn^n)	
		Omega(1)	
		Cannot find theta for logn!	
14	**Properties of Asymptotic notations**		
	General properties -		
	if f(n) is O(g(n)) then a*f(n) is O(g(n))		
	e.g. f(n) = 2n^2 + 5 is O(n^2), then 7f(n) i.e. 14n^2 + 35 is also O(n^2)	This would be true for both Omega and theta n as well	
	Reflexive property -		
	If f(n) is given then f(n) is O(f(n))		
	e.g. f(n) = n^2 then O(n^2)	A function is an upper bound of itself	
		Similarly, a function is a lower bound of itself	
	Transitive property -		
	If f(n) isO(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))		
	e.g. f(n) = n; g(n) = n^2 and h(n) = n^3	True for all notations	
	n is O(n^2) and n^2 is O(n^3) then n is O (n^3)		
	Symmetric property -		

	If f(n) is theta(g(n)) then g(n) is theta(f(n))	True for only theta(n)	
	e.g. f(n) = n^2 g(n) = n^2 ; f(n) = theta(n^2) and g(n) = theta(n^2)		
	Transpose symmetric -	True for BigO and Omega notations	
	if f(n) = O(g(n)) then g(n) is Omega(f(n))		
	e.g. f(n) = n and g(n) is n^2 then n is O(n^2) and n^2 is Omega(n)		
	If f(n) = O(g(n)) and f(n) = Omega(g(n)) then g(n) <= f(n) <= g(n) therefore f(n) = theta(g (n))		
	If f(n) = O(g(n)) and d(n) = O(e(n)) then f(n) + d(n) = O(max(g(n), e(n))		
	e.g. f(n) = n = O(n) , d(n) = n^2 = O(n^2) then f(n) + d(n) = n + n^2 = O(n^2)		
	If f(n) = O(g(n)) and d(n) = O(e(n)) then f(n) * d(n) = O(g(n) * e(n))		
15	**Comparison of functions**		
	First method is substituting values for n and comparing		
	Second method is applying log on both sides		
		Properties of log -	
	Example -	logab = loga + logb	
	f(n) n^2logn; g(n) = n(logn)^10	loga/b = loga - logb	
	Apply log	loga^b = bloga	
	log(n^2log(n)); log(n(logn)^10)	a^(log_cb) = b^(log_ca)	
	log(n^2) + loglogn; logn + loglog^10	a^b = n then b = log_an	
	2logn + loglogn ; logn + 10loglogn		

	here; 2logn is greater than logn and logn is a bigger term than loglogn		
	so, first term is greater than the second one		
	f(n) = 3n^(rootn) ; g(n) = 2^(rootn log_2(n))		
	Applying log		
	3n(rootn) ; (n^rootn)log_2(2)		
	3n(rootn) ; nrootn		
	first term is greater than the second one value wise but asymptatically they are equal		
	f(n) = n^(logn); g(n) = 2^(rootn)		
	apply log,		
	log(n^logn); log(2^rootn)		
	logn*logn ; rootn (log_2(2))		
	log^2n ; rootn		
	canot judge, so apply log again		
	2loglogn; 1/2logn		
	loglogn is smaller than logn		
	thus, second term is greater		
	f(n) = 2^(logn); g(n) = n^(rootn)		
	logn*log_2(2) ; rootn*logn		
	logn ; rootn*logn		
	second term is greater		
	f(n) = 2n; g(n) is 3n		
	both are equal asymptotically		
	f(n) = 2^n; g(n) = 2^(2n)		
	applying log		

	log(2^n); log(2^2n)		
	n; 2n	after applying log, do not cut coefficients	
	second function is greater		
16	**Best, worst and average case analysis**		
	Example -		
a	**Linear search**		
	A = {8, 6, 12, 5, 9, 7, 4, 3, 16, 18} key = 7		
	In linear search, it will start checking for the given key from left hand side		
	total in 6 comparisons, we would get our key		
	Best case - key element is present at first index		
	Best case time - 1 i.e. B(n) = O(1); Omega (1); Theta(1)		
	Worst case - key element is present at the last index		
	Worst case time - n i.e. W(n) = O(n); Omega(n); Theta(n)		
	Average case = all possible case time / no. of cases		
	average case analysis is very difficult for most of the cases		
	Here, average case time = 1 +2 + 3.... + n/2 = n(n+1)/2n = n+1/2		
	A(n) = n+1/2		
b	**Binary search tree**		
	height = logn		
	time taken for a particular key is logn		
	Best case - element present in the root		
	Best case time - k i.e. B(n) = O(1); Omega (!); Theta(1)		

