

<u>S.No.</u>							
	<b>1 Algorithm</b>	<b>Program</b>					
	Design time	Implementation time					
	Domain knowledge	Programmer					
	Any language even English and Maths	Programming language					
	Hardware and software independent	Hardware and operating system dependent					
	Analyze an algorithm	Testing of programs					
	<b>2 Priori Analysis</b>	<b>Posterior Testing</b>					
	Algorithm	Program					
	Independent of language	Language dependent					
	Hardware independent	Hardware dependent					
	Time and space function	watch time and bytes					
	<b>3 Characteristics of algorithm</b>						
	Zero or more inputs						
	Must generate atleast one output						
	Definiteness						
	Finiteness						
	Effectiveness						
	<b>4 How to analyze an algorithm</b>						
	Time						
	Space						
	Network consumption : Data transfer amount						
	Power consumption						
	CPU registers						
	<b>5 Frequency Count Method</b>	Used for time analysis of an algorithm					
	Assign 1 unit of time for each statement						
	For any repetition, calculate the frequency of repetition						
	for(i = 0; i < n; i++) --> condition is checked for n+1 times	2n + 2 units of time ~ n+1 as we see condition i < n only for now					
	any statement within the loop will execute for n times						

	Space complexity depends upon number and kind of variables used					
<b>6</b>	<b>Algorithm : sum(A, n)</b>					
	Single for loop -					
	Time complexity: $O(N)$					
	Space complexity: $O(N)$					
<b>7</b>	<b>Algorithm : Add(A, B, n)</b>	Sum of two square matrices of dimensions $n \times n$				
	Two nested for loops -					
	Time complexity: $O(N^2)$					
	Outer for loop executes for $N+1$ times					
	Inner for loop executes for $N \cdot (N+1)$ times					
	Any statement within inner for loop executes for $(N + 1) \cdot (N + 1)$ times					
	Space complexity: $O(N^2)$					
<b>8</b>	<b>Algorithm : Multiply(A, B, n)</b>					
	Three nested for loops -					
	Time complexity: $O(N^3)$					
	Space complexity: $O(N^2)$					
<b>9</b>	<b>Different algorithm conditions</b>					
	<b>For loops</b>					
	for( $i = n$ ; $i > 0$ ; $i--$ )	$n+1$ times				
	for( $i = 0$ ; $i < n$ ; $i = i + 2$ )	$n/2$ times				
	2 nested for loops where both $i$ and $j$ range from 0 to $n$	$n^2$ times				
	2 nested for loops where $j$ ranges from 0 to $i$	when $i = 0$ ; $j$ loop repeats 0 times; when $i = 1$ ; $j$ loop repeats 1 times; and so on...total number of repetitions: $0 + 1 + 2 + 3 + 4 + \dots + n = O(n^2)$				
	$p = 0$ ; for( $i = 1$ ; $p \leq n$ ; $i++$ ) { $p = p + i$ ; }	$p = k(k+1)/2 \rightarrow$ assuming that the loop exits when $p$ is greater than $n \rightarrow k(k+1) / 2 > n$	$\sim k^2 > n \rightarrow O(\sqrt{n})$			
	for( $i = 1$ ; $i < n$ ; $i = i * 2$ )	will execute for $2^k$ times	$O(\log n)$			
		Assume $i \geq n$ ; $i = 2^k \geq n$				
		$k = \log n$ with base 2				

for(i = n; i >= 1; i = i/2)	i					
	n					
	n/2					
	n/2^2					
	n/2^3					
	.....					
	n/2^k					
	Assume $i < 1 \Rightarrow n / 2^k < 1$		$\sim O(\log n)$ with base 2			
for(i = 0; i * i < n; i++)	$i^2 < n$					
	$i^2 > -n$					
	$i^2 = n \rightarrow i = \text{root}(n)$		$\sim O(\text{root}(n))$			
for(i = 0; i < n; i++) {.....}for(j = 0; j < n; j++){.....}	$O(n)$					
p = 0; for(i = 1; i < n; i*2){.....} for(j = 1; j < p; j*2){.....}	log n times for upper loop; log p times for lower loop		$\sim O(\log(\log n))$			
for(i = 0; i < n; i++) {.....for(j = 0; j < n; j*2){.....}}	Outer loop repeats n times; inner loop repeats logn times		$\sim O(n \log n)$			
for(i = 1; i < n; i = i*3)			$\sim O(\log n)$ with base 3			
<b>While loops</b>						
while vs. do while	do while will execute for minimum one time					
for and while are almost similar	do while will execute as long as the condition is true; for loop will execute until the condition is false					
a = 1;						
while(a < b){ ..... a = a *2;}	1, 2, 2^2, 2^3.....2^k repetitions		$\sim O(\log b)$ with base 2			
	assume $a > b$ ; $2^k > b \Rightarrow k = \log b$ with base 2					
i = n; while(i > 1) {....i = i/2;}			$\sim O(\log n)$ with base 2			
i = 1; k = 1; while(k < n){....k = k + i; i++;}						
	i k					
	1	1				
	2 1 + 1					
	3 2 + 2					
	4 2 + 2 + 3					

	5	$2 + 2 + 3 + 4$					
	.....						
	m	$m(m + 1) / 2$					
	Assume, $k \geq n$	$m(m + 1) / 2 \geq n$	$\sim O(\sqrt{n})$				
	while( $m \neq n$ ) { if( $m > n$ ) $m = m - n$ ; else $n = n - m$ ;		$\sim O(n)$				
	<b>10 Types of time functions</b>						
	$O(1)$ --- constant						
	$O(\log n)$ --- logarithmic						
	$O(n)$ --- linear						
	$O(n^2)$ --- quadratic						
	$O(n^3)$ --- cubic						
	$O(2^n)$ --- exponential						
	<b>11 Order of complexity</b>						
	$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \dots < 2^n < 3^n$ ..... $< n^n$						
	<b>12 Asymptotic Notations</b>						
	Representation of time complexity in simple form which is understandable						
	Big O Notation - works as an upper bound	The function $f(n) = O(g(n))$ iff for all positive constants $c$ and $n_0$ , such that $f(n) \leq c * g(n)$ for all $n \geq n_0$ ; here, $f(n) = O(n)$	e.g. $2n + 3 \leq 10n$ ; All those functions in time order complexity above $n$ become upper bound; below $n$ become lower bound and $n$ is the average bound				

	Big Omega Notation - works as a lower bound	The function $f(n) = \Omega(g(n))$ iff for all positive constants $c$ and $n_0$ , such that $f(n) \geq c \cdot g(n)$ for all $n \geq n_0$ ; here, $f(n) = \Omega(g(n))$	e.g. $2n + 3 \geq 1n$				
	Theta Notation - works as an average bound	The function $f(n) = \theta(g(n))$ iff for all positive constants $c_1$ , $c_2$ and $n_0$ such that $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$	e.g. $f(n) = 2n + 3$ ; $1n \leq 2n + 3 \leq 5n$				
	Most useful is theta notation, then why do we need the other two?	In case we are not able to get the average bound, then we point to its upper or lower bound					
	<b>13 Examples for asymptotic notations</b>						
<b>a</b>	<b><math>f(n) = 2n^2 + 3n + 4</math></b>						
	$2n^2 + 3n + 4 \leq 2n^2 + 3n^2 + 4n^2$ i.e. $9n^2$	$O(n^2)$					
	$2n^2 + 3n + 4 \geq 1n^2$	$\Omega(n^2)$					
	$1n^2 \leq 2n^2 + 3n + 4 \leq 9n^2$	$\Theta(n^2)$					
<b>b</b>	<b><math>f(n) = n^2 \log n + n</math></b>						
	$n^2 \log n \leq n^2 \log n + n \leq 10n^2 \log n$	$O(n^2 \log n)$					
		$\Omega(n^2 \log n)$					
		$\Theta(n^2 \log n)$					
<b>c</b>	<b><math>f(n) = n!</math></b>						
	$1 \leq 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n-1 \cdot n \leq n \cdot n \cdot n \cdot \dots \cdot n$	$O(n^n)$					
		$\Omega(1)$					
		Cannot find theta for $n!$					
<b>d</b>	<b><math>f(n) = \log n!</math></b>						
	$1 \leq \log(1 \cdot 2 \cdot 3 \cdot \dots \cdot n) \leq \log(n \cdot n \cdot n \cdot \dots \cdot n)$	$O(\log n^n)$					
		$\Omega(1)$					
		Cannot find theta for $\log n!$					
	<b>14 Properties of Asymptotic notations</b>						
	General properties -						
	if $f(n)$ is $O(g(n))$ then $a \cdot f(n)$ is $O(g(n))$						
	e.g. $f(n) = 2n^2 + 5$ is $O(n^2)$ , then $7f(n)$ i.e. $14n^2 + 35$ is also $O(n^2)$	This would be true for both $\Omega$ and $\theta$ as well					

Reflexive property -						
If $f(n)$ is given then $f(n)$ is $O(f(n))$						
e.g. $f(n) = n^2$ then $O(n^2)$	A function is an upper bound of itself					
	Similarly, a function is a lower bound of itself					
Transitive property -						
If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ then $f(n)$ is $O(h(n))$						
e.g. $f(n) = n$ ; $g(n) = n^2$ and $h(n) = n^3$	True for all notations					
$n$ is $O(n^2)$ and $n^2$ is $O(n^3)$ then $n$ is $O(n^3)$						
Symmetric property -						
If $f(n)$ is $\theta(g(n))$ then $g(n)$ is $\theta(f(n))$	True for only $\theta(n)$					
e.g. $f(n) = n^2$ $g(n) = n^2$ ; $f(n) = \theta(n^2)$ and $g(n) = \theta(n^2)$						
Transpose symmetric -	True for BigO and Omega notations					
if $f(n) = O(g(n))$ then $g(n)$ is $\Omega(f(n))$						
e.g. $f(n) = n$ and $g(n)$ is $n^2$ then $n$ is $O(n^2)$ and $n^2$ is $\Omega(n)$						
If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ then $g(n) \leq f(n) \leq g(n)$ therefore $f(n) = \theta(g(n))$						
If $f(n) = O(g(n))$ and $d(n) = O(e(n))$ then $f(n) + d(n) = O(\max(g(n), e(n)))$						
e.g. $f(n) = n = O(n)$ , $d(n) = n^2 = O(n^2)$ then $f(n) + d(n) = n + n^2 = O(n^2)$						
If $f(n) = O(g(n))$ and $d(n) = O(e(n))$ then $f(n) * d(n) = O(g(n) * e(n))$						
<b>15 Comparison of functions</b>						
First method is substituting values for $n$ and comparing						
Second method is applying log on both sides						
	Properties of log -					
Example -	$\log ab = \log a + \log b$					
<b><math>f(n) = n^2 \log n</math>; <math>g(n) = n(\log n)^{10}</math></b>	$\log a/b = \log a - \log b$					
Apply log	$\log a^b = b \log a$					

$\log(n^2 \log(n)); \log(n(\log n)^{10})$	$a^{(\log_{cb})} = b^{(\log_{ca})}$					
$\log(n^2) + \log \log n; \log n + \log \log^2 n$	$a^b = n$ then $b = \log_a n$					
$2 \log n + \log \log n; \log n + 10 \log \log n$						
here; $2 \log n$ is greater than $\log n$ and $\log n$ is a bigger term than $\log \log n$						
so, first term is greater than the second one						
<b><math>f(n) = 3n^{(\sqrt{n})}; g(n) = 2^{(\sqrt{n} \log_2(n))}</math></b>						
Applying log						
$3n^{(\sqrt{n})}; (n^{\sqrt{n}}) \log_2(2)$						
$3n^{(\sqrt{n})}; n^{\sqrt{n}}$						
first term is greater than the second one value wise but asymptotically they are equal						
<b><math>f(n) = n^{(\log n)}; g(n) = 2^{(\sqrt{n})}</math></b>						
apply log,						
$\log(n^{\log n}); \log(2^{\sqrt{n}})$						
$\log n \cdot \log n; \sqrt{n} (\log_2(2))$						
$\log^2 n; \sqrt{n}$						
cannot judge, so apply log again						
$2 \log \log n; 1/2 \log n$						
$\log \log n$ is smaller than $\log n$						
thus, second term is greater						
<b><math>f(n) = 2^{(\log n)}; g(n) = n^{(\sqrt{n})}</math></b>						
$\log n \cdot \log_2(2); \sqrt{n} \cdot \log n$						
$\log n; \sqrt{n} \cdot \log n$						
second term is greater						
<b><math>f(n) = 2n; g(n)</math> is <math>3n</math></b>						
both are equal asymptotically						
<b><math>f(n) = 2^n; g(n) = 2^{(2n)}</math></b>						
applying log						
$\log(2^n); \log(2^{2n})$						
$n; 2n$	after applying log, do not cut coefficients					

	second function is greater						
	<b>16 Best, worst and average case analysis</b>						
	Example -						
<b>a</b>	<b>Linear search</b>						
	A = {8, 6, 12, 5, 9, 7, 4, 3, 16, 18} key = 7						
	In linear search, it will start checking for the given key from left hand side						
	total in 6 comparisons, we would get our key						
	Best case - key element is present at first index						
	<b>Best case time - 1 i.e. <math>B(n) = O(1)</math>; <math>\Omega(1)</math>; <math>\Theta(1)</math></b>						
	Worst case - key element is present at the last index						
	<b>Worst case time - n i.e. <math>W(n) = O(n)</math>; <math>\Omega(n)</math>; <math>\Theta(n)</math></b>						
	Average case = all possible case time / no. of cases						
	average case analysis is very difficult for most of the cases						
	Here, average case time = $1 + 2 + 3 + \dots + n/2 = n(n+1)/2n = n+1/2$						
	<b><math>A(n) = n+1/2</math></b>						
<b>b</b>	<b>Binary search tree</b>						
	height = $\log n$						
	time taken for a particular key is $\log n$						
	Best case - element present in the root						
	<b>Best case time - k i.e. <math>B(n) = O(1)</math>; <math>\Omega(1)</math>; <math>\Theta(1)</math></b>						
	Worst case - searching for a leaf element - depends upon the height of the tree						
	<b>Worst case time - <math>\log n</math> i.e. <math>O(\log n)</math></b>						
	min $w(n) = \log n$ ; max $w(n) = n$						
	<b>17 Disjoint sets</b>						
	No common numbers between two sets - intersection is zero						
	Operations - find, union						
	Find - search or check membership						
	Union - Add an edge						
	<b><i>Krisgal algorithm: If you take an edge and both the vertices belong to the same set, then there is a cycle in the graph</i></b>						



	<b>Weighted union</b> is used while adding edges and detecting cycle						
	<b>Collapsing find</b> - process of directly linking node to a direct parent of a set is called collapsing find - reduces the time to find						
	<b>18 Divide and conquer - Strategy 1</b>						
	Strategy - an approach for solving a problem						
	If a problem cannot be solved, divide it into sub-problems and find a solution for each sub problem, combine the solutions. <i>One point to note is that each sub problem should be similar to the original problem only.</i>						
	Recursive in nature						
	Should have one method to combine the solutions of each sub problem						
	<b>19 Problems under Divide and Conquer</b>						
	Binary search						
	Finding maximum and minimum						
	MergeSort						
	QuickSort						
	Strassen's matrix multiplication						
	<b>20 Recurrence relation 1: <math>T(n) = T(n-1) + 1</math></b>						
	void test(int n)						
	{						
	if(n > 0){						
	printf("%d",n);						
	test(n-1)						
	}						
	}						
	test(3)						
	3. test(2)						
	2. test(1)						
	1. test(0)						

	each print statement takes constant time 1 and there are n+ 1 calls made to the function. we can ignore the last call when it is not printing					
	$f(n) = n + 1$ calls ; $O(n)$					
	$T(n) = T(n-1) + 1$ ; if we ignore if condition					
	Let us solve this relation;					
	if we know $T(n-1)$ , we can get $T(n)$					
	$T(n-1) = T(n-2) + 1$					
	$T(n) = [T(n-2) + 1] + 1$					
	$T(n) = T(n-3) + 3$					
	....continue for k times					
	$T(n) = T(n-k) + k$					
	We would stop after k substitutions; now we need to find k					
	Assume $n - k = 0$ ; therefore $n = k$					
	$T(n) = T(n-n) + n$					
	$T(n) = T(0) + n$					
	<b><math>T(n) = n + 1</math> i.e. <math>\theta(n)</math></b>					
	<b>Recurrence relation 2: <math>T(n) = T(n-1) + n</math> (decreasing function)</b>					
21	void test(int n)	$T(n)$				
	{					
	if(n > 0)		1			
	{					
	for(i = 0; i < n; i++)	$n+1$				
	{					
	printf("%d", n);	$n$				
	}					
	test(n-1);	$T(n-1)$				
	}					
	}					
		$T(n) = T(n-1) + 2n + 2$ i.e. $\theta(n)$				
	we can also write $T(n) = T(n-1) + n$ for $n > 0$					
	$T(n) = 1$ for $n = 0$					
	$T(n)$	n time				
	n $T(n-1)$	n-1 time				
	n-1. $T(n-2)$	n - 2 time				

n-2	T(n-3)	n - 3 time					
.....							
T(2)							
2	T(1)	2 units of time					
1	T(0)	1 unit of time					
for T(0) it does nothing		0 unit of time					
time taken -							
		0 + 1 + 2 + ..... + n-1 + n					
<b><i>theta(n^2)</i></b>		<b><i>T(n) = n(n + 1)/2</i></b>					
T(n) = T(n-1) + n							
T(n-1) = T(n-2) + n-1							
thus, T(n) = T(n-2) + (n-1) + n		**remember, don't add the terms					
T(n) = T(n-3) + (n-2) + (n-1) + n							
T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)).....+(n-1) + n		if we continue for k times					
assume n - k = 0; n = k							
Thus, T(n) = T(n-n) + (n - n + 1) + (n - n + 2)..... +(n-1) + n							
T(n) = T(0) + n(n+1)/2							
<b><i>T(n) = 1 + n(n+1)/2</i></b>		<b><i>theta(n^2)</i></b> ; this extra 1 is owing to the calls					
<b>22 Recurrence relation 3: T(n) = T(n-1) + logn</b>							
void test(int n)		T(n)					
{							
if(n>0)							
{							
for(i = 1; i < n; i = i*2)							
{							
printf("%d", i);		log n times					
}							
test(n-1);		T(n-1)					
}							
}							
T(n) = T(n-1) + logn for n > 0							
T(n) = 1 for n = 0							
<b><i>Solve using tree method,</i></b>							

