S.No.			
1	**Algorithm**	**Program**	
	Design time	Implementation time	
	Domain knowledge	Programmer	
	Any language even English and Maths	Programming language	
	Hardware and software independent	Hardware and operating system dependent	
	Analyze an algorithm	Testing of programs	
2	**Priori Analysis**	**Posterior Testing**	
	Algorithm	Program	
	Independent of language	Language dependent	
	Hardware independent	Hardware dependent	
	Time and space function	watch time and bytes	
3	**Characteristics of algorithm**		
	Zero or more inputs		
	Must generate atleast one output		
	Definiteness		
	Finiteness		
	Effectiveness		
4	**How to analyze an algorithm**		
	Time		
	Space		
	Network consumtion : Data transfer amount		
	Power consumption		
	CPU registers		
5	**Frequency Count Method**	Used for time snalysis of an algorithm	
	Assign 1 unit of time for each statement		
	For any repitition, calculate the frequency of repetition		

	for(i = 0; i < n; i++) --> condition is checked for n+1 times	2n + 2 units of time ~ n+1 as we see condition i < n only for now	
	any statement within the loop will execute for n times		
	Space complexity depends upon number and kind of variables used		
6	**Algorithm : sum(A, n)**		
	Single for loop -		
	Time complexity: O(N)		
	Space complexity: O(N)		
7	**Algorithm : Add(A, B, n)**	Sum of two square matrices of dimesions nXn	
	Two nested for loops -		
	Time complexity: O(N^2)		
	Outer for loop executes for N+1 times		
	Inner for loop xecutes for N *(N+1) times		
	Any statement within inner for loop executes for (N + 1) * (N + 1) times		
	Space complexity: O(N^2)		
8	**Algorithm : Multiply(A, B, n)**		
	Three nested for loops -		
	Time complexity: O(N^3)		
	Space complexity: O(N^2)		
9	**Different algorithm conditions**		
	For loops		
	for(i = n; i > 0; i--)	n+1 times	
	for(i = 0; i < n; i = i + 2)	n/2 times	

	2 nested for loops where both i and j range from 0 to n	n^2 times	
	2 nested for loops where j ranges from 0 to i	when i = 0; j loop repeats 0 times; when i = 1; j loop repeates 1 times; and so on...total number of repetitions: 0 + 1+ 2+3+4....+n = O(n^2)	
	p = 0; for(i = 1; p<= n; i++){ p = p + i; }	p = k(k+1)/2 --> assuming that the loop exits when p is greater than n --> k(k+1) / 2 > n	~ k^2 > n --> O(root(n))
	for(i = 1; i < n; i = i *2)	will execute for 2^k times	O(logn)
		Assume i >= n ; i = 2^k >= n	
		k = logn with base 2	
	for(i = n; i >= 1; i = i/2)	i	
		n	
		n/2	
		n/2^2	
		n/2^3	
		.....	
		n/2^k	
		Assume i < 1 => n / 2^k < 1	~ O(logn) with base 2
	for(i = 0; i * i < n; i++)	i*i < n	
		i*i > -n	
		i^2 = n --> i = root(n)	~O(root(n))
	for(i = 0; i < n; i++) {.....}for(j = 0; j < n ; j++){....}	O(n)	
	p = 0; for(i = 1; i < n; i*2){.....} for(j = 1; j < p; j*2){.....}	log n times for upper loop; lop p times for lower loop	~ O(log(logn))
	for(i = 0; i < n; i++) {.....for(j = 0; j < n ; j*2){....}}	Outer loop repeats n times; inner loop repeats logn times	~O(nlogn)
	for(i = 1; i < n; i = i*3)		~O(logn) with base 3
	While loops		
	while vs. do while	do while will execute for minimum one time	
	for and while are almost similar	do while will execute as long as the condition is true; for loop will execute until the condition is false	

	a = 1;		
	while(a < b){ ..... a = a *2;}	1, 2, 2^2, 2^3......2^k repetitions	~O(logb) with base 2
		assume a > b; 2^k > b ==> k = logb with base 2	
	i = n; while(i > 1) {....i = i/2;}		~O(logn) with base 2
	i = 1; k = 1; while(k <n){....k = k + i; i++;}		
	i	k	
	1	1	
	2	1 + 1	
	3	2 + 2	
	4	2 + 2 + 3	
	5	2 + 2 + 3 + 4	
	.....		
	m	m(m + 1) /2	
	Assume, k >= n	m(m + 1)/ 2 >= n	~O(root(n))
	while(m != n) { if(m > n) m = m - n; else n = n - m;}		~O(n)
10	**Types of time functions**		
	O(1) --- constant		
	O(logn) --- logarithmic		
	O(n) --- linear		
	O(n^2) --- quadratic		
	O(n^3) --- cubic		
	O(2^n) --- exponential		
11	**Order of complexity**		
	1 < logn < root(n) < n < nlogn < n^2 < n^3 <.....< 2^n < 3^n .....< n^n		