	Worst case - searching for a leaf element - depends upon the height of the tree		
	Worst case time - logn i.e. O(logn)		
	min w(n) = logn; max w(n) = n		
17	**Disjoint sets**		
	No common numbers between two sets - intersection is zero		
	Operations - find, union		
	Find - search or check membership		
	Union - Add an edge		
	Krisgal algorithm: If you take an edge and both the vertices belong to the same set, then there is a cycle in the graph		
	Weighted union is used while adding edges and detectig cycle		
	Collapsing find - process of directly linking node to a direct parent of a set is called collapsing find - reduces the time to find		
18	**Divide and conquer - Strategy 1**		
	Strategy - an apprach for solving a problem		
	If a problem cannot be solved, divide it into sub-problems and find a solution for each sub problem, combine the solutions. *One point to note is that each sub problem should be similar to the original problem only.*		
	Recursive in nature		
	Should have one method to combine the solutions of each sub problem		
19	**Problems under Divide and Conquer**		
	Binary search		
	Finding maximum and minimum		

	MergeSort		
	QuickSort		
	Strassen's matrix multiplication		
20	**Recurrence relation 1: T(n) = T(n-1) + 1**		
	void test(int n)		
	{		
	if(n > 0){		
	printf("%d",n);		
	test(n-1)		
	}		
	}		
	test(3)		
	3. test(2)		
	2. test(1)		
	1 test(0)		
	each print statement takes constant time 1 and there are n+ 1 calls made to the function. we can ignore the last call when it is not printing		
	f(n) = n + 1 calls ; O(n)		
	T(n) = T(n-1) + 1; if we ignore if condition		
	Let us solve this relation;		
	if we know T(n-1) , we can get T(n)		
	T(n-1) = T(n-2) + 1		
	T(n) = [T(n-2) + 1] + 1		
	T(n) = T(n-3) + 3		
	....continue for k times		
	T(n) = T(n-k) + k		
	We would stop after k substitutions; now we need to find k		

	Assume n - k = 0; therefore n = k		
	T(n) = T(n-n) + n		
	T(n) = T(0) + n		
	T(n) = n + 1 i.e. theta(n)		
21	**Recurrence relation 2: T(n) = T(n-1) + n (decreasing function)**		
	void test(int n)	T(n)	
	{		
	if(n > 0)	1	
	{		
	for(i = 0; i <n; i++)	n+1	
	{		
	printf("%d", n);	n	
	}		
	test(n-1);	T(n-1)	
	}		
	}		
		T(n) = T(n-1) + 2n + 2 i.e. theta(n)	
	we can also write T(n) = T(n-1) + n for n > 0		
	T(n) = 1 for n = 0		
	T(n)	n time	
	n T(n-1)	n-1 time	
	n-1. T(n-2)	n - 2 time	
	n-2 T(n-3)	n - 3 time	
	.....		
	T(2)		
	2 T(1)	2 units of ti me	
	1 T(0)	1 unit of time	
	for T(0) it does nothing	0 unit of time	
	time taken -		

		0 + 1 + 2 +.....+ n-1 + n	
	theta(n^2)	***T(n) = n(n + 1)/2***	
	T(n) = T(n-1) + n		
	T(n-1) = T(n-2) + n-1		
	thus, T(n) = T(n-2) + (n-1) + n	**remember, don't add the terms	
	T(n) = T(n-3) +(n-2) + (n-1) + n		
	T(n) = T(n-k) + (n-(k-1)) + (n-(k-2))....+(n-1) + n	if we continue for k times	
	assume n - k = 0; n = k		
	Thus, T(n) = T(n-n) + (n - n + 1) + (n - n + 2)..... +(n-1) + n		
	T(n) = T(0) + n(n+1)/2		
	T(n) = 1 + n(n+1)/2	***theta(n^2)*** ; this extra 1 is owing to the calls	
22	**Recurrence relation 3: T(n) = T(n-1) + logn**		
	void test(int n)	T(n)	
	{		
	if(n>0)		
	{		
	for(i = 1; i <n; i = i*2)		
	{		
	printf("%d", i);	log n times	
	}		
	test(n-1);	T(n-1)	
	}		
	}		
	T(n) = T(n-1) + logn for n > 0		
	T(n) = 1 for n = 0		
	Solve using tree method,		