	$T(n)$					
	$\log n \quad T(n-1)$					
	$\log(n-1) \quad T(n-2)$					
	$\log(n-2) \quad T(n-3)$					
	.....					
	$\log 2 \quad T(1)$					
	$\log 1 \quad T(0)$					
	$\log n + \log(n-1) + \dots + \log 2 + \log 1$					
	<b><math>\log[n(n-1)(n-2)\dots 2.1] = \log(n!)</math></b>	there is no tight bound for this function but there is an upper bound for it				
	<b><math>O(n \log n)</math></b>					
	<b><i>Solving using induction method.</i></b>					
	$T(n) = T(n-1) + \log n$					
	$T(n) = T(n-2) + \log(n-1) + \log(n)$					
	$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$					
	.....					
	$T(n) = T(n-k) + \log n + \log(n-1) + \dots \log 1$					
	Asume $n-k = 0$					
	$T(n) = T(0) + \log n!$					
	<b><math>T(n) = 1 + \log n!</math></b>					
	<b><math>O(n \log n)</math></b>					
	<b>23 How to get the direct answer for a recurrence relation?</b>					
	$T(n) = T(n-1) + 1$	$O(n)$				
	$T(n) = T(n-1) + n$	$O(n^2)$				
	$T(n) = T(n-1) + \log n$	$O(n \log n)$				
	$T(n) = T(n-1) + n^2$	$O(n^3)$				
	$T(n) = T(n-2) + 1$	$O(n/2) \sim O(n)$				
	$T(n) = T(n-100) + n$	$O(n^2)$				
	$T(n) = 2T(n-1) + 1$	???				
	<b>24 Recurrence relation 4: <math>T(n) = 2T(n-1) + 1</math></b>					
	Test(int n)	$T(n)$				
	{					

if(n > 0)		1				
{						
printf("%d", n);		1				
test(n-1);	T(n-1)					
test(n-1);	T(n-1)					
}						
}						
	$T(n) = 2T(n-1) + 1$					
$T(n) = 2T(n-1) + 1$ for $n > 0$						
$T(n) = 1$ for $n = 0$						
<b>Solve using recursion tree method</b>						
1 T(n-1) T(n-1)			2			
1 T(n-2) T(n-2)	1 T(n-2) T(n-2)		4			
1 T(n-3) T(n-3) 1 T(n-3) T(n-3)	1 T(n-3) T(n-3) 1 T(n-3) T(n-3)		8			
.....						
T(0). T(0)		$2^k$				
$1 + 2 + 2^2 + \dots + 2^k = 2^{k+1} - 1$						
as, $a + ar + ar^2 + \dots + ar^k = a(r^{k+1} - 1)/(r - 1)$						
Assume $n - k = 0$						
<b>thus, <math>2^{n+1} - 1</math></b>	<b><math>O(2^n)</math></b>					
<b>Back substitution method</b>						
$T(n) = 2T(n-1) + 1$						
$T(n) = 4T(n-2) + 2 + 1$						
$T(n) = 8T(n-3) + 4 + 2 + 1$						
.....						
$T(n) = 2^k T(n - k) + 2^k(k-1) + 2^k(k) + \dots + 2^3 + 2^2 + 1$						
Assume $n - k = 0$						
$n = k$						
$T(n) = 2^n T(0) + 1 + 2 + 2^2 + \dots + 2^{n-1}$						
<b><math>T(n) = 2^n + 2^n - 1</math> i.e. <math>2^{n+1} - 1</math></b>						

<b>25 Master theorem for decreasing function</b>						
$T(n) = T(n-1) + 1$	$O(n)$					
$T(n) = T(n-1) + n$	$O(n^2)$					
$T(n) = T(n-1) + \log n$	$O(n \log n)$					
$T(n) = 2T(n-1) + 1$	$O(2^n)$					
$T(n) = 3T(n-1) + 1$	$O(3^n)$					
$T(n) = 2T(n-1) + n$	$O(n2^n)$					
$T(n) = 2T(n-2) + 1$	$O(2^{n/2})$					
<b><math>T(n) = aT(n-b) + f(n)</math></b>						
$a > 0, b > 0$ and $f(n) = O(n^k)$ where $k \geq 0$						
<b>if <math>a = 1, O(n^{k+1})</math> or <math>O(n \cdot f(n))</math></b>						
<b>if <math>a &gt; 1, O(n^k \cdot a^{n/b})</math></b>						
<b>if <math>a &lt; 1, O(n^k)</math> or <math>O(f(n))</math></b>						
<b>26 Dividing functions</b>						
test(int n)	$T(n)$					
{						
if(n > 1)						
{						
printf("%d", n);		1				
test(n/2)	$T(n/2)$					
}						
}						
$T(n) = T(n/2) + 1$ for $n > 1$						
$T(n) = 1$ for $n = 1$						
$T(n)$						
1 $T(n/2)$						
1 $T(n/2^2)$						
1 $T(n/2^3)$						
.....continue for k times						
1 $T(n/2^k)$						

	assume , $n/2^k = 1$						
	thus, we have taken k steps overall						
	<b>since, <math>n/2^k = 1 \Rightarrow k = \log n</math> with base 2</b>	<b><math>O(\log n)</math></b>					
	<b>Solving by substitution method</b>						
	$T(n) = T(n/2) + 1$						
	$T(n) = T(n/2^2) + 2$						
	$T(n) = T(n/2^3) + 3$						
	....						
	$T(n) = T(n/2^k) + k$						
	assume $n/2^k = 1$						
	thus, $k = \log n$ with base 2						
	<b><math>T(n) = T(1) + \log n</math></b>						
	<b><math>O(\log n)</math></b>						
	<b>27 Recurrence relation: <math>T(n) = T(n/2) + n</math></b>						
	$T(n) = T(n/2) + n$ for $n > 1$						
	$T(n) = 1$ for $n=1$						
	$T(n)$						
	$T(n/2) \quad n$						
	$T(n/2^2) \quad n/2$						
	$T(n/2^3) \quad n/2^2$						
	.....						
	$T(n/2^k). \quad n/2^{(k-1)}$						
	$T(n) = n + n/2 + n/2^2 + n/2^3..... + n/2^k$						
	$T(n) = n[1 + 1/2 + 1/2^2 + 1/2^3 + ..... 1/2^k]$						
	<b><math>T(n) = n \cdot 1 = n</math></b>						
	<b><math>O(n)</math></b>						
	Using substitution method						
	$T(n) = T(n/2) + n$						
	.....						
	$T(n) = T(n/2^2) + n/2 + n$						
	.....						

	$T(n) = T(n/2^3) + n/2^2 + n/2 + n$					
	.....					
	$T(n) = T(n/2^k) + n/2^{k-1} + \dots + n/2^2 + n/2 + n$					
	Assume $n/2^k = 1$					
	$k = \log n$ with base 2					
	$T(n) = T(1) + n[1/2^k + 1/2^{k-1} + \dots + 1/2^2 + \dots + 1]$					
	<b><math>T(n) = 1 + 2n \sim O(n)</math></b>					
	<b>28 Recurrence Relation: <math>T(n) = 2T(n/2) + n</math></b>					
	void test(int n)	$T(n)$				
	{					
	if(n > 1)					
	{					
	for(int i = 0; i < n; i++)					
	{					
	stmt	n				
	}					
	test(n/2);	$T(n/2)$				
	test(n/2);	$T(n/2)$				
	$T(n) = 2T(n/2) + n$ for $n > 1$					
	$T(n) = 1$ for $n = 1$					
	<b>Solve using recursion tree method,</b>					
	$T(n)$					
	$T(n/2). \quad T(n/2) \quad n$	n				
	$T(n/2^2) \quad T(n/2^2) \quad T(n/2^2) \quad T(n/2^2) \quad n/2$	n				
	$T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3).$	n				
	.....	n				
	$T(n/2^k).....$					
		n				
	assume $n/2^k = 1$					
	$k = \log n$ with base 2					
	<b><math>T(n) = nk \sim O(n \log n)</math></b>					



	<b>Using backsubstitution method;</b>					
	$T(n) = 2T(n/2) + n$					
	$T(n/2) = 2T(n/2^2) + n/2$					
	$T(n) = 2[2T(n/2^2) + n/2] + n$					
	$T(n) = 2^2T(n/2^2) + n + n$					
	$T(n) = 2^3T(n/2^3) + 3n$					
	....continue for k times					
	$T(n) = 2^kT(n/2^k) + kn$					
	Assume $T(n/2^k) = T(1)$					
	$k = \log n$ with base 2					
	<b>Thus, <math>T(n) = n + n \log n \sim O(n \log n)</math></b>					
<b>29</b>	<b>Masters Theorem for dividing functions</b>					
	$T(n) = aT(n/b) + f(n)$	loga with b				
	$a \geq 1; b > 1; f(n) = \theta(n^k \log^p n)$	k				
	case 1: if loga with base b > k then $\theta(n^{\log_a b})$					
	case 2: if loga with base b = k then					
	if $p > -1$ $\theta(n^k \log^{p+1} n)$					
	if $p = -1$ $\theta(n^k \log \log n)$					
	if $p < -1$ then $\theta(n^k)$					
	case 3: if loga with base b < k					
	then, if $p \geq 0$ , $\theta(n^k \log^p n)$					
	if $p < 0$ , $\theta(n^k)$					
	$T(n) = 2T(n/2) + 1$					
	$a = 2$					
	$b = 2$					
	$f(n) = \theta(n^0 \log^0 n)$					
	$k = 0; p = 0$					
	here, loga with base b > k					
	$\theta(n^1)$ where loga with base b is 1					
	$T(n) = 4T(n/2) + n$					
	log a with base b = 2					

k = 1						
p = 0						
this is an example of case 1						
$\theta(n^2)$						
$T(n) = 8T(n/2) + n$						
log8 with base 2 = 3 > k = 1						
$\theta(n^3)$						
$T(n) = 9T(n/3) + 1$						
loga with base b = 2 > k						
$\theta(n^2)$						
$T(n) = 9T(n/3) + n^2$						
loga with base b = 2 = k	case 2					
$\theta(n^2)$						
$T(n) = 8T(n/2) + n$						
$\theta(n^3)$						
$T(n) = 2T(n/2) + n$						
loga with base b = k = 1; p = 0						
case 2						
$\theta(n \log n)$						
$T(n) = 4T(n/2) + n^2$						
$\theta(n^2 \log n)$						
$T(n) = 4T(n/2) + n^2 \log n$						
$\theta(n^2 \log n^2)$						
$T(n) = 8T(n/2) + n^3$						
$\theta(n^3 \log n)$						
$T(n) = 2T(n/2) + n/\log n$						
log a with base b = k = 1						

	$p = -1$						
	$\theta(n \log \log n)$						
	$T(n) = 2T(n/2) + n/\log n^2$						
	$p = -2$						
	$\theta(n)$						
	$T(n) = 2T(n/2) + n^2$						
	loga with base $b < k$						
	$\theta(n^2)$						
	$T(n) = 2T(n/2) + n^2$						
	$\theta(n^2 \log n)$						
	$T(n) = 2T(n/2) + n^3$						
	loga with base $b < k$						
	$\theta(n^3)$						
<b>30</b>	$T(n) = 2T(n/2) + 1$						
	loga with base $b = 1$						
	$k = 0$						
	loga with base $b > k$						
	$\theta(n^1)$						
	$T(n) = 4T(n/2) + 1$						
	loga with base $b = 2$						
	$k = 0$						
	$\theta(n^2)$						
	$T(n) = 4T(n/2) + n$						
	loga with base $b = 2$						
	$k = 1$						
	$\theta(n^2)$						
	$T(n) = 8T(n/2) + n^2$						
	loga with base $b = 3$						

k = 2						
$\theta(n^3)$						
$T(n) = 16T(n/2) + n^2$						
log a with base b = 4						
k = 2						
$\theta(n^4)$						
$T(n) = T(n/2) + n$						
log a with base b = 0						
k = 1						
$\theta(n)$						
$T(n) = 2T(n/2) + n^2$						
log a with base b = 1						
k = 2						
$\theta(n^2)$						
$T(n) = 2T(n/2) + n^2 \log n$						
log a with base b = 1						
k = 2						
$\theta(n^2 \log n)$						
$T(n) = 4T(n/2) + n^3 \log^2 n$						
log a with base b = 2						
k = 3						
$\theta(n^3 \log^2 n)$						
$T(n) = 2T(n/2) + n^2 / \log n$						
log a with base b = 1						
k = 2						
$\theta(n^2)$						
$T(n) = T(n/2) + 1$						
log a with base b = 0						
k = 0						

	$\theta(\log n)$					
	$T(n) = 2T(n/2) + n$					
	log a with base b = 1					
	k = 1					
	p = 0					
	$\theta(n \log n)$					
	$T(n) = 2T(n/2) + n \log n$					
	log a with base b = 1					
	k = 1					
	p = 1					
	$\theta(n \log^2 n)$					
	$T(n) = 4T(n/2) + n^2$					
	log a with base b = 2					
	k = 2; p = 0					
	$\theta(n^2 \log n)$					
	$T(n) = 4T(n/2) + (n \log n)^2$					
	log a with base b = 2					
	k = 2, p = 2					
	$\theta(n^2 \log^3 n)$					
	$T(n) = 2T(n/2) + n/\log n$					
	log a with base b = 1					
	k = 1; p = -1					
	$\theta(n \log \log n)$					
	$T(n) = 2T(n/2) + n/\log^2 n$					
	log a with base b = 1					
	k = 1; p = -2					
	$\theta(n)$					
<b>31</b>	<b>Root function Recurrence relation</b>					
	$T(n) = T(\sqrt{n}) + 1$ for $n > 2$					

	$T(n) = 1$ for $n = 2$					
	$T(n) = T(\text{root}(n)) + 1$					
	$T(n) = T(n^{(1/2)}) + 1$ .....equation 1					
	using substitution					
	$T(n) = T(n^{(1/2^2)}) + 2$ .....equation 2					
	$T(n) = T(n^{(1/2^3)}) + 3$ .....equation 3					
	$T(n) = T(n^{(1/2^k)}) + k$ .....equation 4					
	assume, $n = 2^m$					
	$T(2^m) = T(2^{(m/2^k)}) + k$					
	assume $T(2^{(m/2^k)}) = T(2)$					
	thus, $m/2^k = 1$					
	$m = 2^k$					
	$k = \log m$ with base 2					
	substituting value of $n$					
	$m = \log n$ with base 2					
	therefore, $k = \log \log n$ with base 2					
	$\theta(\log \log n \text{ with base } 2)$					
	<b>32 Binary Search Iterative Method</b>					
	To perform binary search, the prerequisite is that the list must be in sorted order	$A = \{3, 6, 8, 12, 14, 17, 25, 29, 31, 36, 42, 47, 53, 55, 62\}$				
	we need two index pointers, one is low at the starting point and the other is high at the end point	$l = 1, h = 15$ (lowest and highest index); $mid = 8$				
	$mid = \text{low} + \text{high} / 2$ and we take the floor value	key value = 42; $A[mid] = 29 \rightarrow \text{key} > A[mid]$				
	the key value is on the right hand side as key value is greater than $A[mid]$					
	we will change low to $mid + 1$	$l = 9, h = 15; mid = 9 + 15 / 2 = 12$				
		$A[mid] = 47 > \text{key}$				
	we will change high to $mid - 1$ as $\text{key} < A[mid]$					
		$h = 11, l = 9, mid = 10; A[mid] = 36$				
		$A[mid] < \text{key}$				
	we will change low to $mid + 1$	$l = 11; h = 11; mid = 11; A[mid] = 42$				
	we can return the index as we have found the key value	$A[mid] = \text{key}$				

	therefore, binary search looks faster than linear search. It just took 4 comparisons					
	int BinSearch(A, n, key)					
	{					
	l = 1, h = n					
	mid = l + h / 2 - take floor value					
	while(l <= h){					
	if(key == A[mid])					
	{ return index i.e.element is found}					
	else if(key < A[mid])					
	{h= mid-1;}					
	else {					
	l = mid + 1;}					
	}					
	return 0;					
	}					
	Time taken for binary search = logn					
	min time: O(1)					
	max time: O(logn)					
	avg time = add time for each element and divide by number of elements					
	<b>33 Binarysearch Recursive method</b>					
	Alogirthm RBinarySearch(l,h,key)	T(n)				
	{					
	if(l==h)	1				
	{					
	if(A[l]== key)					
	{					
	return l;					
	}					
	else					
	{					
	return 0;					