12	**Asymptotic Notations**		
	Representation of time omplexity in simple form which is understandable		
	Big O Notation - works as an upper bound	The function f(n) = O(g(n)) iff for all positive constants c and n_0, such that f(n) <= c * g (n) for all n >= n_0; here, f(n) = O(n)	e.g. 2n + 3 <= 10n; All those functions in time order complexity above n become upper bound; below n become lower bound and n is the average bound
	Big Omega Notation - works as a lower bound	The function f(n) = Omega(g(n)) iff for all positive constants c and n_0, such that f(n) >= c * g(n) for all n >= n_0; here, f(n) = Omega(n)	e.g. 2n + 3 >= 1n
	Theta Notation - works as an average bound	The function f(n) = theta(g(n)) iff for all positive constants c1, c2 and n0 such that c1 * g(n) <= f(n) <= c2 * g(n)	e.g. f(n) 2n + 3; 1n <= 2n + 3 <= 5n
	Most useful is theta notation, then why do we need the other two?	In case we are not able to get the average bound, then we point to its upper or lower bound	
13	**Examples for asymptotic notations**		
a	**f(n) = 2n^2 + 3n + 4**		
	2n^2 + 3n + 4 <= 2n^2 + 3n^2 + 4n^2 i.e. 9n^2	O(n^2)	
	2n^2 + 3n + 4 >= 1n^2	Omega(n^2)	
	1n^2 <= 2n^2 + 3n + 4 <= 9n^2	Theta(n^2)	
b	**f(n) = n^2logn + n**		
	n^2logn <= n^2logn + n <= 10n^2logn	O(n^2logn)	
		Omega(n^2logn)	
		Theta(n^2logn)	
c	**f(n) = n!**		
	1 <= 1*2*3*4....*n-1*n <= n*n*n*n*......*n	O(n^n)	
		Omega(1)	
		Cannot find theta for n!	

d	**f(n) = logn!**		
	1 <= log(1*2*3.....*n) <= log(n*n*n*n....*n)	O(logn^n)	
		Omega(1)	
		Cannot find theta for logn!	
14	**Properties of Asymptotic notations**		
	General properties -		
	if f(n) is O(g(n)) then a*f(n) is O(g(n))		
	e.g. f(n) = 2n^2 + 5 is O(n^2), then 7f(n) i.e. 14n^2 + 35 is also O(n^2)	This would be true for both Omega and theta n as well	
	Reflexive property -		
	If f(n) is given then f(n) is O(f(n))		
	e.g. f(n) = n^2 then O(n^2)	A function is an upper bound of itself	
		Similarly, a function is a lower bound of itself	
	Transitive property -		
	If f(n) isO(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))		
	e.g. f(n) = n; g(n) = n^2 and h(n) = n^3	True for all notations	
	n is O(n^2) and n^2 is O(n^3) then n is O(n^3)		
	Symmetric property -		
	If f(n) is theta(g(n)) then g(n) is theta(f(n))	True for only theta(n)	
	e.g. f(n) = n^2 g(n) = n^2 ; f(n) = theta(n^2) and g(n) = theta (n^2)		
	Transpose symmetric -	True for BigO and Omega notations	
	if f(n) = O(g(n)) then g(n) is Omega(f(n))		
	e.g. f(n) = n and g(n) is n^2 then n is O(n^2) and n^2 is Omega(n)		