	T(n)		
	logn T(n-1)		
	log(n-1) T(n-2)		
	log(n-2) T(n-3)		
	.....		
	log2 T(!)		
	log 1 T(0)		
	logn + log(n-1) + ...+ log2 + log1		
	log[n(n-1)(n-2)....2.1] = log(n!)	there is no tight bound for this function but there is an upper bound for it	
	O(nlogn)		
	Solving using induction method.		
	T(n) = T(n-1) + logn		
	T(n) = T(n-2) + log(n-1) + log(n)		
	T(n) = T(n-3) + log(n-2) + log(n-1) + logn		
	.....		
	T(n) = T(n-k) + logn + log(n-1) +....log1		
	Asume n-k = 0		
	T(n) = T(0) + logn!		
	T(n) = 1 + logn!		
	O(nlogn)		
23	***How to get the direct answer for a recurrence relation?***		
	T(n) = T(n-1) + 1	O(n)	
	T(n) = T(n-1) + n	O(n^2)	
	T(n) = T(n-1) + logn	O(nlogn)	
	T(n) = T(n-1) + n^2	O(n^3)	
	T(n) = T(n-2) + 1	O(n/2) ~ O(n)	

	T(n) = T(n-100) + n	O(n^2)	
	T(n) = 2T(n-1) + 1	???	
24	**Recurrence relation 4: T(n) = 2T(n-1) + 1**		
	Test(int n)	T(n)	
	{		
	if(n > 0)	1	
	{		
	printf("%d", n);	1	
	test(n-1);	T(n-1)	
	test(n-1);	T(n-1)	
	}		
	}		
		T(n) = 2T(n-1) + 1	
	T(n) = 2T(n-1) + 1 for n> 0		
	T(n) = 1 for n = 0		
	Solve using recursion tree method		
	1 T(n-1) T(n-1)		2
	1 T(n-2) T(n-2)	1 T(n-2) T(n-2)	4
	1 T(n-3) T(n-3) 1 T(n-3) T(n-3)	1 T(n-3) T(n-3) 1 T(n-3) T(n-3)	8
	.....		
	T(0). T(0)		2^k
	1 + 2 + 2^2 .....+2^k = 2^(k+1) - 1		
	as, a + ar + ar^2.....ar^k = a(r^(k+1) - 1)/(r -1)		
	Assume n - k = 0		
	thus, 2^(n+1) - 1	***O(2^n)***	

	Back substitution method		
	T(n) = 2T(n-1) + 1		
	T(n) = 4T(n-2) + 2 + 1		
	T(n) = 8T(n-3) + 4 + 2 + 1		
	.....		
	T(n) = 2^kT(n - k) + 2^(k-1) + 2^(k).....2^3 + 2^2 + 1		
	Assume n - k = 0		
	n = k		
	T(n) = 2^nT(0) + 1 + 2 + 2^2.... + 2^n-1		
	T(n) = 2^n + 2^n - 1 i.e. 2^(n+1) - 1		
25	**Master theorem for decreasing function**		
	T(n) = T(n-1) + 1	O(n)	
	T(n) = T(n-1) + n	O(n^2)	
	T(n) = T(n-1) + logn	O(nlogn)	
	T(n) = 2T(n-1) + 1	O(2^n)	
	T(n) = 3T(n-1) + 1	O(3^n)	
	T(n) = 2T(n-1) + n	O(n2^n)	
	T(n) = 2T(n-2) + 1	O(2^n/2)	
	T(n) = aT(n-b) + f(n)		
	a > 0, b > 0 and f(n) = O(n^k) where k >= 0		
	if a = 1, O(n^k+1) or O(n*f(n))		
	if a > 1, O(n^k * a^n/b)		
	if(a < 1) O(n^k) or O(f(n))		
26	**Dividing functions**		
	test(int n)	T(n)	
	{		