	}					
	else					
	{					
	mid = l + h / 2 //taking floor value	1				
	if(key == A[mid])	1				
	{return mid;}					
	if(key < A[mid])	1				
	{					
	return RBinarySearch(l, mid - 1, key)	T(n/2)				
	}					
	else					
	{					
	return RBinarySearch(mid+1, h, key)	T(n/2)				
	}					
	}					
		T(n) = 1; n = 1				
		T(n) = T(n/2) + 1 for n > 1				
		theta(logn)				
	<b>34 Heaps</b>					
<b>a</b>	<b>Representation of a binary tree using an array</b>					
	T {A, B, C, D, E, F, G}					
	if a node is at index i;					
	its left child is at node 2*i					
	its right child is at node 2*i + 1					
	its parent is at node i/2					
	if there are missing nodes, we leave a blank in its place in the array					
<b>b</b>	<b>Full binary tree</b>					
	In its height, it has maximum number of nodes and if we wish to add a node, height would increase					
	Max no. of nodes = $2^h - 1$					
<b>c</b>	<b>Complete binary tree</b>					
	there is no missing element from first element to the last element in array representation of the binary tree					



	Every full binary tree is also a complete binary tree					
	A complete binary tree is a full binary tree until height $h - 1$					
	Height of a complete binary tree would be minimum i.e. $\log n$					
<b>d</b>	<b>Heap</b>					
	Heap is a complete binary tree					
	Max Heap: every node has value greater than all its descendants {50, 30, 20, 15, 10, 8, 16}					
	Min Heap: every node has value smaller or equal to than all its descendants {10, 30, 20, 35, 40, 32, 25}					
	<b>35 Insert operation in a max heap</b>					
	Insert 60 in the above given max heap					
	this value should be inserted in the last free space in the array					
	i.e. left child of the left most leaf node					
	Then, adjust the elements to make it as a heap					
	So, compare and move 60 up the levels and in the array check at $i/2$ indices where initially $i$ would be the last empty index where 60 was inserted					
	Time taken would be equal to the number of swaps					
	this depends upon the height of the tree i.e. $\log n$ , hence $O(\log n)$					
	minimum time is of no swaps $O(1)$ ; max would be $O(\log n)$					
	<b>36 Delete operation in a max heap</b>					
	From the heap, we need to remove the root / top most element only					
	The last element in the complete binary tree would come in its place					
	Adjust the elements to maintain heap order					
	From the root towards the leaf, adjust					
	Compare the children ( $2i$ and $2i + 1$ ) and whichever child is greater than compare with the parent					
	Time taken depends upon the height; max could be $O(\log n)$					
	Whenever you delete from max heap, you get the next max element and in case of min heap, it would be the next min element					
	<b>37 HeapSort</b>					

	For a given set of numbers, create a heap					
	Delete all the elements from the heap					
	Total N elements we have inserted; each element we assume is moved up to the root; so time taken $O(N\log N)$					
	Then we delete the elements					
	Store deleted elements in the array in free space in the end					
	Deletion also takes $O(N\log N)$ time					
	Thus, heapsort takes $O(N\log N)$					
	<b>38 Heapify</b>					
	The process of creating heap but direction is opposite than creating a heap					
	$O(N)$					
	<b>39 Priority Queue</b>					
	elements will have priority and they would be inserted and deleted as per the priority order					
	For min heap, smaller the no. higher the priority					
	For max heap, greater the no. higher the priority					
	$O(\log N)$ for insertion and/or deletion					
	<b>40 TwoWay MergeSort - Iterative method</b>	Algorithm Merge(A, B, m, n)				
	merging two sorted lists to get a sorted result	{i = 1, j = 1, k = 1;				
	A = {2, 8, 15, 18} i	while(i <= m && j <= n){				
	B = { 5, 9, 12, 17} j	if(A[i] < B[j])				
	Compare A(i) with B(j) to get C(k) and move to next location	{				
	m + n elements are obtained , thus theta(m + n)	C[k++] = A[i++];				
		}				
		else {				
		C[k++] = B[j++];				
		}				
		for(; i <= m; i++){				
		C[k++] = A[i];				
		}				
		for(; j <= n; j++){				
		C[k++] = B[j];				
		}				

		}					
<b>41</b>	<b>Merging more than two lists</b>						
	M-way merging						
	A = {4, 6, 12}						
	B = {3, 5, 9}						
	C = {8, 10, 16}						
	D = {2, 4, 18}						
	One way is that we merge A and B; C and D and then finally merge the two resulting lists --> so we perform merge three times here						
	Another way is that we first merge A and B; then we merge resulting list with C ; and the resulting list with D						
	Two-way mergesort is an iterative process whereas mergeSort is a recursive process						
	A = {9, 3, 7, 5, 6, 4, 8, 2} - given an array and we have to sort them using 2-way mergesort						
<b>1st pass</b>	We would consider each element as a sorted list and merge	merged n elements in this pass					
	First select two lists 3 and 9; then merge them - 3, 9						
	Similarly, we select two lists 7 and 5 , merge them - 5 and 7						
	Another lists we get are {4, 6} and {2, 8}						
	Now, we have 4 lists with two elements each						
<b>2nd pass</b>	When we merged we kept the resulting 4 lists in another array B; B = {{3, 9}, {5, 7}, {4, 6}, {2, 8}}	merged n elements in this pass					
	We merge two lists each						
<b>3rd pass</b>	C = {{3, 5, 7, 9}, {2, 4, 6, 8}}	merged n elements in this pass					
	we merge the above two lists to get a single sorted list						
	D = {2, 3, 4, 5, 6, 7, 8, 9}						
	log(no of elements) = no. of passes						
	<b>Time complexity: O(n(logn))</b>						
<b>42</b>	<b>MergeSort</b>						
	A = {9, 3, 7, 5, 6, 4, 8, 2}	Algorithm MergeSort(l, h){	T(n)				
	If there is a single element, we can consider it as a base or small problem {Divide and conquer}						
		if(l < h){					

		mid = (l + h) / 2;	1				
		MergeSort(l, mid);	T(n/2)				
		MergeSort(mid + 1, h);	T(n/2)				
		Merge(l, mid, h);	n				
		}	T(n) = 2T (n/2) + n for n > 1				
		}	T(n) = 1 for n = 1				
	<b>time complexity: theta(nlogn)</b>		using master's theorem, a = 2, b = 2, k = 1				
	merging is done in post order traversal		loga with base b = 1 = k				
			thus, it is case 2				
			<b>theta(nlogn)</b>				
<b>43 Pros of MergeSort</b>		<b>Cons of MergeSort</b>					
	works great for Large size lists	Extra space (not inplace sort)					
	suitable for Linked List	no small problem					
	supports external sorting	recursive and uses a stack (need n + logn space) i.e. space complexity: O(n + logn) where n is the extra space and logn is the stack space					
	stable: the order of duplicates is maintained						
		insertion sort (O(n^2))					
		mergesort O(nlogn)					
		for small problems, n <= 15; insertionsort works better --> use insertion sort					
<b>43 QuickSort</b>							
	students arranging themselves in increasing order of heights						
	<b>10</b> 80 90 60 30 20						
	5 6 3 4 2 1 <b>9</b>						
	4 6 7 <b>10</b> 16 12 13 14						
	A = {10, 16, 8, 12, 15, 6, 3, 9, 5, INFINITY}	partition(l, h){					
	select first element as a pivot	pivot = A[l];					

	pivot = 10	i = l; j = h;					
	we need to find the sorted position for 10	while(i<j){do					
	i starting from pivot and j starting from infinity	{					
	i would check for elements greater than 10; j would check for elements smaller than pivot	i++;					
	we are using the partitioning procedure	} while(A[i]<= pivot);					
	increment i until next value is greater than 10 and decrement j until next value is smaller than pivot; stop and swap	do					
	{10, 5, 8, 9, 3, 6, 15, 12, 16}	{					
	send pivot element at j position	j--;					
	now, we can sort the two lists around the partitioning position by performing quicksort recursively	}while(A[j] > pivot);					
		if(i<j){					
	QuickSort(l, h)	swap(A[i], A[j]);					
	{	}					
	if(l < h)	swap(A[l], A[j]);					
	{	return j;					
	j = partition(l, h);	}					
	QuickSort(l, j);						
	QuickSort(j+ 1, h);						
	}						
	}						
	<b>44 QuickSort Analysis</b>						
	suppose it is partitioning in the middle of 1 and 15th index						
	then, two partitions: [1, 7] ; [9, 15]						
	further partitions: [1, 3]; [5, 7]; [9, 11]; [13, 15]						
	at each level , n elements are being handled						
	and there are logn levels						
	thus time complexity for best case: O(nlogn)						
	median : middle element of a sorted list						
	best case of quicksort is that the partitioning occurs exactly at the middle						
	worstcase: if we have an already sorted list						
	time complexity for worstcase: O(n^2)						
	to handle this, try taking middle element as a pivot						

2. select random element as a pivot						
<b>45 Strassen's matrix multiplication</b>						
A = [a11 a12						
a21. a22]						
B = [b11 b12						
b21 b22]						
Cij = Summing up Aik*Bkj						
for(i = 0; i < n ; i++){						
for(j = 0; i < n ; j++){						
C[i,j]= 0;						
for(k=0;k<n;k++){						
C[i,j] += A[i, k]*B[k, i];						
}						
}						
}						
C11 = a11*b11 + a12*b21						
C21 = a11*b12 + a12*b22	A = [a11]					
C21 = a21*b11 + a22*b21	B = [b11]					
c22 = a21*b12 + a22*b22	C = [a11*b11]					
for [2*2] matrix, we would use above formula	for [1*1] matrix, use above formula					
we assume that the matrix has dimensions of power of 2	Algorithm MM(A, B, n)					
	{					
	if(n <= 2					
8 times the function is calling itself	{					
T(n) = 8T(n/2) + n^2 for n > 1	C = 4 formula stated above;					
a = 8, b = 2, log a with base b = 3	}					
k = 2	else					
it is case 1 of master's theorem	{					
theta(n^3)	mid = n/2					
	MM(A11, B11, n/2) + MM(A12, B21, n/2);					
	MM(A11, B12, n/2) + MM(A12, B22, n/2);					
	MM(A21, B11, n/2) + MM(A22, B21, n/2);					
	MM(A22, B22, n/2) + MM(A21, B12, n/2);					

		}					
		}					
	Strassen's approach -						
	has given 4 different formulas with 7 multiplications	$P = (A_{11} + A_{22})(B_{11} + B_{22})$					
	$C_{11} = A_{11} * B_{11} + A_{12} * B_{21}$	$Q = (A_{21} + A_{22}) B_{11}$					
	$C_{21} = A_{11} * B_{12} + A_{12} * B_{22}$	$R = A_{11}(B_{12} - B_{22})$					
	$C_{21} = A_{21} * B_{11} + A_{22} * B_{21}$	$S = A_{22}(B_{21} - B_{11})$					
	$C_{22} = A_{21} * B_{12} + A_{22} * B_{22}$	$T = (A_{11} + A_{12})B_{22}$					
		$U = (A_{21} - A_{11})(B_{11} + B_{12})$					
		$V = (A_{12} - A_{22})(B_{21} + B_{22})$					
		$C_{11} = P + S - T + V$					
		$C_{12} = R + T$					
		$C_{13} = Q + S$					
		$C_{22} = P + R - Q + U$					
		$T(n) = 7T(n/2) + n^2$ for $n > 2$					
		$T(n) = 1$ for $n \leq 2$					
		using master's theorem,					
		$O(n^{\log_7 \text{ with base } 2}) = O(n^{2.81})$					
	<b>Strategies used for solving optimization problems - Greedy Method, Dynamic programming, branch and bound</b>						
	<b>46 Greedy method</b>						
	Design which we can adopt to solve similar problems		Greedy method says that each problem should be solved in stages - each stage we give an input, check if the solution is feasible then we pick it up and move to next stage				

	Solving optimization problems		Algorithm Greedy(a, n) a = {a1, a2, a3, a4, a5}; n = 5				
	<b>Optimization problem:</b> Problems which require either minimum or maximum result		{				
	Suppose we have a problem P where we need to travel from source A to destination B, we can have several solutions such as walking on foot, travel by an airplane, ride on a bike, travel by a bus, drive on a car, go by a train and so on..... Now, we notice that we also have some constraints. The solutions that satisfy the conditions given in a problem are called <b>feasible solutions</b>	Minimum cost journey - " <b>Minimization problem</b> "; then feasible solutions giving minimum cost are called <b>optimal solutions</b> . <b>There can be many feasible solutions but only one optimal solution</b>	for i = 1 to n do {x = select(a):				
	Example: selecting a car to purchase	Example: Hire a person for your company	if feasible(x) then				
	Method 1: Looking at all the models available in the city	Method 2: Conduct an assessment center to filter people at each stage and get the best person	{				
	Method 2: Checking for the features of the cars and filtering and selecting based on your preferences - greedy method	So, the person may not be the best but the approach is greedy here as we are using our criteria and constraints to choose the best person	solution = solution + x;				
	<b>47 Knapsack problem</b>		}				
	n = 7; m = 15	Bag capacity is 15 kgs and we have been given 7 objects. we have to fill this bag with these objects. Profit is the gain we get by transferring this object. Problem is a container loading problem. Problem is filling the container with the objects as the capacity of container is limited	}				
<b>Objects</b>	{0 1 2 3 4 5 6 7 }	Optimization and maximization problem	}				
<b>Profits</b>	{P 10 5. 15 7 6 18 3 }	Constraints : Bag weight limit					
<b>Weights</b>	{W 2 3 5 7 1 4 1 }						
<b>Profit by weight</b>	{P/W 5 1.3 3 1 6 4.5 3}						
<b>0&lt;=x&lt;= 1</b>	Objects are divisible i.e. we can take just half kg of object 1 and may be 2 kgs of object 2 and so on						
<b>x</b>	()						
	x1 x2 x3 x4 x5 x6 x7						
<b>Method 1</b>	Take the thing that have maximum profit						



<b>Method 2</b>	Take things with smaller weight so that you can put in more things						
<b>Method 3</b>	Take things that have highest profit by weight						
	Let's use method 3						
	First, I include object 5 that has maximum profit by weight. Then I check remaining weight I can put in. We can still put in 14 kgs. Then we select all the quantity of object 1. Remaining weight limit 12 kgs. Add all of object 6. Remaining weight limit 8 kgs. Add all of object 3. Remaining weight limit 3 kgs. Add all of object 7. Remaining weight limit is 2 kgs. Add 2/3 of object 2 as we have only 2 kgs limit remaining.						
<b>x</b>	( 1    2/3    1    0    1    1    1 )						
	Calculate total profit and verify weight						
	<b>Total weight</b> = $1*2 + 2/3*3 + 1*5 + 0*7 + 1*1 + 1*4 + 1*1 = 15$	//Multiplying x elements by Weight w for each object					
	<b>Total profits</b> = $1*10 + 2/3*5 + 1*15 + 1*6 + 1*18 + 1*3 = 54.6$	//Multiplying x elements by Profit P for each object					
<b>48</b>	<b>0/1 Knapsack problem</b>						
	Objects are indivisible and fractions are not allowed i.e. either you include the whole thing or you do not include it at all						
<b>49</b>	<b>Job sequencing with deadlines</b>	n = 5 (tasks)					
<b>Jobs</b>	J1    J2    J3    J4    J5						
<b>Profits</b>	20    15    10    5    1						
<b>Deadlines</b>	2    2    1    3    3						
	Assume that there is a machine, on which each job has to be processed and each job takes 1 unit of time ( hour) for completion						
	Set of the jobs which can be completed within their deadlines such that profit is maximized	Constraints: deadlines must be met					
<b>deadlines</b>	0-----1-----2-----3	maximum 3 slots / jobs					
<b>time slots</b>	9am-----10am-----11am-----12am						
<b>Jobs chosen</b>	J2            J1            J4						
<b>Profits</b>	$15 + 20 + 5 = 40$						
<b>Sequence</b>	J1 --> J2 ---> J4    J2 --> J1 --> J4						



Example:												
List	x1	x2	x3	x4	x5							
Sizes	20	30	10	5	30							
Increasing order of sizes	5	10	20	30	30							
Lists	x4	x3	x1	x2	x5							
	First x4 and x3 are merged, cost = 15; then result is merged with x1; cost = 35; x2 and x5 are merged with cost = 60; the two resulting lists are merged with cost = 95											
Total cost	15 + 35 + 60 + 95 = 205											
	3*5 + 3*10 + 2*20 + 2*30 + 2*30 = 205					//multiplying distance of each node and size of each node						
52	Huffman Coding											
	Compression technique used to reduce the size of data or message											
Message	BCCABBDDEAECBBAEDDCC											
	Length = 20											
	it has to be sent using ASCII codes (8- bit)											
	A 65 01000001					Size = 8*20 = 160 bits						
	B 66 01000010											
	C 67											
	D 68											
	E 69											
	Can we use our own codes instead of ASCII codes?											
	Fixed size method											
Character	A	B	C	D	E							
Count	3	5	6	4	2	Total count = 20						
Code	000	001	010	011	100							
message	BCCABBDDEAECBBAEDDCC											
bit code	001010....					size = 20*3 = 60 bits						
						5*8 = 40 bits for ASCII code translations						
						5*3 = 15 bits --> our assigned codes						

		40 + 15 = 55 bits					
		message: 60 bits					
		chart: 55 bits					
		total message size: 115 bits					
		so, the message size reduced from 160 bits to 115 bits					
		thus, 40% reduction in size with fixed sized code					
	Huffman coding - variable sized code	element that appears more / often should have a smaller sized code					
<b>character</b>	A    B    C    D    E						
<b>count</b>	3    5    6    4    2						
<b>code</b>							
	first, arrange the letters with increasing count / frequency						
<b>character</b>	E    A    D    B    C						
<b>count</b>	2    3    4    5    6						
<b>code</b>	000    001    01    10    11	Merge two smaller ones, we get 5, then combine with D, we get 9. Combine B and C, we get 11. Finally, combine two resulting lists, we get 20.					
<b>bit count</b>	6    9    8    10    12	On left side paths, mark as 0 and on right side mark as 1					
<b>total bits for message</b>	45 bits	Bit count for message can also be obtained from the tree, by counting number of edges for a letter and multiplying by the number of occurrences for that letter in the message i.e. summation of distance and frequency of a letter					
<b>ASCII codes for chart</b>	5*8 = 40 bits						
<b>assigned codes</b>	12 bits						
<b>total bits for tree/table</b>	52 bits						
<b>Size of total msg</b>	52 + 45 = <b>97 bits</b>						
<b>Message transferred</b>	001111101101111001011000111110100010100000110	A tree or a table would be needed along with it					
<b>Decoding</b>	BCCD...						