	If f(n) = O(g(n)) and f(n) = Omega(g(n)) then g(n) <= f(n) <= g (n) therefore f(n) = theta(g(n))		
	If f(n) = O(g(n)) and d(n) = O(e(n)) then f(n) + d(n) = O(max(g (n), e(n))		
	e.g. f(n) = n = O(n) , d(n) = n^2 = O(n^2) then f(n) + d(n) = n + n^2 = O(n^2)		
	If f(n) = O(g(n)) and d(n) = O(e(n)) then f(n) * d(n) = O(g(n) * e (n))		
15	**Comparison of functions**		
	First method is substituting values for n and comparing		
	Second method is applying log on both sides		
		Properties of log -	
	Example -	logab = loga + logb	
	f(n) n^2logn; g(n) = n(logn)^10	loga/b = loga - logb	
	Apply log	loga^b = bloga	
	log(n^2log(n)); log(n(logn)^10)	a^(log_cb) = b^(log_ca)	
	log(n^2) + loglogn; logn + loglog^10	a^b = n then b = log_an	
	2logn + loglogn ; logn + 10loglogn		
	here; 2logn is greater than logn and logn is a bigger term than loglogn		
	so, first term is greater than the second one		
	f(n) = 3n^(rootn) ; g(n) = 2^(rootn log_2(n))		
	Applying log		
	3n(rootn) ; (n^rootn)log_2(2)		
	3n(rootn) ; nrootn		
	first term is greater than the second one value wise but asymptatically they are equal		

	f(n) = n^(logn); g(n) = 2^(rootn)		
	apply log,		
	log(n^logn); log(2^rootn)		
	logn*logn ; rootn (log_2(2))		
	log^2n ; rootn		
	canot judge, so apply log again		
	2loglogn; 1/2logn		
	loglogn is smaller than logn		
	thus, second term is greater		
	f(n) = 2^(logn); g(n) = n^(rootn)		
	logn*log_2(2) ; rootn*logn		
	logn ; rootn*logn		
	second term is greater		
	f(n) = 2n; g(n) is 3n		
	both are equal asymptotically		
	f(n) = 2^n; g(n) = 2^(2n)		
	applying log		
	log(2^n); log(2^2n)		
	n; 2n	after applying log, do not cut coefficients	
	second function is greater		
16	**Best, worst and average case analysis**		
	Example -		
a	**Linear search**		
	A = {8, 6, 12, 5, 9, 7, 4, 3, 16, 18} key = 7		
	In linear search, it will start checking for the given key from left hand side		

	total in 6 comparisons, we would get our key		
	Best case - key element is present at first index		
	Best case time - 1 i.e. B(n) = O(1); Omega(1); Theta(1)		
	Worst case - key element is present at the last index		
	Worst case time - n i.e. W(n) = O(n); Omega(n); Theta(n)		
	Average case = all possible case time / no. of cases		
	average case analysis is very difficult for most of the cases		
	Here, average case time = 1 +2 + 3.... + n/2 = n(n+1)/2n = n+1/2		
	A(n) = n+1/2		
b	**Binary search tree**		
	height = logn		
	time taken for a particular key is logn		
	Best case - element present in the root		
	Best case time - k i.e. B(n) = O(1); Omega(!); Theta(1)		
	Worst case - searching for a leaf element - depends upon the height of the tree		
	Worst case time - logn i.e. O(logn)		
	min w(n) = logn; max w(n) = n		
17	**Disjoint sets**		
	No common numbers between two sets - intersection is zero		
	Operations - find, union		
	Find - search or check membership		
	Union - Add an edge		
	Krisgal algorithm: If you take an edge and both the vertices belong to the same set, then there is a cycle in the graph		
	Weighted union is used while adding edges and detectig cycle		

	Collapsing find - process of directly linking node to a direct parent of a set is called collapsing find - reduces the time to find		
18	**Divide and conquer - Strategy 1**		
	Strategy - an apprach for solving a problem		
	If a problem cannot be solved, divide it into sub-problems and find a solution for each sub problem, combine the solutions. *One point to note is that each sub problem should be similar to the original problem only.*		
	Recursive in nature		
	Should have one method to combine the solutions of each sub problem		
19	**Problems under Divide and Conquer**		
	Binary search		
	Finding maximum and minimum		
	MergeSort		
	QuickSort		
	Strassen's matrix multiplication		
20	**Recurrence relation 1: T(n) = T(n-1) + 1**		
	void test(int n)		
	{		
	if(n > 0){		
	printf("%d",n);		
	test(n-1)		
	}		
	}		
	test(3)		
	3. test(2)		