	if(n > 1)		
	{		
	printf("%d", n);	1	
	test(n/2)	T(n/2)	
	}		
	}		
	T(n) = T(n/2) + 1 for n > 1		
	T(n) = 1 for n = 1		
	T(n)		
	1 T(n/2)		
	1 T(n/2^2)		
	1 T(n/2^3)		
	.....continue for k times		
	1 T(n/2^k)		
	assume , n/2^k = 1		
	thus, we have taken k steps overall		
	since, n/2^k = 1 => k = logn with base 2	***O(logn)***	
	Solving by substitution method		
	T(n) = T(n/2) + 1		
	T(n) = T(n/2^2) + 2		
	T(n) = T(n/2^3) + 3		
	....		
	T(n) = T(n/2^k) + k		
	assume n/2^k = 1		
	thus, k = logn with base 2		
	T(n) = T(1) + logn		

	O(logn)		
27	**Recurrence relation: T(n) = T(n/2) + n**		
	T(n) = T(n/2) + n for n > 1		
	T(n) = 1 for n=1		
	T(n)		
	T(n/2) n		
	T(n/2^2) n/2		
	T(n/2^3) n/2^2		
	.....		
	T(n/2^k). n/2^(k-1)		
	T(n) = n + n/2 + n/2^2 + n/2^3..... + n/2^k		
	T(n) = n[1 + 1/2 + 1/2^2 + 1/2^3 +.....1/2^k]		
	T(n) = n*1 = n		
	O(n)		
	Using substitution method		
	T(n) = T(n/2) + n		
	.....		
	T(n) = T(n/2^2) + n/2 + n		
	.....		
	T(n) = T(n/2^3) + n/2^2 + n/2 + n		
	.....		
	T(n) = T(n/2^k) + n/2^k-1 ....+n/2^2 + n/2 + n		
	Assume n /2^k = 1		
	k = logn with base 2		
	T(n) = T(1) + n[1/2^k-1.....+1/2^2....+1]		
	T(n) = 1 + 2n ~ O(n)		

28	**Recurrence Relation: T(n) = 2T(n/2) + n**		
	void test(int n)	T(n)	
	{		
	if(n > 1)		
	{		
	for(int i = 0; i <n ; i++)		
	{		
	stmt	n	
	}		
	test(n/2);	T(n/2)	
	test(n/2);	T(n/2)	
	T(n) = 2T(n/2) + n for n > 1		
	T(n) = 1 for n = 1		
	Solve using recursion tree method,		
	T(n)		
	T(n/2). T(n/2) n	n	
	T(n/2^2) T(n/2^2) T(n/2^2) T(n/2^2) n/2	n	
	T(n/2^3). T(n/2^3). T(n/2^3). T(n/2^3). T (n/2^3). T(n/2^3). T(n/2^3). T(n/2^3).	n	
	.....	n	
	T(n/2^k).....		
		n	
	assume n / 2^k = 1		
	k = logn with base 2		
	T(n) = nk ~ O(nlogn)		
	Using backsubstitution method;		

	T(n) = 2T(n/2) + n		
	T(n/2) = 2T(n/2^2) + n/2		
	T(n) = 2[2T(n/2^2) + n/2] + n		
	T(n) = 2^2T(n/2^2) + n +n		
	T(n) = 2^3T(n/2^3) + 3n		
	....continue for k times		
	T(n) = 2^kT(n/2^k) + kn		
	Asume T(n/2^k) = T(1)		
	k = logn with base 2		
	Thus, T(n) = n +nlogn ~O(nlogn)		
29	**Masters Theorem for dividing functions**		
	T(n) = aT(n/b) + f(n)	loga with b	
	a>=1; b > 1; f(n) = theta(n^k* log^pn)	k	
	case 1: if loga with base b > k then theta(n^ (loga with base b))		
	case 2: if loga with base b = k then		
	if p > -1 theta(n^klog^(p+1)n)		
	if p = -1 theta(n^kloglogn))		
	if p < -1 then theta(n^k)		
	case 3: if loga with base b < k		
	then, if p >= 0, theta(n^klog^pn)		
	if p < 0 , theta(n^k)		
	T(n) = 2T(n/2) + 1		
	a = 2		
	b = 2		
	f(n) = theta(n^(0) * log n ^ 0)		
	k = 0; p = 0		
	here, loga with base b > k		