<b>53</b>	<b>Minimum Cost Spanning Tree</b>						
	$G = (V, E)$						
	$V = \{1, 2, 3, 4, 5, 6\}$	$ V  = n = 6$					
	$E = \{(1,2), (2,3), (3,4), (4,5), (5, 6), (6,1)\}$	$ V  - 1 = 5 \text{ edges}$					
	the tree should not have a cycle						
	S is a subset of G, WHERE IN $S = (V', E')$	$V' = V;  E'  =  V  - 1$					
	Number of edges in graph = 6 out of which I have to select 5 edges for spanning tree - thus i can select in <b>6C5 ways</b>	Suppose we have 7 edges, out of which the seventh edge (3,5) divides the graph into two cycles of less tha 6 vertices, then we can select 5 edges for spanning tree in <b>7C5 - 2 ways</b>					
<b>General formula</b>	$ E C( V -1) - \text{no. of cycles}$						
	Now, if we have a weighted graph, I wish to know the number of possible spanning tree						
	Vertices = 4						
	Edges = 3						
	cost = 14						
	similarly , depending upon the edges we select, cost may vary each time						
	Can I found the minimum cost spanning tree?						
<b>Method 1</b>	Try all possible spanning trees and get the minimum cost spanning tree						
<b>Method 2</b>	Prim's algorithm (Greedy method)						
<b>Method 3</b>	Krskal's algorithm (Greedy method)						
<b>Method 2:</b>	<b>Prim's algorithm</b>						
	Select the minimum cost edge from the graph first	(6,1) ; w = 10					
	Then, following this select minimum cost edge but make sure it is connected to previously chosen vertices	(5, 6); w = 25					
		(5, 4); w = 22					
		(4, 3); w = 12					
		(3, 2); w = 16					
		(2, 7); w = 14					
	Now, if we add costs of all the chosen edges, total cost = 99						

	For non connected graphs we cannot find the minimum cost spanning tree or spanning tree						
<b>Method 3</b>	<b>Kruskal's method</b>						
	Always select smallest cost edge						
	(1,6); w = 10						
	(3, 4); w = 12						
	(2, 7); w = 14						
	(2, 3); w = 16						
	(4, 5); w = 22						
	(5, 6); w = 25						
	total cost = 99						
	vertices count :  V	To get a minimum cost edge each time, min heap can be used					
	edges count:  V  - 1	theta(nlogn)					
	theta( V  E )						
	theta(n.e) = theta(n^2)						
	for non-connected graphs, spanning tree cannot be found						
	Kruskal algo may give spanning tree for those non connected componr=ents but bot for the graph as a whole						
	if in a certain graph, certain edges' weights are missing, then use the given weights of remaining edges to guess the weight						
<b>54</b>	<b>Dijkstra algorithm</b>						
	Single source shortest path to all the vertices						
	find the shortest path to a vertex annd update it to other vertices. this updation is called relaxation						
	<b>Relaxation</b>						
	if(d[u] +c(u,v) < d[v]){ d[v] = d[u] + c(u,v)}						
	no of vertices =  V						
	at most no. of vertices relaxing =  V						

	worst case time of Dijkstra algorithm: $\theta(n^2)$										
	Example - starting vertex is 1										
<b>selected vertex</b>	2	3	4	5	6						
<b>4</b>	50	45	<b>10</b>	infinity	infinity						
<b>5</b>	50	45	<b>10</b>	<b>25</b>	infinity						
<b>2</b>	45	45	<b>10</b>	<b>25</b>	infinity						
<b>3</b>	45	45	<b>10</b>	<b>25</b>	infinity						
<b>6</b>	45	45	<b>10</b>	<b>25</b>	infinity						
	Another example - starting vertex is 1										
<b>selected vertex</b>	{2,	3,	4}								
<b>2</b>	{3,	infinity,	5}								
<b>4</b>	{3,	infinity,	5}								
<b>3</b>	{3,	7,	5}								
	{3,	7,	5}								
	Another example - starting vertex is 1										
<b>selected vertex</b>	{2,	3,	4}								
<b>2</b>	{3,	infinity,	5}								
<b>4</b>	{3,	infinity,	5}								
<b>3</b>	{3,	7,	5}								
	{-3,	7,	5}								
	Dijkstra algorithm might work or might not work in case of an edge having negative weightage										
<b>55</b>	<b>Dynamic programming</b>										
	<b>Dynamic programming vs greedy method</b>										

	In Greedy method, we try to follow a predefined procedure that gives us the best / optimal result. The procedure is already known for optimization. But, in dynamic programming, we try to get all the solutions and then decide the best solution. Mostly dynamic programming questions are solved using recursive procedures. They follow a principle of optimality. In greedy method, decision is taken just once and followed through whereas in dynamic programming, decision is taken at each step					
	<b>Example:</b>					
	<b>Fibonacci series</b>					
	fib(n) = 0 if n = 0	T(n) = 2T(n-1) + 1{Approximating T(n-2) ~ T(n-1) here}				
	fib(n) = 1 if n = 1	Time taken would be O(2^n) by using Master's theorem				
	fib(n) = fib(n-2) + fib(n-1) if n > 1	Why can't we reduce the function calls to reduce the time taken?				
		For this, we would take a global array and initially fill it with -1				
	int fib(n) {	F = {-1,-1,-1,-1,-1}				
	if(n <= 1){	Then, as the function calls f(1), mark it as 1				
	return n;}	Then f(0) is marked as 1				
	return fib(n-2) + fib(n-1);	Then use the stored result to get the rest.				
	}	Finally, F would get updated as we solve: F = {0, 1, 1, 2, 3, 5}				
		Total 6 calls are made then i.e. n+1 calls i.e. O(n)				
		This is called result of memorization				
	From the above example, we can see reduction in number of calls from O(2^n) to O(n) using memorization. It follows top down approach. The same problem can be solved using tabular method (iterative process) as shown below:					
	int fib(int n) {					
	if(n <= 1) {					
	return n;}					
	F[0] = 0; F[1] = 1;					
	for(int i = 2; i <= n; i++){					
	F[i] = F[i-2] + F[i-1];					
	}					
	return F[n];					
	}					



	F= {0, 1, 1, 2, 3, 5}																	
	This is a bottom - up approach i.e. starting from F[0] and moving to F[n]																	
	<b>56 Multistage Graph</b>																	
	<i>A multistage graph is a directed weighted graph. The vertices are divided into stages such that the edges are connecting vertices from one stage to next stage only. First stage and last stage will have only one vertex to represent start and end point. This is usually used to represent resource allocation.</i>																	
	The objective of the problem is that I have to select a path which gives me minimum cost.												//it is a minimization or optimization problem					
	Dynamic programming works on principle of optimality. Principle of optimality says that a problem can be solved in a sequence of decisions.																	
	From first stage I have to select one optimal vertex that leads to minimum cost and I have to take this decision at each stage. Thus, I can apply dynamic programming here.																	
<b>V</b>	1	2	3	4	5	6	7	8	9	10	11	12						
<b>Cost</b>	16	7	9	18	15	7	5	7	4	2	5	0	cost(5, 12) = 0 ; here 5 is the stage and 12 is the vertex					
<b>d</b>	2/3	7	6	8	8	10	10	10	12	12	12	12	cost(4, 9) = 4					
													cost(4, 10) = 2					
	Formula for multistage graph:												cost(4, 11) = 5					
	<b>cost(lth stage, jth vertex no.) = cost(i, j) = min{C(j, i) + cost(i+1, i)}</b>												cost(3, 6) = min{ C(6, 9) + cost(4, 9) , C(6, 10) + cost(4, 10)} = min{6 + 4, 5 + 2} = 7					
													Similarly, cost(3, 7) = min{8, 5} = 5					
	Now, we will solve it by going in forward direction and taking decisions based on above data;												cost(3, 8) = min{7, 11} = 7					
	d(1,1) = 2												Similarly, cost(2, 2) = min{C(2, 6) + cost(3, 6), C(2, 7) + cost(3, 7), C(2, 8) + cost(3, 8)} = min{11, 7, 8} = 7					
	d(2,2) = 7												cost(2, 3) = min{9, 12} = 9					
	d(3, 7) = 10												cost(2, 4) = min{18} = 18					
	d(4, 10) = 12												cost(2, 5) = min{16, 15} = 15					
	<b>Path: 2---&gt; 7---&gt; 10----&gt; 12</b>												cost(1,1) = min{16, 16, 21, 17} = 16					
	d(1, 1) = 3																	
	d(2, 3) = 6																	
	d(3, 6) = 10																	
	d(4, 10) = 12																	
	<b>Path: 3---&gt; 6---&gt; 10----&gt; 12</b>																	

	So, we have two paths with same cost.															
57	Multistage Graph (Program)															
	Cost adjacency Matrix									main(){						
	0	1	2	3	4	5	6	7	8	int stages = 4, min;						
0	0	0	0	0	0	0	0	0	0	int n = 8;						
1	0	0	2	1	3	0	0	0	0	int cost[9], d[9], path[9];						
2	0	0	0	0	0	2	3	0	0	int c[9][9] = {{0,0,0,0,0,0,0,0,0},						
3	0	0	0	0	0	6	7	0	0	{0,0,2,1,3,0,0,0,0}, {0,0,0,0,0,2,3,0,0}, .....}						
4	0	0	0	0	0	6	8	9	0	cost[n] = 0;						
5	0	0	0	0	0	0	0	0	6	for(int i = n-1; i >=1; i--){						
6	0	0	0	0	0	0	0	0	4	min = 32767;						
7	0	0	0	0	0	0	0	0	5	for(int k = i + 1; k <= n; k++){						
8	0	0	0	0	0	0	0	0	0	if(C[i][k] != 0 && C[i][k] +C[k] < min){						
										min = C[i][k] +C[k] ;						
										d[i] = k;						
	0	1	2	3	4	5	6	7	8							
cost	--	9	7	11	12	6	4	5	0	}						
d	--	2	6	6	5	8	8	8	--	}						
path	--	1	2	6	8					cost[i] = min;						
										}						
	Path is calculated using the following formula: p[i] d[p[i-1]] ; p [2] = d[p[2-1]] = d[1] = 2									p[1] = 1; p[stages] = n;						
	time complexity: O(n^2)									for(i = 2; i < stages; i++){						
										p[i] = p[d[i-1]];						
58	All Pairs Shortest Path															
	If I use Dijkstra algorithm on each vertex to find the shortest path of all, the time complexity would be O(n^3).															
A0 =	1	2	3	4												
1	0	3	INF	7												
2	8	0	2	INF												
3	5	INF	0	1												
4	2	INF	INF	0												
	Considering vertex 1 as intermediate vertex									A0[2,3]	A0[2,1] + A0[1,3]					
A1 =	1	2	3	4	2	<	8	+	INF							



	For generating a single matrix C after single multiplication of 2 matrices of order (5X4) and (5X3) , we would need to do 60 multiplications									
	((A1.A2).A3).A4 or (A1.A2).(A3.A4) or ....there could be several ways then, how to choose the right way?									
	T(n) = 2n(n-1)/2 trees are possible thus, with 3 nodes ; T(3) = 5									
	Using tabular approach (bottom up approach),									
<b>m</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	in m[1,1] i.e. A1 , nothing is multiplied, hence it can be taken as zero					
	<b>1</b>	0	120	88	158	m[1,2] = A1 . A2				
	<b>2</b>	-	0	48	104	(5X4) (4X6)				
	<b>3</b>	-	-	0	84	Total cost of multiplication = 5 * 4 * 6 = 120				
	<b>4</b>	-	-	-	0					
<b>s</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	m[1,3] = A1.A2.A3					
	<b>1</b>	-	1	1	3	Two possibilities: A1.(A2.A3) or (A1.A2).A3				
		-	-	2	3	(5X4) (4X6) (6X2)				
	<b>2</b>	-	-	-	3	for A1.(A2.A3) --> m[1,1] + m[2,3] + (5*4*2)	for (A1.A2). A3 --> m[1,2] + m[3,3] + (5*6*2)			
	<b>3</b>	-	-	-	3	0 + 48 + 40	120 + 0 + 60			
	<b>4</b>	-	-	-	-	88	180			
	Similarly, m[2,4]									
	formula:				A2.(A3.A4) (A2.A3).A4					
	m[i,j] = m[i,k] + m[k+1, j] + di-1 * dk * dj				(4X6) (6X2)(2X7) (4X6) (6X2)(2X7)					
	<b>we are generating n(n -1)/2 elements</b>				for A2.(A3.A4) --> m[2,2] + m[3,4] + (4*6*7)	for (A2.A3). A4 --> m[2,3] + m[4,4] + (4*2*7)				
	<b>time complexity = O(n^3)</b>				0 + 84 + 168	48 + 0 + 56				
					252	104				
					m[1,4]					

		$\min\{m[1,1] + m[2,4] + (5*4*7), m[1,2] + m[3,4] + (5*6*7), m[1,3] + m[4,4] + (5*2*7)\}$					
		$\min\{0+104+140, 120 + 84+210, 88+70\}$					
		$\min\{244, 414, 158\}$					
60	<b>Matrix chain multiplication - A few pointers</b>						
	<b>Condition of the multiplication:</b> The number of columns in the first matrix involved in the multiplication must be equal to the number of rows in the second matrix						
<b>A=</b>	a11   a12   a13	2X3 dimension					
	a21   a22   a23						
<b>B=</b>	b11   b12	3X2 dimension					
	b21   b22						
	b31   b32						
<b>A*B =</b>	$a_{11}*b_{11} + a_{12}*b_{21} + a_{13}*b_{31}$ $a_{11}*b_{12} + a_{12}*b_{22} + a_{13}*b_{32}$	12 multiplications ( $2*3*2$ )					
	$a_{21}*b_{11} + a_{22}*b_{21} + a_{23}*b_{31}$ $a_{21}*b_{12} + a_{22}*b_{22} + a_{31}*b_{32}$	2X2 dimensions of the resultant matrix					
	<b>A1 X A2 X A3 {Multiplication of more than two matrices}</b>						
	2X3   3X4   4X2						
	d0 d1. d1 d2   d2 d3						
	Same answer by the two following methods (Associative property)						
	<b>Method 1:</b> (A1 X A2) X A3	<b>Method 2:</b> A1 X (A2 X A3)					
	2X3 3X4 i.e ( $2*3*4$ ) = 24 multiplications for A1 X A2	2X3   3X4   4X2					
	Now, (A1 X A2) X A3 would require ( $2*4*2$ ) = 16 multiplications	A2 X A3 requires ( $3*4*2$ ) = 24 multiplications					
	Thus, altogether 40 multiplications are required	A1 X (A2 X A3) requires ( $2*3*2$ ) = 12 multiplications					
		Altogether, 36 multiplications are needed here.					
	Now, dynamic programming asks us to find all the possible methods for matrix multiplication and check which one costs the minimum --> thsi implies that for 10 matrices, there would be numerous methods and we would have to check for all before proceeding with any one of them. Thus, we need a formula to check all that						
	We need to find C[1,3]						
	<b>Method 1:</b> (A1 X A2) X A3	<b>Method 2:</b> A1 X (A2 X A3)					
	C[1,2] = 24;   C[3,3] = 0	C[1,1] = 0; C[2,3] = 24					