	2. test(1)		
	1 test(0)		
	each print statement takes constant time 1 and there are n+ 1 calls made to the function. we can ignore the last call when it is not printing		
	f(n) = n + 1 calls ; O(n)		
	T(n) = T(n-1) + 1; if we ignore if condition		
	Let us solve this relation;		
	if we know T(n-1) , we can get T(n)		
	T(n-1) = T(n-2) + 1		
	T(n) = [T(n-2) + 1] + 1		
	T(n) = T(n-3) + 3		
	....continue for k times		
	T(n) = T(n-k) + k		
	We would stop after k substitutions; now we need to find k		
	Assume n - k = 0; therefore n = k		
	T(n) = T(n-n) + n		
	T(n) = T(0) + n		
	T(n) = n + 1 i.e. theta(n)		
21	**Recurrence relation 2: T(n) = T(n-1) + n (decreasing function)**		
	void test(int n)	T(n)	
	{		
	if(n > 0)	1	
	{		
	for(i = 0; i <n; i++)	n+1	
	{		
	printf("%d", n);	n	
	}		
	test(n-1);	T(n-1)	

	}		
	}		
		T(n) = T(n-1) + 2n + 2 i.e. theta(n)	
	we can also write T(n) = T(n-1) + n for n > 0		
	T(n) = 1 for n = 0		
	T(n)	n time	
	n T(n-1)	n-1 time	
	n-1. T(n-2)	n - 2 time	
	n-2 T(n-3)	n - 3 time	
	....		
	T(2)		
	2 T(1)	2 units of ti me	
	1 T(0)	1 unit of time	
	for T(0) it does nothing	0 unit of time	
	time taken -		
		0 + 1 + 2 +.....+ n-1 + n	
	theta(n^2)	***T(n) = n(n + 1)/2***	
	T(n) = T(n-1) + n		
	T(n-1) = T(n-2) + n-1		
	thus, T(n) = T(n-2) + (n-1) + n	**remember, don't add the terms	
	T(n) = T(n-3) +(n-2) + (n-1) + n		
	T(n) = T(n-k) + (n-(k-1)) + (n-(k-2))....+(n-1) + n	if we continue for k times	
	assume n - k = 0; n = k		
	Thus, T(n) = T(n-n) + (n - n + 1) + (n - n + 2)..... +(n-1) + n		
	T(n) = T(0) + n(n+1)/2		
	T(n) = 1 + n(n+1)/2	***theta(n^2)*** ; this extra 1 is owing to the calls	
22	**Recurrence relation 3: T(n) = T(n-1) + logn**		
	void test(int n)	T(n)	

	{		
	if(n>0)		
	{		
	for(i = 1; i <n; i = i*2)		
	{		
	printf("%d", i);	log n times	
	}		
	test(n-1);	T(n-1)	
	}		
	}		
	T(n) = T(n-1) + logn for n > 0		
	T(n) = 1 for n = 0		
	Solve using tree method,		
	T(n)		
	logn T(n-1)		
	log(n-1) T(n-2)		
	log(n-2) T(n-3)		
	......		
	log2 T(!)		
	log 1 T(0)		
	logn + log(n-1) + ...+ log2 + log1		
	log[n(n-1)(n-2)....2.1] = log(n!)	there is no tight bound for this function but there is an upper bound for it	
	O(nlogn)		
	Solving using induction method.		
	T(n) = T(n-1) + logn		
	T(n) = T(n-2) + log(n-1) + log(n)		
	T(n) = T(n-3) + log(n-2) + log(n-1) + logn		

	.....		
	T(n) = T(n-k) + logn + log(n-1) +....log1		
	Asume n-k = 0		
	T(n) = T(0) + logn!		
	T(n) = 1 + logn!		
	O(nlogn)		
23	***How to get the direct answer for a recurrence relation?***		
	T(n) = T(n-1) + 1	O(n)	
	T(n) = T(n-1) + n	O(n^2)	
	T(n) = T(n-1) + logn	O(nlogn)	
	T(n) = T(n-1) + n^2	O(n^3)	
	T(n) = T(n-2) + 1	O(n/2) ~ O(n)	
	T(n) = T(n-100) + n	O(n^2)	
	T(n) = 2T(n-1) + 1	???	
24	**Recurrence relation 4: T(n) = 2T(n-1) + 1**		
	Test(int n)	T(n)	
	{		
	if(n > 0)	1	
	{		
	printf("%d", n);	1	
	test(n-1);	T(n-1)	
	test(n-1);	T(n-1)	
	}		
	}		
		T(n) = 2T(n-1) + 1	
	T(n) = 2T(n-1) + 1 for n> 0		
	T(n) = 1 for n = 0		