	theta(n^1) where loga with base b is 1		
	T(n) = 4T(n/2) + n		
	log a with base b = 2		
	k = 1		
	p = 0		
	this is an example of case 1		
	theta(n^2)		
	T(n) = 8T(n/2) + n		
	log8 with base 2 = 3 > k = 1		
	theta(n^3)		
	T(n) = 9T(n/3) + 1		
	loga with base b = 2 > k		
	theta(n^2)		
	T(n) = 9T(n/3) + n^2		
	loga with base b = 2 = k	case 2	
	theta(n^2)		
	T(n) = 8T(n/2) + n		
	theta(n^3)		
	T(n) = 2T(n/2) + n		
	loga with base b = k = 1; p = 0		
	case 2		
	theta(nlogn)		
	T(n) = 4T(n/2) + n^2		

	theta(n^2logn)		
	T(n) = 4T(n/2) + n^2logn		
	theta(n^2logn^2)		
	T(n) = 8T(n/2) + n^3		
	theta(n^3logn)		
	T(n) = 2T(n/2) + n/logn		
	log a with base b = k = 1		
	p = -1		
	theta(nloglogn)		
	T(n) = 2T(n/2) + n/logn^2		
	p = -2		
	theta(n)		
	T(n) = 2T(n/2) + n^2		
	loga with base b < k		
	theta(n^2)		
	T(n) = 2T(n/2) + n^2		
	theta(n^2logn)		
	T(n) = 2T(n/2) + n^3		
	loga with base b < k		
	theta(n^3)		
30	T(n) = 2T(n/2) + 1		
	loga with base b = 1		

	k = 0		
	loga with base b > k		
	theta(n^1)		
	T(n) = 4T(n/2) + 1		
	loga with base b = 2		
	k = 0		
	theta(n^2)		
	T(n) = 4T(n/2) + n		
	loga with base b = 2		
	k = 1		
	theta(n^2)		
	T(n) = 8T(n/2) + n^2		
	loga with base b = 3		
	k = 2		
	theta(n^3)		
	T(n) = 16T(n/2) + n^2		
	loga with base b = 4		
	k = 2		
	theta(n^4)		
	T(n) = T(n/2) + n		
	log a with base b = 0		
	k = 1		
	theta(n)		
	T(n) = 2T(n/2) + n^2		

	loga with base b = 1		
	k = 2		
	theta(n^2)		
	T(n) = 2T(n/2) + n^2logn		
	loga with base b = 1		
	k = 2		
	theta(n^2logn)		
	T(n) = 4T(n/2) + n^3log^2n		
	loga with base b = 2		
	k = 3		
	theta(n^3log^2n)		
	T(n) = 2T(n/2) + n^2 / logn		
	log a with base b = 1		
	k = 2		
	theta(n^2)		
	T(n) = T(n/2) + 1		
	log a with base b = 0		
	k = 0		
	theta(logn)		
	T(n) = 2T(n/2) + n		
	log a with base b = 1		
	k = 1		
	p = 0		
	theta(nlogn)		

	T(n) = 2T(n/2) + nlogn		
	log a with base b = 1		
	k = 1		
	p = 1		
	theta(nlog^2n)		
	T(n) = 4T(n/2) + n^2		
	log a with base b = 2		
	k = 2; p = 0		
	theta(n^2logn)		
	T(n) = 4T(n/2) + (nlogn)^2		
	log a with base b = 2		
	k = 2, p = 2		
	theta(n^2 log^3n)		
	T(n) = 2T(n/2) + n/logn		
	log a with base b = 1		
	k = 1; p = -1		
	theta(nloglogn)		
	T(n) = 2T(n/2) + n/log^2n		
	log a with base b = 1		
	k = 1; p = -2		
	theta(n)		
31	**Root function Recurrence relation**		
	T(n) = T(root(n) + 1) for n>2		
	T(n) = 1 for n = 2		