	$C[1,2] + C[3,3] + d_0 \cdot d_2 \cdot d_3 = 40$	$C[1,1] + C[2,3] + d_0 \cdot d_1 \cdot d_3 = 36$					
	<b><math>C[i,j] = C[i, k] + C[k+1, j] + d_{i-1} \cdot d_k \cdot d_j</math></b>						
	After generalization,						
	$C[i,j] = \min \{ C[i,k] + C[k+1,j] + d_{i-1} \cdot d_k \cdot d_j \}$						
	where $i \leq k \leq j$						
	$A_1 \times A_2 \times A_3 \times A_4$						
	$d_0 \ d_1 \ d_1 \ d_2 \ d_2 \ d_3 \ d_3 \ d_4$						
	Check which method works the best for the above matrix chain multiplication	$2n \ C \ n / n + 1$ multiplications are possible where $n = \text{no. of matrices} - 1$					
	1. $A_1 (A_2 (A_3 A_4))$	Modified formula: $2(n-1) \ C \ (n-1) / n$					
	2. $A_1 ((A_2 A_3) A_4)$	Now, for $n = 4$ , $2 \cdot 3 \cdot 3 / 4 = 6 \cdot 5 \cdot 4 / 3 \cdot 2 \cdot 1 / 4 = 5$					
	3. $(A_1 A_2)(A_3 A_4)$	$n = 5$ , 14 multiplications					
	4. $(A_1 (A_2 A_3)) A_4$						
	5. $((A_1 A_2) A_3) A_4$						
	Applying the formula:						
<b>4-1 = 3 values</b>	$C[1,4] = \min \{ k = 1; C[1,1] + C[2,4] + d_0 \cdot d_1 \cdot d_4,$						
	$k = 2; C[1,2] + C[3,4] + d_0 \cdot d_2 \cdot d_4,$						
	$k = 3; C[1,3] + C[4,4] + d_0 \cdot d_3 \cdot d_4 \}$						
	$1 \leq k \leq 4$						
	1. $A_1 (A_2 A_3 A_4)$						
	2. $(A_1 A_2) (A_3 A_4)$						
	3. $(A_1 A_2 A_3) A_4$						
	here, $C[1,1] = 0; C[4,4] = 0$						
<b>4-2 = 2 values</b>	$C[2,4] = \min \{ k = 2; C[2,2] + C[3,4] + d_1 \cdot d_2 \cdot d_4$						
	$k = 3; C[2,3] + C[4,4] + d_1 \cdot d_3 \cdot d_4 \}$						
	$2 \leq k \leq 4$						

<b>4-3 = 1 value</b>	$C[3,4] = C[3,3] + C[4,4] + d_2 \cdot d_3 \cdot d_4$								
	A1 X A2 X A3 X A4								
	3X2   2X4   4X2   2X5								
	d0 d1   d1 d2 d2   d3   d3 d4								
	As we notice that there is a repetition of the values needed , such as C[3,4] or C[4,4], there is unnecessary calculation, we are repeating, thus we should use a table (4X4)								
<b>C table</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	$C[1,2] = \min\{k=1; C[1,1] + C[2,2] + d_0 \cdot d_1 \cdot d_2\}$				
	<b>1</b>	0	24	28	58	Thus, $C[1,2] = 3 \cdot 2 \cdot 4 = 24$			
	<b>2</b>	-	0	16	36	$C[2,3] = \min\{k = 2; C[2,2] + C[3,3] + d_1 \cdot d_2 \cdot d_3\}$			
	<b>3</b>	-	-	0	40	Thus, $C[2,3] = 2 \cdot 4 \cdot 2 = 16$			
	<b>4</b>	-	-	-	0	$C[3,4] = d_2 \cdot d_3 \cdot d_4 = 4 \cdot 2 \cdot 5 = 40$			
						$C[1,3] = \min\{k=1; C[1,1] + C[2,3] + d_0 \cdot d_1 \cdot d_3\}$			
<b>k table</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>		$k = 2; C[1,2] + C[3,3] + d_0 \cdot d_2 \cdot d_3\}$			
	<b>1</b>	-	1	1	3	$C[1,3] = \min\{16 + 3 \cdot 2 \cdot 2, 24 + 3 \cdot 4 \cdot 2\}$			
	<b>2</b>	-	-	2	3	$C[1,3] = \min\{28, 48\} = 28$			
	<b>3</b>	-	-	-	3				
	<b>4</b>	-	-	-	-	$C[2,4] = \min\{C[2,2] + C[3,4] + d_1 \cdot d_2 \cdot d_4, C[2,3] + C[4,4] + d_1 \cdot d_3 \cdot d_4\}$			
						$C[2,4] = \min\{40 + 2 \cdot 4 \cdot 5, 16 + 2 \cdot 2 \cdot 5\}$			
						The result shows that we need to do minimum of 58 multiplications to get the result of A1 X A2 X A3 X A4			
						The k table will give the paranthesization			
						$((A1) (A2 A3))(A4)$			
						How much time it has taken ? $1+2+3+4 = 4(5) / 2$ i.e. $n(n+1)/2 \sim n^2$			
						we also tried n posisble values of k to compute this value, thus time taken is $n^2 \cdot n$ i.e. $O(n^3)$			
						$k = 2; C[1,2] + C[3,4] + d_0 \cdot d_2 \cdot d_4,$			
						$k=3; C[1,3] + C[4,4] + d_0 \cdot d_3 \cdot d_4\}$			
						$C[1,4] = \min\{36 + 3 \cdot 2 \cdot 5, 24 + 40 + 3 \cdot 4 \cdot 5, 28 + 3 \cdot 2 \cdot 5\}$			
						$C[1,4] = \min\{66, 124, 58\} = 58$			
<b>61 Matrix chain multiplication Program</b>									
	A1 X A2 X A3 X A4					main{			
	5X4   4X6   6X2   2X7					int n = 5;			





Relaxation means between a pair of vertices u and v if there is an edge, then check if:	for (4,3) --> infinity + 3 < infinity only, so no change					
if( $d[u] + C(u,v) < d[v]$ ) {	for (1,4), $0 + 5 < \text{infinity}$ , thus vertex 4 is updated to 5					
$d[v] = d[u] + C(u,v)$	for (1,2) $0 + 4 < \text{infinity}$ , thus vertex 2 is updated to 4					
edgeList --> (1,2)(1,3)(1,4)(2,5)(3,2)(3,5)(4,3)(4,6)(5,7)(6,7)	Second iteration:					
Now, I have to relax these edges for $ V  - 1$ i.e. 6 times	for (3,2), vertex 2 is already 4, which is less than $d[u] + C(u,v)$ in this case					
Initially, mark the distance for source vertex as 0 and for the rest of the vertices as infinity	for (4,3) vertex 3 is updated to $5+3 = 8$					
Now, let's relax edge (1,2)	for (1,4)--> no change					
here, $d[u] = 0$ ; $d[v] = \text{infinity}$ ; $C(u,v) = 6$	for(1,2) --> no change					
$0 + 6 < \text{infinity}$ ; thus $d[v] = 6$	Third iteration					
for vertex 2, distance is 6;	for(3,2), $8 - 10 = -2 < d[v]$ which is 4 right now; thus updated for vertex 2 as -2					
similarly, relaxing (1,3); thus its distance is updated to 5 from infinity	for the rest of the edges there won't be any change					
in similar way, (1,4) is relaxed, following that the distance of vertex 4 is updated to 5 from infinity	results obtained :					
Now, relaxing (2,5); $d[u] = 6$ ; $C(u,v) = -1$ ; $d[v] = \text{infinity}$	vertex 1 --> 0					
the distance (2,5) is updated to $6 - 1 = 5$	vertex 2 --> -2					
similarly, relaxing (3,2); $d[u] = 5$ , $C(u,v) = -2$ , $d[v] = 6$	vertex 3 --> 8					
$5 - 2 = 3 < 6$ thus, $d[v]$ is updated here to 3 i.e. at vertex 2, distance is updated to 3	vertex 4 --> 5					
Now, relaxing (3,5), $d[u] = 5$ , $C(u,v) = 1$ , $d[v] = 5$ here $d[u] + C(u,v)$ is not smaller than $d[v]$ hence the distance of vertex 5 is not modified	Now, if I relax one more time extra, there's no change					
Moving to (4,3), relaxing it --> $d[u] = 5$ , $C(u,v) = -2$ ; $d[v] = 5$ , $d[u] + C(u,v) = 3 < d[v]$ hence distance of vertex 3 is updated to 3	<b>Drawback of Bellman Ford algorithm:</b>					
following this (4,6) is relaxed again and checked, $d[u] = 5$ , $C(u,v) = -1$ ; thus distance of vertex 6 is updated to $5-1 = 4$	let us an edge(2,4) in the above example					
Now, relaxing (5,7), $d[u] = 5$ ; $C(u,v) = 3$ ; $d[v] = \text{infinity}$	we see even after N-1 iterations, there is one vertex changing, we note that there's a problem					
thus, $d[v]$ is updated to 8 for edge (5,7)	the reason is that there is a cycle of edges where total weight of edges is negative i.e. $5 + 3 + (-10) = -2$ , thus graph cannot be solved					
moving to (6,7), $d[u] = 4$ , $C(u,v) = 3$ ; $d[v] = 8$	hence, for a negative weighted cycle, the bellman ford algorithm fails					

d[v] is updated to 7 here for edge (6,7)						
let us continue second time;						
there won't be any change in (1,2), (1,3), (1,4)						
when we check for (2,5); $d[u] = 3$ $C(u,v) = -1$ $d[v] = 5$ ; $d[v]$ for edge (2,5) is updated to 2						
similarly, for edge (3,2), the value is change to $3 - 2 = 1$ ; earlier it was 3						
(4,3) and (4,6), there's no change						
for (5,7) $d[u]$ has changed from 5 to 2; thus $d[v]$ changes to $2 + 3 = 5$						
for (6,7) there won't be any change						
let us continue third time;						
there won't be any change in (1,2), (1,3), (1,4),						
when we check for (2,5); $d[u] = 1$ $C(u,v) = -1$ $d[v] = 2$ ; $d[v]$ for edge (2,5) is updated to 0						
for (3,2) there own't be any change						
for (3,5) again thee won't be any change						
for (4,3), (4,6) --> no change						
for (5,7) $d[v]$ gets updated to $0 + 3 = 3$						
for (6,7) --> no change						
let us check for fourth time,						
we notice for all edges --> no change						
results obtained:						
vertex 1 --> 0						
vertex 2 --> 1						
vertex 3 --> 3						
vertex 4 --> 5						
vertex 5 --> 0						
vertex 6 --> 4						
vertex 7 --> 3						
so, finally these are the shortest paths						
<b>time complexity: <math>O( E ( V  - 1)) \sim O( V  E ) \sim O(N^2)</math></b>						
If it is a complete graph, that is between every two vertex there is an edge, then number of edges is $N(N - 1) / 2$						
i.e. $ E  = N(N - 1) / 2$						
then time complexity = $O( E  V ) O(N((N - 1) / 2)(N - 1)) \sim O(N^3)$						

63	0/1 Knapsack Problem																		
	m = 8; n = 4 where m is the capacity of the bag																		
	P = {1, 2, 5, 6}																		
	w = (2, 3, 4, 5)																		
	Have to give a solution of what all should be included to maximize profit																		
	x can be either 0 or 1; we cannot take fraction of an object																		
	x would be like {1, 0, 1....}																		
	sum of profits is maximized i.e. $\max\{\sum(P_i x_i)\}$																		
	$\sum(w_i x_i) \leq m$																		
	2^4 solutions possible i.e. n^4 solutions																		
	let us use tabulation method																		
	as the capacity is 8 so we would take 0-8 columns and 0-5 rows																		
V			0	1	2	3	4	5	6	7	8								
P	W	0	0	0	0	0	0	0	0	0	0								
1	2	1	0	0	1	1	1	1	1	1	1								
2	3	2	0	0	1	2	2	3	3	3	3								
5	4	3	0	0	1	2	5	5	6	7	7								
6	5	4	0	0	1	2	5	6	6	7	8								
	V[i,w] = max{V[i-1, w], V[i -1, w-w[i] ]+ P[i]}																		
	V[4,1] = max{V[3,1] , V[3, 1 - 5] + 6} = max{V[3,1], V[3,-4] + 6}																		
	V[3,-4] is an undefined location; thus, V[4,1] is equal to V[3,1]																		
	FROM THE ABOVE FORMULA WE DEDUCE THAT UPTO 5TH WEIGHT, WE WOULD BE GETTING A NEGATIVE NUMBER																		
	so, we fill these values as it is from previous row until column 4																		
	for 5th column, V[4,5] = max{V[3,5], V[3,0] + 6} = max{5, 0 + 6} = 6																		
	V[4,6] = max{V[3,6] , V[3, 1] + 6} = max{6, 6} --> so take 6 only																		
	V[4,7] = max{V[3,7], V[3,2] + 6} = max{7,7} = 7																		
	V[4,8] = max{V[3,8], V[3,3] +6} = max{7, 2 + 6} = max{7,8} = 8																		

	Now, we need to write down $x_1$ , $x_2$ , $x_3$ and $x_4$ values						
	let us come with maximum profit i.e. 8 which we got only by including 4th object						
	thus $x_4 = 1$ ; remaining profit = $8 - 6 = 2$						
	now, check for object 3, if there is a value 2; check if the value 2 is there in for object 2 as well at the same place, if yes, then object 3 was not taken i.e. $x_3 = 0$ ; if the same 2 is there for object 1 then $x_2 = 0$ else $x_2 = 1$						
	here, $x_3 = 0$						
	$x_2 = 1$						
	$x_1 = 0$ as the remaining profit is 0						
	$x = \{0 \ 1 \ 0 \ 1\}$ for maximizing the profit						
	<b>let us use sets method</b>						
	we will prepare sets of P and w						
	$s_0 = \{(0,0)\}$						
	$s_0(1) = \{(1,2)\}$ here, we added first object to elements of set $s_0$						
	$s_1 = \{(0,0), (1,2)\}$ merged the above two sets to get $s_1$						
	$s_1(1) = \{(2,3), (3,5)\}$ , added second object to elements of $s_1$						
	$s_2 = \{(0,0), (1,2), (2,3), (3,5)\}$ merged the above two sets to get $s_2$						
	$s_2(1) = \{(5,4), (6,6), (7,7), (8,9)\}$						
	$s_3 = \{(0,0), (1,2), (2,3), (3,5), (5,4), (6,6), (7,7)\}$ removed (8,9) as it exceeded the permitted limit						
	in the above set, we notice that as profit increases, weight increases, but at (5,4) profit has increased from (3,5) whereas weight has decreased						
	thus, we discard (3,5) with lesser profit {dominance rule}						
	so, $s_3 = \{(0,0), (1,2), (2,3), (5,4), (6,6), (7,7)\}$						
	now, considering the fourth object,						
	$s_3(1) = \{(6,5), (7,7), (8,8), (11,9), (12,11), (13,12)\}$						
	$s_4 = \{(0,0), (1,2), (2,3), (5,4), (6,6), (7,7), (8,8)\}$ merged above two sets and removed elements using dominance rule and those exceeding the weight limits						
	time taken is almost $(2^n)$						
	maximum order is (8,8)						
	(8,8) belongs to $s_4$						

	check whether it belongs to s3 ----> it does not, hence object 4 is included					
	now (8,8) - (6,5) = (2,3)					
	(2,3) --> check if it belongs to s3 --> yes, check whether it belongs to s2 -> yes, thus object 3 is not included --> now, check if it belongs to s1 --> no, that means object 2 is included and object 1 is not					
	2-2, 3-3 = (0,0)					
	(0,0) belongs to set 1 and set 0 as well therefore object 1 is not included					
	x = {0 1 0 1}					
<b>64</b>	<b>0/1 Knapsack Dynamic Programming</b>					
		main()				
	P = {0, 1, 2, 5, 6}	{				
	wt = {0, 2, 3, 4, 5}	int P[5] = {0, 1, 2, 5, 6}				
		int wt[5] = {0, 2, 3, 4, 5}				
	n = 4, m = 8	int m = 8, n = 4;				
		int k[5][9];				
	i = n ; j = m;					
	while(i > 0 && j > 0) {	for(int i = 0; i <= n; i++)				
	if(k[i][j] == k[i-1][j])	{				
	{	for(int w = 0; w <= m; w++)				
	cout << i << "=0" << endl; i--;	{				
	}	if(i==0    w == 0)				
	else {	{				
	cout << i << "=1" << endl; i--; j = j - wt[i];	k[i][w] = 0;				
	}	} else if (wt[i] <= w)				
	}	{				
		k[i][w] = max{P[i] + k				
		[i-1][w - wt[i]], k[i-1][w];				
		}				
		else				
		{				
		k[i][w] = k[i-1][w];				
		}				
		}				
		cout<<k[n][w];				