	T(n) = T(root(n)) + 1		
	T(n) = T(n^(1/2)) + 1 .....equation 1		
	using substitution		
	T(n) = T(n^(1/2^2)) + 2.....equation 2		
	T(n) = T(n^(1/2^3)) +3.....equation 3		
	T(n) = T(n^(1/2^k)) + k.....equation 4		
	assume, n = 2^m		
	T(2^m) = T(2^(m/2^k)) + k		
	assume T(2^(m/2^k)) = T(2)		
	thus, m/2^k = 1		
	m = 2^k		
	k = log m with base 2		
	substituting value of n		
	m = logn with base 2		
	therefore, k = loglogn with base 2		
	theta(loglogn with base 2)		
32	**Binary Search Iterative Method**		
	To perform binary search, the prerequisite is that the list must be in sorted order	A = {3, 6, 8, 12, 14, 17, 25, 29, 31, 36, 42, 47, 53, 55, 62}	
	we need two index pointers, one is low at the starting point and the other is high at the end point	l = 1, h = 15 (lowest and highest index); mid = 8	
	mid = low + high / 2 and we take the floor value	key value = 42; A[mid] = 29 --> key > A [mid]	
	the key value is on the right hand side as key value is greater than A[mid]		
	we will change low to mid + 1	l = 9, h = 15; mid = 9 + 15 / 2 = 12	
		A[mid] = 47 > key	
	we will change high to mid - 1 as key < A [mid]		

		h = 11, l = 9, mid = 10; A[mid] = 36	
		A[mid] < key	
	we will change low to mid + 1	l = 11; h = 11; mid = 11; A[mid] = 42	
	we can return the index as we have found the key value	A[mid] = key	
	therefore, binary search looks faster than linear search. It just took 4 comparisons		
	int BinSearch(A, n, key)		
	{		
	l = 1, h = n		
	mid = l + h / 2 - take floor value		
	while(l <= h){		
	if(key == A[mid])		
	{ return index i.e.element is found}		
	else if(key < A[mid])		
	{h= mid-1;}		
	else {		
	l = mid + 1;}		
	}		
	return 0;		
	}		
	Time taken for binary search = logn		
	min time: O(1)		
	max time: O(logn)		
	avg time = add time for each element and divide by number of elements		
33	**Binarysearch Recursive method**		
	Alogirthm RBinarySearch(l,h,key)	T(n)	

	{		
	if(l==h)	1	
	{		
	if(A[low]== key)		
	{		
	return l;		
	}		
	else		
	{		
	return 0;		
	}		
	else		
	{		
	mid = l + h / 2 //taking floor value	1	
	if(key == A[mid])	1	
	{return mid;}		
	if(key < A[mid])	1	
	{		
	return RBinarySearch(l, mid - 1, key)	T(n/2)	
	}		
	else		
	{		
	return RBinarySearch(mid+1, h, key)	T(n/2)	
	}		
	}		
		T(n) = 1; n =1	
		T(n) = T(n/2) + 1 for n > 1	
		theta(logn)	

34	**Heaps**		
a	**Representation of a binary tree using an array**		
	T {A, B, C, D, E, F, G}		
	if a node is at index i;		
	its left child is at node 2*i		
	its right child is at node 2*i + 1		
	its parent is at node i/2		
	if there are missing nodes, we leave a blank in its place in the array		
b	**Full binary tree**		
	In its hieght, it has maximum number of nodes and if we wish to add a node, height would increase		
	Max no. of nodes = 2^h - 1		
c	**Complete binary tree**		
	there is no missing element from first element to the last element in array representation of the binary tree		
	Every full binary tree is also a complete binary tree		
	A complete binary tree is a full binary tree until height h - 1		
	Height of a complete binary tree would be minimum i.e. logn		
d	**Heap**		
	Heap is a complete binary tree		
	Max Heap: every node has value greater than all its descendants {50, 30, 20, 15, 10, 8, 16}		
	Min Heap: every node has value smaller or equal to than all its descendants {10, 30, 20, 35, 40, 32, 25}		

35	**Insert operation in a max heap**		
	Insert 60 in the above given max heap		
	this value should be inserted in the last free space in the array		
	i.e. left child of the left most leaf node		
	Then, adjust the elements to make it as a heap		
	So, compare and move 60 up the levels and in the array check at i/2 indices where initially i would be the last empty index where 60 was inserted		
	Time taken would be equal to the number of swaps		
	this depends upon the height of the tree i.e. logn, hence O(logn)		
	minimum time is of no swaps O(1); max would be O(logn)		
36	**Delete operation in a max heap**		
	From the heap, we need to remove the root / top most element only		
	The last element in the complete binary tree would come in its place		
	Adjust the elements to maintain heap order		
	From the root towards the leaf, adjust		
	Compare the children (2i and 2i +1) and whichever child is greater than compare with the parent		
	Time taken depends upon the height; max could be O(logn)		
	Whenever you delete from max heap, you get the next max element and in case of min heap, it would be the next min element		
37	**HeapSort**		