			}						
<b>65 Optimal Binary Search Tree</b>									
keys --> 10, 20, 30, 40, 50, 60, 70									
time taken for searching a particular key in a BST is logn where n is the number of nodes and logn is the minimum height of the tree									
if the target key is not in the tree, the search would be unsuccessful									
2 <sup>n</sup> C n / n + 1 binary searches are possible for n keys									
cost is dependent upon number of comparisons needed									
A balanced binary search tree would require the minimum number of comparisons									
In an optimal binary search tree problem, in addition to the number of comparisons we also consider the frequency of search of those keys									
<b>keys</b>	1	2	3	4	C[0,2]				
	10	20	30	40	10    20				
<b>frequency</b>	4	2	6	3	4    2				
<b>j</b>									
	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>				
<b>i</b>	<b>0</b>	0	4	8 <sub>1</sub>	20 <sub>3</sub>	26 <sub>3</sub>	I = j - i = 0		
	<b>1</b>	0	0	2	10 <sub>3</sub>	16 <sub>3</sub>	0 - 0 = 0		
	<b>2</b>	0	0	0	6	12 <sub>3</sub>	1 - 1 = 0		
	<b>3</b>	0	0	0	0	3	2 - 2 = 0		
	<b>4</b>	0	0	0	0	0	3 - 3 = 0		
						4 - 4 = 0			
w(0,4) = sum(f(i)) where 0<=i <= 4						set up cost for all diagonala values as 0;			
calculating For first case, C[0,2] = C[0,0] + C[1,2] + w[0,2]						I = j - i = 1			
C[0,2] = 0 + 2 + 6						C[0,1] i.e. considering only key 1			
C[0,2] = 8						cost is 4			
Similarly, calculating cost C[0,2] in the second case = C[0,1] + C[2,2] + w[0,2]						similarly, C[1,2] i.e. considering only second key			
here, C[0,2] = 4 + 0 + 6 = 10						cost is 2			
						Similarly, C[2,3] = 6 and C[3,4] = 3			

C[1,3] (CASE 1 when 20 is the root and 30 is it's right node) = $2*1 + 6*2 = 14$	Now, $l = j - i = 2$ i.e. we would consider 2 keys at a time					
C[1,3] (CASE 2 when 30 is the root and 20 is its left node) = $6*1 + 2*2 = 10$	i.e C[0,2] , C[1,3] and C[2,4]					
	C[0,2]: key 10 and 20 with frequency 4 and 2 respectively					
C[2,4] (CASE 1 when 30 is the root and 40 is its right node) = $6*1 + 3*2 = 12$	Two possibilities: 10 in the root and 20 it's right node OR 20 in the root and 10 it's left node					
C[2,4] (CASE 2 when 40 is the root and 30 is on the left) = $3*1 + 6*2 = 15$	for first case: Cost = $4*1 + 2*2 = 8$					
	for second case: Cost = $2*1 + 4*2 = 10$					
$l = j - i = 3$	minimum cost is 8 and the root is 1					
C[0,3] , C[1,4] for C[0,3], $w[0,3] = 12$						
For C[0,3] , (CASE 1 : 10 as a root and 20 it's right node, 30 is the right node of 20) = $C[0,0] + C[1,3] + w[0,3] = 0 + 10 + 12 = 22$						
(CASE 2, 10 as a root and 30 it's right node, 20 is the left node of 30)						
(CASE 3 when 20 is the root, 10 it's left child and 30 it's right child) = $C[0,1] + C[2,3] + 12 = 4 + 6 + 12 = 22$						
(CASE 4 when 30 is the root, 20 is the left child and 10 is its left child) = $C[0,2] + C[3,3] + 12 = 8 + 0 + 12 = 20$						
(CASE 5 when 30 is the root, 10 is its left child and 20 is the right child of 10)						
C[1,4] ; $w[1,4] = 11$						
$C[1,4] = \min\{C[1,1] + C[2,4] + 11; C[1,2] + C[3,4] + 11; C[1,3] + C[4,4] + 11\}$						
$C[1,4] = \min\{0 + 12 + 11; 2 + 3 + 11; 10 + 0 + 11\} = \min\{23, 16, 21\} = 16$						
$l = j - i = 4 = 4 - 0$ ; $w[0,4] = 15$						
$C[0,4] = \min\{C[0,0] + C[1,4] + 15; C[0,1] + C[2,4] + 15; C[0,2] + C[3,4] + 15; C[0,3] + C[4,4] + 15\} = \min\{0 + 16 + 15; 4 + 12 + 15; 8 + 3 + 15; 20 + 0 + 15\} = \min\{31, 31, 26, 35\} = 26$						

	root[0,4] = 3					
	then left child is root[0,2] and right child is root[3,4]					
	root[0,2] is 1 and root[3,4] is 4; root[3,4] is further subdivided into root[3,3] and root[4,4]					
	root[0,2] is further divided into root[0,0] and root[1,2]					
	root[1,2] is the second key and its further divided into root[1,1] and root[2,2]					
	$C[i,j] = \min\{C[i,k-1] + C[k,j]\} + w(i,j)$ where $i < k \leq j$					
66	<b>Optimal Binary Search Tree Successful And Unsuccessful Probability</b>					
	Unsuccessful nodes can be represented by dummy nodes					
	If there are n keys, there could be n+1 square dummy nodes					
	Successful search probability represents probability of getting a given key in the lot whereas unsuccessful search probability represents a range of values of the key					
	$\text{cost}[0,n] = P_i * \text{level}(a_i) + Q_i * (\text{level}(e_i) - 1)$					
	this cost value is calculated over $1 \leq i \leq n$ for successful searches ( $P_i$ ) and $0 \leq i \leq n$ for unsuccessful searches ( $Q_i$ )					
	the above cost is minimized for optimal binary search tree					
	Is it possible that we find out the best tree without calculating the cost of all the trees					
	Let's use dynamic programming to find out the minimum cost arrangement without actually computing cost for each binary search tree possible					
	$C[i,j] = \min\{C[i,k-1] + C[k,j]\} + w(i,j)$ where $i < k \leq j$					
	Let us consider it for just three nodes					
	i.e. $C[0,3] = \min\{C[0,0] + C[1,3] + w[0,3], C[0,1] + C[2,3] + w[0,3], C[0,2] + C[3,3] + w[0,3]\}$ where $0 < k \leq 3$					
	$C[0,0] = C[3,3] = 0$					
	$C[1,3] = \min\{C[1,1] + C[2,3], C[1,2] + C[3,3]\} + w[1,3]$ where $1 < k \leq 3$					



	Here, $C[1,1] = C[3,3] = 0$										
	Values we need, $C[0,0]$ , $C[1,1]$ , $C[2,2]$ , $C[3,3]$ , $C[0,1]$ , $C[1,2]$ , $C[2,3]$ , $C[0,2]$ , $C[1,3]$ , $C[0,3]$										
	$i, j$										
$j - i = 0$	$C[0,0]$	$C[1,1]$	$C[2,2]$	$C[3,3]$		$w[0,2] = q_0 + p_1 + q_1 + p_2 + q_2$					
$j - i = 1$	$C[0,1]$	$C[1,2]$	$C[2,3]$			$w[0,3] = q_0 + p_1 + q_1 + p_2 + q_2 + p_3 + q_3$					
$j - i = 2$	$C[0,2]$	$C[1,3]$				thus, $w[0,3] = w[0,2] + p_3 + q_3$					
$j - i = 3$	$C[0,3]$					$w[i, j] = w[i, j-1] + p_i + q_j$					
	we basically need to find the cost, weight and the root at each stage										
	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>j --&gt;</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>keys</b>	10	20	30	40		$j - i = 0$	$w_{00} = q_0 = 2$ $C_{00} = 0$ $r_{00} = 0$	$w_{11} = q_1 = 3$ $C_{11} = 0$ $r_{11} = 0$	$w_{22} = q_2 = 1$ $C_{22} = 0$ $r_{22} = 0$	$w_{33} = q_3 = 1$ $C_{33} = 0$ $r_{33} = 0$	$w_{44} = q_4 = 1$ $C_{44} = 0$ $r_{44} = 0$
<b>p<sub>i</sub></b>	3	3	1	1		$j - i = 1$	$w_{01} = w[0,0] + p_1 + q_1 = 8$ $C_{01} = 8$ $r_{01} = 1$	$w_{12} = 7$ $C_{12} = 7$ $r_{12} = 2$	$w_{23} = 3$ $C_{23} = 3$ $r_{23} = 3$	$w_{34} = 3$ $C_{34} = 3$ $r_{34} = 4$	
<b>q<sub>i</sub></b>	2	3	1	1	1	$j - i = 2$	$w_{02} = 12$ $C_{02} = 19$ $r_{02} = 1$	$w_{12} = 9$ $C_{12} = 12$ $r_{12} = 2$	$w_{24} = 5$ $C_{24} = 8$ $r_{24} = 3$		
						$j - i = 3$	$w_{03} = 14$ $C_{03} = 25$ $r_{03} = 2$	$w_{14} = 11$ $C_{14} = 19$ $r_{14} = 2$			
						$j - i = 4$	$w_{04} = 16$ $C_{04} = 32$ $r_{04} = 2$				
	Finally , the tree looks like the below tree:										
	$r[0,4] = 20$										
$r[0,1] = 10$						$r[2,4] = 30$					
						$r[3,4] = 40$					
<b>67</b>	<b>Travelling Salesman Problem Using Dynamic Programming</b>										
	A directed weighted graph is given and every edge is having weight										
	we have to start from some vertex and travel all the vertices and return back to the starting vertex and the cost of travelling should be minimum										

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>						
<b>1</b>	0	10	15	20						
<b>2</b>	5	0	9	10						
<b>3</b>	6	13	0	12						
<b>4</b>	8	8	9	0						
	$g(i,s) = \min \{ C_{ik} + g(k, s - \{k\}) \}$ where $k$ belongs to $s$									
	$g(1, \{2,3,4\}) = \min \{ C_{1k} + g(k, \{2,3,4\} - \{k\}) \}$ where $k$ belongs to $\{2,3,4\}$				Now, this is a recursive formula for which result can be obtained by generating recursive tree					
	vertex 1									
<b>here, costs can be taken from given table</b>	vertex 2 $g(1,2) = \min \{ C_{12} + g(2, \{3,4\}) \}$ ; $C_{12} = 10$				vertex 3 $g(1,3) = \min \{ C_{13} + g(3, \{2,4\}) \}$ ; $C_{13} = 15$	vertex 4 $g(1,4) = C_{14} + g(4, \{2,3\})$ ; $C_{14} = 20$				
	vertex 3 $C_{23} + g(3, \{4\})$    vertex 4 $C_{24} + g(4, \{3\})$ ; $C_{23} = 9, C_{24} = 10$				vertex 2 $C_{32} + g(2, \{4\})$    vertex 4 $C_{34} + g(4, \{2\})$ ; $C_{32} = 13, C_{34} = 12$	vertex 2 $C_{42} + g(2, \{3\})$    vertex 3 $C_{43} + g(3, \{2\})$ ; $C_{42} = 8, C_{43} = 9$				
	$g(3,\{4\}) = C_{34} + g(4, \text{phi i.e. nothing})$ ; $g(4, \{3\}) = C_{43} + g(3, \text{phi})$ ; $C_{34} = 12, C_{43} = 9$ ; $g(4, \text{phi}) \sim g(4,1) = 8$ ; similarly, $g(3, \text{phi}) = 6$				$g(2, \{4\}) = C_{24} + g(4, \text{phi})$ ; $g(4,\{2\}) = C_{42} + g(2, \text{phi})$ ; $C_{24} = 10, C_{42} = 8, g(4, \text{phi}) = 8, g(2, \text{phi}) = 5$	$g(2,\{3\}) = C_{23} + g(3, \text{phi})$ ; $g(3, \{2\}) = C_{32} + g(2, \text{phi})$ ; $C_{23} = 9, C_{32} = 13$ ; $g(3, \text{phi}) = 6, g(2, \text{phi}) = 5$				
	we use tabular method to solve these from bottom to top actually,									
	$g(2,\text{phi}) = 5$									
	$g(3, \text{phi}) = 6$									
	$g(4, \text{phi}) = 8$									
	$g(2,\{3\}) = 15$									
	$g(2,\{4\}) = 18$									
	$g(3,\{2\}) = 18$									
	$g(3,\{4\}) = 20$									
	$g(4,\{2\}) = 13$									

	$g(4, \{3\}) = 15$									
	$g(2, \{3, 4\}) = \text{minimum of the options} = 25$									
	$g(3, \{2, 4\}) = 25$									
	$g(4, \{2, 3\}) = 23$									
	$g(1, \{2, 3, 4\}) = 35$									
	<b>68 Reliability Design Problem</b>									
	we have to set up a system									
	system consists of certain devices									
	Reliability here means what is the probability of a device working perfectly / good				Example:					
DEVICES	D1	D2	D3	D4	$D_i$	$C_i$	$r_i$	$u_i$		
COST	C1	C2	C3	C4	D1	30	0.9	2		
RELIABILITY	$r_1$	$r_2$	$r_3$	$r_4$	D2	15	0.8	3		
	0.9	0.9	0.9	0.9	D3	20	0.5	3		
					$C = 105$					
	Reliability of the entire system = product of individual reliability of the devices = $0.9^4 = 0.6561$ i.e. 35% chance that the system might experience a failure				At least one copy of each device must be taken, so $C_1 + C_2 + C_3 = 30 + 15 + 20 = 65$					
	In case of failure, system is designed to have parallel copies of devices as a back up				Remaining amount = $C - \sum(C_i) = 105 - 65 = 40$					
	here, $r_1 = 0.9$				if I spend the entire remaining amount on D1, I would be able to purchase 1 D1 device					
	$1 - r_1 = 1 - 0.9 = 0.1$				<b><math>u_i = [C - \sum(C_i) / C_i] + 1</math> copies of any device <math>D_i</math></b>					
	$(1 - r_1)^3 = 0.1^3 = 0.001$				for D2, $40/15 = 2 + 1 = 3$ copies					
	probability that three copies are working perfectly = $1 - (1 - r_1)^3 = 0.999$				for D3, $40/20 = 2 + 1 = 3$ copies					
	Reliability has improved by using a parallel system				Solving this is similar to the set method of 0/1 knapsack problem, let's delve deeper					
	How many devices should I buy within the given cost such that the system's reliability is maximized				$(R, C)$					
					$s_0 = \{(1, 0)\}$					
					consider D1, $s_{11} = \{(0.9, 30)\}$					
					if we take 2 copies of the first device D1, $s_{12} = \{(1(1-0.9)^2)\} = 1 - 0.1^2 = 0.99$					
					thus, $s_{12} = \{(0.99, 60)\}$					

		$s_1 = \{(0.9, 30), (0.99, 60)\}$					
		consider D2 one copy, $s_{2_1} = \{(0.72, 45), (0.792, 75)\}$					
		Taking 2 copies of D2, $s_{2_2} = 1 - (1 - r_2)^2 = 1 - (1 - 0.8)^2 = 1 - 0.04 = 0.96$					
		$\{(0.864, 60), (0.9504, 90)\}$					
		in the last case, remaining cost = $105 - 90 = 15$ ; we cannot purchase D3 as its cost is 20 which is more than our remaining amount; so we would discard that set option as it is unfeasible					
		consider 3 copies of D2, reliability = $1 - (1 - r_2)^3 = 1 - (1 - 0.8)^3 = 1 - 0.008 = 0.992$					
		$s_{2_3} = \{(0.8928, 75), (\dots, 105)\}$ --> second case is discarded as it is unfeasible					
		$s_2 = \{(0.72, 45), (0.792, 75), (0.864, 60), (0.8928, 75)\}$					
		in $s_2$ , the third case has cost decreased but reliability is increased, thus we would use dominance rule to remove / discard the third case					
		Now, consider one copy of D3, $s_{3_1} = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$					
		$r_3 = 0.5$ ; $1 - (1 - r_3)^2 = 1 - 0.25 = 0.75$					
		$s_{3_2} = \{(0.54, 85), (0.648, 100)\}$ the third case has cost exceeding 105 thus it has been discarded					
		Consider three copies; $1 - (1 - r_3)^3 = 0.875$					
		$s_{3_3} = \{(0.63, 105)\}$					
		$s_3 = \{(0.36, 65), (0.432, 80), (0.4464, 95), (0.54, 85), (0.648, 100), (0.63, 105)\}$					
		Using dominance rule, $s_3 = \{(0.36, 65), (0.432, 80), (0.54, 85), (0.648, 100)\}$					
		Maximum reliability = 0.648 and the cost would be 100					
		for getting the number of copies of devices, go backward from 0.648, 100 ordered pair					
		which shows 1 copy of D1, 2 copies of D2, 2 copies of D3					
69	<b>Longest Common Subsequence (LCS)</b>						
	What is LCS?						