	For a given set of numbers, create a heap		
	Delete all the elements from the heap		
	Total N elements we have inserted; each element we assume is moved up to the root; so time taken O(NlogN)		
	Then we delete the elements		
	Store deleted elements in the array in free space in the end		
	Deletion also takes O(NlogN) time		
	Thus, heapsort takes O(NlogN)		
38	**Heapify**		
	The process of creating heap but direction is opposite than creating a heap		
	O(N)		
39	**Priority Queue**		
	elements will have priority and they would be inserted and deleted as per the priority order		
	For min heap, smaller the no. higher the priority		
	For max heap, greater the no. higher the priority		
	O(logN) for insertion and/or deletion		
40	**TwoWay MergeSort - Iterative method**	Algorithm Merge(A, B, m, n)	
	merging two sorted lists to get a sorted result	{i = 1, j = 1, k = 1;	
	A = {2, 8, 15, 18} i	while(i <= m && j<=n){	
	B = { 5, 9, 12, 17} j	if(A[i] < B[j])	
	Compare A(i) with B(j) to get C(k) and move to next location	{	
	m + n elements are obtained , thus theta(m + n)	C[k++] = A[i++];	

		}	
		else {	
		C[k++] = B[j++];	
		}	
		for(; i <=m; i++){	
		C[k++]=A[i];	
		}	
		for(;j<= n;j++){	
		C[k++] = B[j];	
		}	
		}	
41	**Merging more than two lists**		
	M-way merging		
	A = {4, 6, 12}		
	B = {3, 5, 9}		
	C = {8, 10, 16}		
	D = {2, 4, 18}		
	One way is that we merge A and B; C and D and then finally merge the two resulting lists --> so we perform merge three times here		
	Another way is that we first merge A and B; then we merge resulting list with C ; and the resulting list with D		
	Two-way mergesort is an iterative process whereas mergeSort is a recursive process		
	A = {9, 3, 7, 5, 6, 4, 8, 2} - given an array and we have to sort them using 2-way mergesort		
Ist pass	We would consider each element as a sorted list and merge	merged n elements in this pass	
	First select two lists 3 and 9; then merge them - 3, 9		

	Similarly, we select two lists 7 and 5 , merge them - 5 and 7		
	Another lists we get are {4, 6} and {2, 8}		
	Now, we have 4 lists with two elements each		
2nd pass	When we merged we kept the resulting 4 lists in another array B; B = {{3, 9}, {5, 7}, {4, 6}, {2, 8}}	merged n elements in this pass	
	We merge two lists each		
3rd pass	C = {{3, 5, 7, 9}, {2, 4, 6, 8}}	merged n elements in this pass	
	we merge the above two lists to get a single sorted list		
	D = {2, 3, 4, 5, 6, 7, 8, 9}		
	log(no of elements) = no. of passes		
	Time complexity: O(n(logn))		
42	**MergeSort**		
	A = {9, 3, 7, 5, 6, 4, 8, 2}	Algorithm MergeSort(l, h){	T(n)
	If there is a single element, we can consider it as a base or small problem {Divide and conquer}		
		if(l < h){	
		mid = (l + h) / 2;	1
		MergeSort(l, mid);	T(n/2)
		MergeSort(mid + 1, h);	T(n/2)
		Merge(l, mid, h);	n
		}	T(n) = 2T(n/2) + n for n > 1
		}	T(n) = 1 for n = 1
	time complexity: theta(nlogn)		using master's theorem, a = 2, b = 2, k = 1
	merging is done in post order traversal		loga with base b = 1 = k
			thus, it is case 2
			theta(nlogn)

43	**Pros of MergeSort**	**Cons of MergeSort**	
	works great for Large size lists	Extra space (not inplace sort)	
	suitable for Linked List	no small problem	
	supports external sorting	recursive and uses a stack (need n + logn space) i.e. space complexity: O(n + logn) where n is the extra space and logn is the stack space	
	stable: the order of duplicates is maintained		
		insertion sort (O(n^2))	
		mergesort O(nlogn)	
		for small problems, n <= 15; insertionsort works better --> use insertion sort	
43	**QuickSort**		
	students arranging themselves in increasing order of heights		
	10 80 90 60 30 20		
	5 6 3 4 2 1 **9**		
	4 6 7 **10** 16 12 13 14		
	A = {10, 16, 8, 12, 15, 6, 3, 9, 5, INFINITY}	partition(l, h){	
	select first element as a pivot	pivot = A[l];	
	pivot = 10	i = l; j = h;	
	we need to find the sorted position for 10	while(i<j){do	
	i starting form pivot and j starting from infinity	{	
	i would check for elements greater than 10; j would heck for elements smaller than pivot	i++;	
	we are using the partitioning procedure	} while(A[i]<= pivot);	
	increment i until next vlue is greater than 10 and decrement j until next value is smaller than pivot; stop and swap	do	
	{10, 5, 8, 9, 3, 6, 15, 12, 16}	{	
	send pivot element at j position	j--;	

	now, we can sort the two lists around the partitioning position by performing quicksort recursively	}while(A[j] > pivot);	
		if(i<j){	
	QuickSort(l, h)	swap(A[i], A[j]);	
	{	}	
	if(l < h)	swap(A[l], A[j]);	
	{	return j;	
	j = partition(l, h);	}	
	QuickSort(l, j);		
	QuickSort(j+ 1, h);		
	}		
	}		
44	**QuickSort Analysis**		
	suppose it is partitioning in the middle of 1 and 15th index		
	then, two partitions: [1, 7] ; [9, 15]		
	further partitions: [1, 3]; [5, 7]; [9, 11]; [13, 15]		
	at each level , n elements are being handled		
	and there are logn levels		
	thus time complexity for best case: O(nlogn)		
	median : middle element of a sorted list		
	best case of quicksort is that the partitioning occurs exactly at the middle		
	worstcase: if we have an already sorted list		
	time complexity for worstcase: O(n^2)		
	to handle this, try taking middle element as a pivot		
	2. select random element as a pivot		

45	**Strassen's matrix multiplication**		
	A = [a11 a12		
	a21. a22]		
	B = [b11 b12		
	b21 b22]		
	Cij = Summing up Aik*Bkj		
	for(i = 0; i < n ; i++){		
	for(j = 0; i < n ; j++){		
	C[i,j]= 0;		
	for(k=0;k<n;k++){		
	C[i,j] += A[i, k]*B[k, i];		
	}		
	}		
	}		
	C11 = a11*b11 + a12*b21		
	C21 = a11*b12 + a12*b22	A = [a11]	
	C21 = a21*b11 + a22*b21	B = [b11]	
	c22 = a21*b12 + a22*b22	C = [a11*b11]	
	for [2*2] matrix, we would use above formula	for [1*1] matrix, use above formula	
	we assume that the matrix has dimensions of power of 2	Algorithm MM(A, B, n)	
		{	
		if(n <= 2	
	8 times the function is calling itself	{	
	T(n) = 8T(n/2) + n^2 for n > 1	C = 4 formula stated above;	
	a = 8, b = 2, log a with base b = 3	}	
	k = 2	else	
	it is case 1 of master's theorem	{	
	theta(n^3)	mid = n/2	

		MM(A11, B11, n/2) + MM(A12, B21, n/2);	
		MM(A11, B12, n/2) + MM(A12, B22, n/2);	
		MM(A21, B11, n/2) + MM(A22, B21, n/2);	
		MM(A22, B22, n/2) + MM(A21, B12, n/2);	
		}	
		}	
	Strassen's approach -		
	has given 4 different formulas with 7 multiplications	P = (A11 + A22)(B11 + B22)	
	C11 = A11*B11 + A12*B21	Q = (A21 + A22) B11	
	C21 = A11*B12 + A12*B22	R = A11(B12 - B22)	
	C21 = A21*B11 + A22*B21	S = A22(B21 - B11)	
	C22 = A21*B12 + A22*B22	T = (A11 + A12)B22	
		U = [A21 - A11)(B11 + B12)	
		V = (A12 - A22)(B21 + B22)	
		C11 = P + S - T + V	
		C12 = R + T	
		C13 = Q + S	
		C22 = P + R- Q + U	
		T(n) = 7T(n/2) + n^2 for n > 2	
		T(n) = 1 for n<= 2	
		using master's theorm,	
		O(n^(log7 with base 2)) = O(n^2.81)	