	LCS Using Recursion						
	LCS Using Memoization (to save time as recursion is time consuming)						
	LCS Using Dynamic Programming						
	String 1: a b c d e f g h i j	String1: a b d a c e					
	String 2: e c d g i	String 2: b a b c e					
	c d g i is the longest subsequence	b a c e and a b c e are the longest subsequences					
	<b>LCS Using recursion</b>						
<b>A</b>	b    d    \0	int LCS(i,j)					
		{					
<b>B</b>	a    b    c    d    \0	if (A[i] == '\0'    B[j] == '\0')					
		return 0;					
		else if (A[i] == B[j])					
		return 1 + LCS(i + 1, j + 1);					
		else					
		return max(LCS(i+1, j), LCS(i, j+1));					
		}					
	Recursion tree for the above example -						
	A[0] , B[0] : b, a : 2						
<b>Call1</b>	A[1], B[0] : d, a : 1	A[0], B[1]: b, b : 2					
<b>Call2</b>	A[2], B[0] : \0, a          A[1], B[1] : d, b : 1	1 + A[1], B[2] : 1 + d, c					
<b>Call3</b>	0          A[2], B[1] : \0, b          A[1], B[2] : d, c : 1	1 + A[2], B[2]          1 + A[1], B[3]					
<b>Call 4</b>	0          0          A[2], B[2] : \0, c          A[1], B[3] : d, d : 1	1 + \0          1 + 1					
	0          0          0          1 + A[2], B[4] : 1 + \0 : 1						
	The purpose of the above problem was to depict that there is an issue of overlapping. But, there is no reason for repeat recalls if we can store the answer of previous stage. Thus, we use memoization						
	<b>LCS using memoization</b>						
	<b>a    b    c    d    \0</b>						

		0	1	2	3	4								
b	0	2	2	-	-	-								
d	1	1	1	1	1	-								
l	0	0	0	0	-	0								
		Memoization can reduce the number of function calls : O(m X n) where m and n are the counts in String 1 and 2 respectively												
		LCS using dynamic programming												
		DP would follow bottom up approach and fill out the table top to bottom whereas in the previous approach we were following top down approach and filling the table in bottom up manner						if(A[i] == B[j])						
		A : b d						{						
		B : a b c d						LCS[i,j] = 1 + LCS[i - 1, j - 1];						
								} else						
								{						
		0	1	2	3	4	LCS[i,j] = max(LCS(i - 1, j], LCS[i, j - 1])							
	0	0	0	0	0	0	}							
b	1	0	0	1	1	1								
d	2	0	0	1	1	2								
		if we trace our way back from 2 (d,d) above then we see that we go backwards diagonally to 1 (b,c) , then horizontally to 1 (b, b) and then diagonally back to 0 (0,a); thus there are two times we moved diagonally back i.e. b d is the LCS												
str 1		s t o n e												
str 2		l o n g e s t												
			l	o	n	g	e	s	t					
		0	1	2	3	4	5	6	7					
o	0	0	0	0	0	0	0	0	0					
s	1	0	0	0	0	0	0	1	1					
t	2	0	0	0	0	0	0	1	2					
e	3	0	0	1	1	1	1	1	2					
n	4	0	0	1	2	2	2	2	2					



	How to find an articulation point in a graph?	Algorithm:					
	DFS: 1 2 3 4 5 6	Conduct depth first search and find depth first search spanning tree					
	d: 1 6 3 2 4 5	Mention the discovery times for each vertex					
	L: 1 1 1 1 3 3	We need to find the lowest discovery number to find any vertex by taking a back edge					
	u, v: parent, child then the lowest number of v i.e. $L[v] \geq d[u]$ then u is an articulation point. This condition is true for all except the root.	Find out articulation point					
	The above algorithm is true for v = 5 and u = 3; hence 3 is the articulation point						
	If root has more than one child, then root is also an articulation point						
	<b>72 Backtracking</b>						
	<b>Brute force approach</b>						
	When you have multiple solutions and you want to get all those solutions , then you use backtracking						
	Example: B1B2G1 (Three students) in 3 chairs ..... ..	All possible arrangements 3! ways					
	<b>State space tree</b>						
	B1 (in chair 1)						
	B2 (in chair 2)						
	G1 (in chair 3)						
	for another arrangement, take out B2 and G1; then G1 can sit in chair 2 and B2 in chair 3						
	Now, no more solutions with B1 in chair 1; then take out B1 as well; put B2 in chair 1						
	B1 in chair 2 and G1 in chair 3; B1 in chair 3 and G1 in chair 2						
	Again, no more solutions with B2 in chair 1, so take B2 out and put G1 in chair 1						
	B1 in chair 2 and B2 in chair 3; B2 in chair 2 and B1 in chair 3						
	thus, altogether 6 solutions	Now, in backtracking problems, usually we have certain constraints and we select solutions that work for them; for instance, G1 cannot sit in any middle positions					
		Then, possible solutions are: B1B2G1; B2B1G1; G1B1B2; G1B2B1					
		The rest of the <b>nodes are killed</b> when we applied the <b>bounding function</b>					



	Same brute force approach is used by <b>branch and bound strategy</b> ; both strategies also generate state space tree as well...but <b>back tracking follows depth first search</b> whereas <b>branch and bound follows breadth first search</b> approach						
	Branch and bound state space tree is generated level-wise						
	<b>73 N-Queens Problem</b>						
	A chess board is given (4X4) for the case of simplicity even though a standard chess board is 8X8. We are given 4 queens Q1Q2Q3Q4	Queen's moves can be diagonal or horizontal or vertical; we need to place the 4 queens such that they are not under attack i.e. they are not in the same row, column or diagonal					
	<b>1                  2                  3                  4</b>	We have more than one solution and we want all possible solutions					
	.....                  .....                  ....                  ....	I can place them in $16C4$ ways. But, A queen cannot be kept anywhere on the board but first queen in Row 1; second in row 2 and so on.. We can avoid keeping more than one queen in single column					
<b>1</b>							
<b>2</b>	.....                  .....                  ....                  ....						
<b>3</b>	.....                  .....                  ....                  ....						
<b>4</b>	.....                  .....                  ....                  ....						
	State space tree first without taking care of diagonal attacks						
	Q1 in column1 --> Q2 in column2 --> Q3 in column 3 --> Q4 in column 4						
	Now, move back and move Q3 in column 4 and Q4 in column 3						
	Now, move back and move Q2 in column 3 --> Q3 in column 2 --> Q4 in column 4						
	Again , move back and move Q3 in column 4 and Q4 in column 2						
	Now, move back and move Q2 in column 2 --> 2 possibilities Q3 in column 2 and Q4 in column 3; Q3 in column 3 and Q4 in column 2						
	Now, roll back completely, move Q1 in column 2 and we get similar to above set of solutions						
	Thus, total possible cells we are checking: $1 + 4 + 4*3 + 4*3*2 + 4*3*2!$ nodes when we have avoided same rows and same columns but we are allowing diagonals	65 nodes					

	1 + Summation(Product(N - j)) where j ranges from 0 to i and i ranges from 0 to 3; here N is 4 ; for 8X8 board, N would be 8					
	Now, let us solve this with bounding function i.e. condition - not same row, same column , same diagonal					
	State space tree					
	Q1 in column 1 --> Q2 in column 2 --> under attack --> kill the node					
	Q1 in column 1 --> Q2 in column 3 --> Q3 in column 2 --> under attack --> kill the node					
	Q1 in column 1 --> Q2 in column 3 --> Q3 in column 4 --> under attack --> kill the node					
	Q1 in column 1 --> Q2 in column 4 --> Q3 in column 2 --> Q4 in column 3 --> under attack --> kill the node					
	Q1 in column 1 --> Q2 in column 4 --> Q3 in column 3 --> Q4 in column 2 --> under attack --> kill the node					
	Q1 in column 2 --> Q2 in column 1 --> under attack --> kill the node					
	Q1 in column 2 --> Q2 in column 3 --> under attack --> kill the node					
	<b>Q1 in column 2 --&gt; Q2 in column 4 --&gt; Q3 in column 1 --&gt; Q4 in column 3</b>	solution 1				
	<b>Q1 in column 3 --&gt; Q2 in column 1 --&gt; Q3 in column 4 --&gt; Q4 in column 2</b>	solution 2 ( mirror image of solution 1)				
<b>74</b>	<b>Sum of subsets problem</b>					
	w[1:6] = {5, 10, 12, 13, 15, 18}	Six weights are given; we have to take a subset such that their sum is 30				
	n = 6; m = 30	Total is 73 here for given weights				
	x = ..... .....					
	1    2    3    4    5    6	Each xi value can be 0/1				
	State space tree					
	Either first weight is included    Or first weight is not included					
	Either second weight is included    Or second weight is not included					
	Either third weight is included    Or second third is not included					
	...so, we get 7 levels --> 2^6 paths i.e. 2^n paths	thus, it is time consuming but we try to kill the nodes if they do not satisfy the bounding function				
	0, 73					

5, 68 (first weight is included)						
15, 58 (second weight is included)						
27, 46 (third weight is included)						
if I include fourth weight --> 40, 33 --> exceeds the bounding condition --> kill this node	Summation( $w_i x_{i=1 \text{ to } k}$ ) + $w_{k+1} \leq m$					
Try without including 4th object, 27, 33						
Try including 5th object --> 43, 18 --> kill this node						
Try without 5th object --> 27, 18						
Now, try including 6th object --> 45, 0 --> exceeding --> kill the node						
Try without 6th object --> 27, 0 --> meaningless --> not enough weights	Summation( $w_i x_{i=1 \text{ to } k}$ ) + $w_{i=k+1 \text{ to } n} > m$					
Try excluding object 3; 15, 46						
Try including 4th object --> 28, 33						
Try including 5th object --> 43, 18 --> exceeding --> kill the node						
Try without 5th object and include 6th object --> 44, 0 --> kill the node						
Try excluding object 4 as well : 15, 33						
Try including 5th object : 30, 18	Solution 1: Object 1, 2 and 5					
Several other possible solutions by following depth first search in this backtracking problem						
<b>75 Graph coloring problem</b>						
A graph is given and some colors are given. We need to color the vertices of the graph such that no two neighboring vertices are of the same color						
So, let us start from vertex 1 (Red color)	we have 5 vertices and three colors: R, G, B					
vertex 2 (neighbors 1 and 3) : G color						
vertex 3 (neighbors 2, 4 and 5) : R color						
vertex 4 (neighbors 3 and 5) : G color						
vertex 5 (neighbors 4, 1, 2 and 3): B color						
1    2    3    4    5						
R    G    R    G    B	m Coloring Decision Problem (can the graph be colored or not) or Chromatic color problem					

	G R G R B	m Coloring Optimization Problem (minimum how many colors are needed)					
	if we had just two colors, it can not be colored						
	we just want to know if the graph can be colored by the given number of colors						
	4 vertices: 1,2 , 3, 4						
	3 colrs: RGB						
	State space tree						
	vertex 1 can be R , G , or B						
	following that vertex 2 can be R, G, B and so on	Right now, we are not applying adjacency condition					
		Total number of nodes generated: $1 + 3 + 3*3 + 3*3*3 + 3*3*3*3 = 1 + 3 + 9 + 27 + 81 = 3^5 - 1 / 3 - 1$					
		Total time spent is $3^{n+1}$ i.e. $C^{n+1}$					
	Now, let us consider adjacency condition						
	vertex 1: R color						
	vertex 2 (neighbors 1 and 3): G color						
	vertex 3 (neighbors 2 and 4): R color						
	vertex 4 (neighbors 1 and 3): G or B color						
	R G R G    R G R B						
	R G B G						
	R B R B    R B R G						
	R B G B						
	I have a map with different regions. We need to color the map such that the adjacent areas are not of the same color. Minimum number of times the sheet needs to be passed through a printer						
	For each region of a map, we mention it as a vertex and we would draw an edge for neighboring regions						
	Then, we can solve the m-coloring problem in the graph and implement those colors for the map						
	<b>76 Hamiltonian cycle</b>						
	If a graph is given , we have to start from any one vertex and travel through all the vertices and reach back to the starting vertex --> this forms a cycle --> we need to check if there is a hamiltonian cycle possible --> find all possibilities						

	Graph given may be directed or non-directed but it must be connected						
	It's an exponential time taking problem						
	If the order of the vertices is the same, even though the starting vertex varies, it is still considered the same hamiltonian cycle						
	If there is an <b>articulation point</b> (junction point) in a graph, then hamiltonian cycle is not possible in the graph						
	If there is a <b>pendant vertex</b> in a graph, then hamiltonian cycle is not possible in the graph						
	Adjacency matrix for the graph:	<b>Algorithm</b>					
<b>G</b>	<b>1 2 3 4 5</b>	{					
	<b>1</b> 0 1 1 0 1	do					
	<b>2</b> 1 0 1 1 1	{					
	<b>3</b> 1 1 0 1 0	NextVertex(k);					
	<b>4</b> 0 1 1 0 1	if(x[k] == 0)					
	<b>5</b> 1 1 0 1 0	{					
		return;					
	Array to determine if the cycle has been found:	}					
<b>x</b>	..... .....	if(k == n)					
	1 2 3 4 5	{					
	Initially all these values are zero;	print(x[1:n]);					
	we do not want repetitions, so we fix the starting vertex; let us take it as 1	} else					
<b>x</b>	1 0 0 0 0	{					
	State space tree:	Hamiltonian(k+1)					
	vertex 1	}					
	try to put 1 in vertex 2; but 1 is already at vertex 1; so put 2 at vertex 2 --> check if there is an edge from 1 to 2 --> adjacency matrix shows that there is an edge	} while(true);					
<b>x</b>	1 2 0 0 0	}					
	For next position i.e. vertex 3, try putting 1 or 2 but they are already there so use 3; check if there is an edge between 2 and 3 --> yes, there is	Algorithm NextVertex(k)					
<b>x</b>	1 2 3 0 0	{					
	Similarly, check if vertex 4 is connected to vertex 3 by an edge --> yes	do					
<b>x</b>	1 2 3 4 0	{					
	Now, 4 is connected with 5 with an edge	x[k] = (x[k] + 1)mod(n+1);					

x	1	2	3	4	5	if(x[k] == 0) return;					
	Now, from 5 to 1 there is an edge, hence a cycle is formed and first solution is obtained					if(G[x[k - 1], x[k]] not equals 0){					
	we can check for cycle again by using vertices in another order					for j = 1 to k - 1 do if (x[j] == x[k]) break;					
x	1	2	5	4	3	if(j == k)					
	all possibilities: 4!					if(k < n or (k == n) && G[x[n], x[1]] not equals zero					
	for n vertices graph , (n - 1)!, thus time complexity is O (n^n)					return;					
						} while(true);					
						}					
										</	

	Further expansion is not possible as the fifth node has already been placed						
	similar manner, J3 and J4 are popped and placed as child of J2						
	If we use queue --> FIFO Branch and bound						
	If we use stack --> LIFO Branch and Bound						
	Now, let us look at Least cost branch and bound method						
	we will start with vertex 1 and compute cost at each node						
	1, Cost = infinity						
	J1, cost: 25						
	J2, cost = 12						
	J3, cost 19						
	J4, cost = 30						
	let us explore the node with the minimum cost i.e. J2						
	<b>78 Job sequencing with deadline problem</b>						
	Maximization problem to maximize profit but branch and bound can only solve minimization problems, thus we have converted profits to penalty and problem now becomes to minimize the overall penalty						
<b>Jobs</b>	1	2	3	4			
<b>Penalty</b>	5	10	6	3			
<b>Deadline</b>	1	3	2	1			
<b>Time taken for completion</b>	1	2	1	1			
	By generating a state space tree:	each node will be specified by upper bound (sum of all the penalties for the jobs that have not been included till now); cost (sum of penalties of jobs included till now)					
	1: U = INFINITY, COST: 0						
	2: J1 is included; cost: 0, u: 19						
	3: J2 is included and we are not doing J1: u: 14, cost = 5						

	4: J3 is included and we are not doing J1 and J2: cost = 15; u: 14	here, cost has increased from the previous node whereas the upper value remains the same, thus we kill this node					
	5: J4 is included and we are not doing J1, J2 and J3; COST = 21 (HIGHER THAN THE PREVIOUS NODE, THUS KILL THIS NODE)						
	Now, let's move to the next level in the state space tree						
	Let us include J1 and further three scenarios: J2 is included; J3 is included; J4 is included						
	6: J2 is included, cost = 0; u = 9						
	7: J3 is included, cost = 10 (2nd job is not included); u = 9 --> kill this node						
	8: J4 is included whereas J2 and J3 are excluded ; cost = 16 --> kill this node						
	for J2 , 2 possibilities: J3 is included; J4 is included						
	9: J1 is not included, J2 is included, J3 is included: cost = 5; u = 8						
	10: J1 is not included, J2 is included, J3 is not included, J4 is included: cost = 11; u = 8 --> kill this node						
	11: J1 is included --> J2 is included --> J3 is included						
	but, if we complete J1 and J2, 3 hours are used and there is no time left for J3 --> this job cannot be done; similarly J4 cannot be done						
	12: J1 is not included, J2 is included, J3 is included, J4 is included; cost = 5; u = 5; but they cannot be done this manner based on the time and deadlines						
	Thus, optimum cost we got is 5 and penalty is 8: doing J2 and J3						
	<b>79 0/1 Knapsack problem</b>						
<b>Profit</b>	10    10    12    18						
<b>Weight</b>	2      4      6      9						
	m = 15; n = 4						
	Maximize profit when you can fill the bag with complete weight or not take it at all						



	Branch and bound can solve only minimization problems, so to convert this maximization problem to a minimization one, we would convert positive profits to negative ones and then minimize them overall						
	We will use LC Branch and bound (always exploring that node whose cost is minimum)						
	$u = \text{summation}(\text{Pixi}) \leq m$						
	$C = \text{summation}(\text{Pixi})$ with fraction						
	We would generate a state space tree but we need to decide what kind of solution we want - a subset or a fixed-sized solution of 0's and 1's. So, here we are seeking a fixed sized solution						
	Let us generate state space tree						
	upper = infinity						
	$C = 10 + 10 + 12$						
	$2 + 4 + 6$ i.e total comes out to 12						
	we can include 3 more kgs						
	thus, $C = 10 + 10 + 12 + 18/9 * 3 = -38$						
	$u = -32$						
<b>u</b>	-32						
<b>C</b>	-38						
	Now, I am including the first object then we get the same solution, $u = -32$ , $v = -38$ ; but if we exclude object 1, then						
	$C = 10 + 12 + 18/9 * 5 = -32$ ; upper = -22						
	$4 + 6 + 5$ kgs						
	Now, this cost is not greater than the previous upper, so no need to kill this node						
	we will follow least cost branch and bound; and go ahead with the least cost branch which is including object 1 as the cost is -38						
	it can be further explored, if we include object 2 after including object 1 or not						
	if we include object 2, then the cost and upper bound remain the same as nothing has been excluded						
	on the other hand, if we exclude object 2, the new cost and upper bound value in this scenario is as follows:						
	$u = -22$						

	$C = 10 + 12 + 18/9 * 7 = -36$					
	$2 + 6 + 7$					
	Now, the cost is not greater than $u = -32$ , thus we would not kill it					
	Now, least cost is -38 when we include both objects 1 and 2, so we would explore that path more					
	if Object 3 is included, cost remains the same i.e. $C = -38$ , $u = -32$					
	if Object 3 is excluded, cost is as follows:					
	$C = 10 + 10 + 18 = -38$					
	$2 + 4 + 9$					
	$u = -38$					
	here, we got the smaller upperbound, so upper bound gets updated to -38					
	also look at the cost of alive nodes, and kill those nodes whose Cost is greater than -38					
	Now, if we take further the scenario that all objects are included, it is not feasible as weight exceeds the bag limit					
	so, we would explore the scenario when object 3 is excluded while the rest are included					
	$C = -38$ ; $u = -38$					
	and if we consider not including 4th object, then $C = -20$ ; this cost is greater than -38 so kill the node					
	<b>thus, answer is <math>x\{1, 1, 0, 1\}</math></b>					
	<b>maximum profit is 38 and weight is 15</b>					
	<b>80 Travelling salesperson branch and bound</b>					
	A weighted graph is given and we need to find the shortest tour such that it travels through all the vertices and returns back to the starting vertex					
	Now, if we make the state space tree for a 4-vertex graph and we start with vertex 1, then the first level would be three choices - vertex 2, 3 or 4; It will further have 2 choices each on each path, for example - vertex 1 - vertex 2 - then can take vertex 3 or 4 further it will take the remaining vertex	Now, in this case, we are using depth first search which is normally used in back tracking problems where we wish to find all possible solutions. then we can discard or replace solutions that have the least tour cost / weight				
	We would use level order for applying branch and bound	but, this problem if solved by back tracking, it is time consuming				
	For every node, we would find the cost, if cost is greater than the upper bound set, then that node is killed					

	Let us solve an example:										
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>						
<b>1</b>	INF	20	30	10	11						
<b>2</b>	15	INF	16	4	2						
<b>3</b>	3	5	INF	2	4						
<b>4</b>	19	6	18	INF	3						
<b>5</b>	16	4	7	16	INF						
	Now, we would first reduce this matrix by writing the minimum row values and subtracting the minimum values from all the values										
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>						
<b>1</b>	INF	10	20	0	1		10				
<b>2</b>	13	INF	14	2	0		2				
<b>3</b>	1	3	INF	0	2		2				
<b>4</b>	16	3	15	INF	0		3				
<b>5</b>	12	0	3	12	INF		4				
						Total cost of reduction = 21					
	Now, write the minimum value of each column and reduce them										
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>						
<b>1</b>	INF	10	17	0	1						
<b>2</b>	12	INF	11	2	0						
<b>3</b>	0	3	INF	0	2						
<b>4</b>	15	3	12	INF	0						
<b>5</b>	11	0	0	12	INF						
	1	0	3	0	0	Total cost of reduction = 25					
	Now, minimum cost of tour would be at least 25 (i.e. cost of reduction) but to find it corectly, we would implement a state space tree										
	from vertex 1 --> we cen go to vertex 2, or 3, or 4 or 5										
	Cost of first matrix would be 25; upper = infinity										
	for every node we would find the cost and will update it only once we reach the leaf node rather than updating it each time										
	Now, vertex 1 --> vertex 2, make the first row and second column as infinity										
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>						

<b>1</b>	INF	INF	INF	INF	INF						
<b>2</b>	12	INF	11	2	0						
<b>3</b>	0	INF	INF	0	2						
<b>4</b>	15	INF	12	INF	0						
<b>5</b>	11	INF	0	12	INF						
we note that it is still a reduced matrix, then the cost is $C(1,2)$ + cost of reduction, $r + rCap$											
$10 + 25 + 0 = 35$											
Now, let's calculate the cost for vertex 1 --> vertex 3											
Make first row as infinity, third column as infinity, make path 3 to 1 also as infinity											
<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>							
<b>1</b>	INF	INF	INF	INF	INF	Checking row and column minimums and subtracting if some row or column has non- zero minimum					
<b>2</b>	12	INF	INF	2	0						
<b>3</b>	INF	3	INF	0	2						
<b>4</b>	15	3	INF	INF	0						
<b>5</b>	11	0	INF	12	INF						
	11	0	inf	0	0	Cost of further reduction: 11					
<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>							
<b>1</b>	INF	INF	INF	INF	INF						
<b>2</b>	1	INF	INF	2	0						
<b>3</b>	INF	3	INF	0	2	Net cost : $C(1,3) + r + rCap = 17 + 25 + 11$ $= 53$					
<b>4</b>	4	3	INF	INF	0						
<b>5</b>	0	0	INF	12	INF						
similarly, we calculated the cost of the remaining two vertices											
vertex 1 --> 4, cost = 25											
vertex 1 --> 5, cost = 31											
Now, if we consider FIFO branch and bound, then we would consider vertex 1 --> 2, if we consider LC branch and bound, then we would consider vertex 1 --> 4											
from vertex 4 --> can further go to vertex 2, 3 or 5											
we will use this matrix that was obtained from vertex 1 --> 4											

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>						
<b>1</b>	INF	INF	INF	INF	INF						
<b>2</b>	12	INF	11	INF	0						
<b>3</b>	0	3	INF	INF	2						
<b>4</b>	INF	3	12	INF	0						
<b>5</b>	11	0	0	INF	INF						
now, vertex 4 --> 2; set row 4 and column 2 as infinity, also set 1,2 as infinity as we cannot go back from 2 to vertex 1											
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>						
<b>1</b>	INF	INF	INF	INF	INF						
<b>2</b>	12	INF	11	INF	0		0				
<b>3</b>	0	INF	INF	INF	2		0				
<b>4</b>	INF	INF	INF	INF	INF						
<b>5</b>	11	INF	0	INF	INF		0				
it is already reduced , thus cost = (4,2) + C(4) + rCap = 3 + 25 + 0 = 28											
For other two vertices, cost is C = 50 (vertex 1 --> 4 --> 3) and C = 36 (vertex 1 --> 4 --> 5)											
Select the minimum cost one --> vertex 1 --> 4 --> 2											
Now, remaining vertices are 3 and 5											
for vertex 3, make second row and third column infinity											
also, make 3,1 as infinity											
then, we make it a reduced matrix											
Cost = C(2,3) + C(2) + rCap = 11 + 28 + 13 = 52											
Now, cost for vertex 1 --> 4 --> 2 --> 5; C is 28											
considering minimum cost one, i.e. vertex 5 , vertex 1 --> 4 --> 2 --> 5											
the last one remaining is vertex 3 only											
Cost comes out to be 28											
thus, shortest dist. route : vertex 1 --> 4 --> 2 --> 5 --> 3											
<b>81</b>	<b>NP -Hard and NP-Complete</b>										
	<b>Polynomial time</b>					<b>Exponential time</b>					

	Linear search - $n$	0/1 Knapsack - $2^n$					
	Binary search - $\log n$	Travelling salesperson - $2^n$					
	Insertion sort - $n^2$	Sum of subsets - $2^n$					
	Merge sort - $n \log n$	Graph coloring - $2^n$					
	Matrix multiplication - $n^3$	Hamiltonian cycle - $2^n$					
	We want faster methods to solve these problems.						
	Frameworks made for doing research on exponential time problems						
	Those frameworks are NP-Hard and NP-Complete						
	When you are unable to solve exponential time problems, you at least try to show similarities between the problems, such that if one is solved, the other can also be solved	Either solve it, or atleast relate it					
	When you are unable to write polynomial time deterministic algorithms for exponential time problems, then you should go about writing polynomial time non-deterministic algorithms						
	Deterministic algorithm means that each and every statement has been written by us and we know how it works						
	In a non deterministic algorithm also, most of the statements might be deterministic , just some statements we might not be sure or aware of, we can leave them blank and fill it when we get hold of those problems	This helps in preserving the work done until now for future researchers who might be able to fill in the gaps					
	<b>How to write non-deterministic algorithms?</b>						
	Example:						
	Algorithm NSearch(A, n, key)	here, choice(), success(), failure() are all the ways / statements we use in non-deterministic algorithms					
	j = choice();	we assume that these are taking $O(1)$ time					
	if(key == A[j])	Initially, we do not know the time taken and they are non-deterministic ...once in future, we get to know them, then it is filled in by deterministic statements					
	{	Entire algorithm takes $O(1)$ time - fastest					
	write(j);	How the choice() is working we do not know, once we get to know about it, it becomes a technique / deterministic					
	success();						
	}						
	write(0);						
	failure();						

	<b>P</b> is the set of those deterministic algorithms which take polynomial time	<b>NP</b> is the set of those non-deterministic algorithms which might take polynomial time					
	P is shown as a subset of NP as they were once unknown in the past and have been found with time.'						
	Cook's theorem tries to prove that P is a subset of NP						
	<b>CNF - Satisfiability:</b> To find the relationship b/w exponential time taking problems						
	x: {x1, x2, x3}						
	CNF propositional calculus formula; $CNF = (x1 \vee x2 \vee x3) \wedge (x1 \text{ complement} \vee x2 \text{ complement} \vee x3 \text{ complement})$						
	Satisfiability problem is to find out for which values of xi the above formula is true.						
	x1      x2      x3						
	0      0      0						
	0      0      1						
	0      1      0						
	0      1      1						
	1      0      0						
	1      0      1						
	1      1      0						
	1      1      1						
	I should try all the above values for the CNF and it takes ( $2^n$ ) or exponential time						
	If we make a state space tree, the path from the root to the leaf of the tree gives us the solution						
	Also, to understand how to find the relationships between these exponential time problems, we take a 0/1 knapsack problem						
	$P = \{10, 8, 12\}$ w - { 5, 4, 3} m = 8						
	Profit should be maximized and capacity of the bag should not be exceeded						
	x1, x2, x3 can take $2^3$ values as shown above						
	Thus, satisfiability has also been added to exponential time problems and if you solve the state space tree in polynomial time, then use it to find solution for CNF						
	Satisfiability is the base problem						
	<b>NP - Hard problem (Satisfiability)</b>						

	We take some instance of satisfiability and from that we convert it into 0/1 knapsack problem. We can say that if either one is solved and it can be used to solve the other in polynomial time. This shows that they are related to each other..						
	If satisfiability is reducing to 0/1 knapsack problem, then 0/1 knapsack problem is also knapsack problem						
	Reduction has a transitive property						
	Satisfiability $\rightarrow$ L1, then L1 $\rightarrow$ L2						
	For satisfiability, we have non-deterministic polynomial time algorithm. Then that problem is also called NP-Complete.						
	Satisfiability $\rightarrow$ L; for any of these problems, if this problem is reduced by Satisfiability, then if we are able to write non-deterministic algorithm, then L also becomes NP-Complete						
	L -- we need to show that this is directly or indirectly related to satisfiability $\rightarrow$ it becomes part of NP Hard class, if we also write a non-deterministic algorithm $\rightarrow$ it becomes part of NP-Complete						
	Satisfiability is the subset of NP-Hard and if satisfiability is used to reduce Graph coloring or 0/1 knapsack then they also become NP Hard						
	Intersection of NP Hard and non-deterministic algorithm problems is NP complete						
	P is a subset of NP (non-deterministic algorithms) and this P keeps on expanding as the research yields results with time						
	If we are able to prove that $P = NP$ , then it can be proved that whatever non-deterministic we are working on today, it will become deterministic in future						
	<b>Cook's algorithm:</b> satisfiability lies in P ie it becomes deterministic, if and only if $P = NP$						
82	<b>NP-Hard Graph Problem &amp; Clique decision making problem (CDP)</b>						
	<b>Complete graph</b> - for every vertex, there is an edge connecting to all the other vertices						
	$ V  = n$ , then number of edges $ E  = n(n-1)/2$						
	What is a clique? Sub graph of a graph which is actually a complete graph is called a clique						
	A problem which requires answer yes or no is a decision problem						



What is the maximum clique size in the graph? -- optimization problem						
If a decision problem is an NP Hard problem , then the optimization problem is also an NP Hard problem, but we first need to prove that clique decision problem is an NP Hard problem.						
Select a known LP Hard problem and from that relate and prove the other problem LP Hard problem.						
We should take example I1 of problem L1 and I2 of problem L2 , such that if I2 can be solved in polynomial time, then I1 can also be solved in polynomial time						
And, vice versa should also be true. The conversion from L1 to L2 can also be done in polynomial time.						
CDP problem (L2) -- want to show as NP Hard problem	Satisfiability problem (L1) -- known as NP Hard problem					
We have to prove that satisfiability reduces to CDP						
For this we have to take an example of satisfiability i.e. a conjunctive normal form formula	From that formula we should prepare a graph having some clique					
I will take a satisfiability example formula with three variables x1, x2 and x3						
$F = (x1 \vee x2) \wedge (\bar{x1} \vee \bar{x2} \vee x3)$						
There are basically three clauses C1, C2 and C3						
$F = \bigwedge_i C_i$ where i takes values from 1 to k						
Take the formula and make a graph out of it						
$G \vee = \{ \langle a, i \rangle \mid a \text{ belongs to } C_i \}$						
How to connect edges?						
Rule 1: Do not connect vertices from the same clause						
Rule 2: You cannot connect a clause with its negation i.e. x1, i cannot be connected to $\bar{x1}$ , i						
$E = \{ \langle \langle a, i \rangle, \langle b, j \rangle \rangle \mid i \text{ not equals to } j \text{ and } b \text{ not equals to } \bar{a} \}$						
We note that there is a clique of 2 vertices as well as 3 vertices.						
Hence, it is proved that if you take a satisfiability formula with k distinct clauses, then a k-clique sized graph is formed using it.						
Now, make the vertices of the clique as true	x1    x2    x3					
Now, C1, C2 and C3 all three are 1 and thus the formula F is also 1 i.e. true	0      1      1					
this proves that a satisfiability formula results in a clique decision problem, thus as we know a satisfiability formula / problem is an NP Hard problem, hence clique decision making is also an NP Hard problem						

	To further prove it NP Complete, we need to write a non-deterministic polynomial time algorithm						
<b>83</b>	<b>Knuth - Morris - Pratt (KMP) algorithm</b>						
	Used for pattern matching						
<b>String</b>	a b c d e f g h						
<b>Pattern</b>	d e f						
	Several such algorithms exist, we need to see a basic algorithm Naive and find its drawback and see how KMP algorithm handles those drawbacks						
	Write the pattern directly below the string letters. Compare the pattern with the first letters. If it does not match, then shift the entire pattern by one letter						
	If there is a match, then no need to shift, check the next letter in both pattern and string and do the same for all the letters in the pattern						
	Now, we note that in case of mismatch, j shifts back to index 0 and i shifts back to index 1; this is a waste of time as i had already moved ahead						
	We want to avoid this backtracking / shifting of i in basic Naive algorithm						
	Now, let us look at the worst drawback of basic Naive algorithm						
	suppose we have a string: a a a a a a b and a pattern: a a a b						
	here, a a a is repeating , we are checking again and again, moving back and checking it						
	this basic Naive algorithm is not checking the pattern and just blindly comparing. thus, time complexity is $O(mn)$						
	<b>Terminology of KMP algorithm:</b> here, <b>pattern: abcdabc</b>						
<b>prefix</b>	a, ab, abc, abcd						
<b>suffix</b>	c, bc, abc, dabc						
	Now, compare and see if there is a prefix matching with a suffix						
	abc is one such prefix and suffix						
	Basically, we wish to check beginning part of the pattern is again appearing anywhere else in the pattern or not						
	In KMP algorithm ,for a pattern , we generate a Pi table (that is for longest prefix)						
	that pi is also called as longest prefix which is same as some suffix	LPS is also another name					