	Solve using recursion tree method		
	1 T(n-1) T(n-1)		2
	1 T(n-2) T(n-2)	1 T(n-2) T(n-2)	4
	1 T(n-3) T(n-3) 1 T(n-3) T(n-3)	1 T(n-3) T(n-3) 1 T(n-3) T(n-3)	8
	....		
	T(0). T(0)		2^k
	1 + 2 + 2^2 ......+2^k = 2^(k+1) - 1		
	as, a + ar + ar^2.....ar^k = a(r^(k+1) - 1)/(r -1)		
	Assume n - k = 0		
	thus, 2^(n+1) - 1	***O(2^n)***	
	Back substitution method		
	T(n) = 2T(n-1) + 1		
	T(n) = 4T(n-2) + 2 + 1		
	T(n) = 8T(n-3) + 4 + 2 + 1		
	.....		
	T(n) = 2^kT(n - k) + 2^(k-1) + 2^(k).....2^3 + 2^2 + 1		
	Assume n - k = 0		
	n = k		
	T(n) = 2^nT(0) + 1 + 2 + 2^2.... + 2^n-1		
	T(n) = 2^n + 2^n - 1 i.e. 2^(n+1) - 1		
25	**Master theorem for decreasing function**		
	T(n) = T(n-1) + 1	O(n)	
	T(n) = T(n-1) + n	O(n^2)	
	T(n) = T(n-1) + logn	O(nlogn)	
	T(n) = 2T(n-1) + 1	O(2^n)	
	T(n) = 3T(n-1) + 1	O(3^n)	

	T(n) = 2T(n-1) + n	O(n2^n)	
	T(n) = 2T(n-2) + 1	O(2^n/2)	
	T(n) = aT(n-b) + f(n)		
	a > 0, b > 0 and f(n) = O(n^k) where k >= 0		
	if a = 1, O(n^k+1) or O(n*f(n))		
	if a > 1, O(n^k * a^n/b)		
	if(a < 1) O(n^k) or O(f(n))		
26	**Dividing functions**		
	test(int n)	T(n)	
	{		
	if(n > 1)		
	{		
	printf("%d", n);	1	
	test(n/2)	T(n/2)	
	}		
	}		
	T(n) = T(n/2) + 1 for n > 1		
	T(n) = 1 for n = 1		
	T(n)		
	1 T(n/2)		
	1 T(n/2^2)		
	1 T(n/2^3)		
	......continue for k times		
	1 T(n/2^k)		

	assume , n/2^k = 1		
	thus, we have taken k steps overall		
	since, n/2^k = 1 => k = logn with base 2	***O(logn)***	
	Solving by substitution method		
	T(n) = T(n/2) + 1		
	T(n) = T(n/2^2) + 2		
	T(n) = T(n/2^3) + 3		
	....		
	T(n) = T(n/2^k) + k		
	assume n/2^k = 1		
	thus, k = logn with base 2		
	T(n) = T(1) + logn		
	O(logn)		
27	**Recurrence relation: T(n) = T(n/2) + n**		
	T(n) = T(n/2) + n for n > 1		
	T(n) = 1 for n=1		
	T(n)		
	T(n/2) n		
	T(n/2^2) n/2		
	T(n/2^3) n/2^2		
	......		
	T(n/2^k). n/2^(k-1)		
	T(n) = n + n/2 + n/2^2 + n/2^3..... + n/2^k		
	T(n) = n[1 + 1/2 + 1/2^2 + 1/2^3 +.....1/2^k]		
	T(n) = n*1 = n		
	O(n)		

	Using substitution method		
	T(n) = T(n/2) + n		
	......		
	T(n) = T(n/2^2) + n/2 + n		
	......		
	T(n) = T(n/2^3) + n/2^2 + n/2 + n		
	....		
	T(n) = T(n/2^k) + n/2^k-1 ....+n/2^2 + n/2 + n		
	Assume n /2^k = 1		
	k = logn with base 2		
	T(n) = T(1) + n[1/2^k-1.....+1/2^2....+1]		
	T(n) = 1 + 2n ~ O(n)		
28	**Recurrence Relation: T(n) = 2T(n/2) + n**		
	void test(int n)	T(n)	
	{		
	if(n > 1)		
	{		
	for(int i = 0; i <n ; i++)		
	{		
	stmt	n	
	}		
	test(n/2);	T(n/2)	
	test(n/2);	T(n/2)	
	T(n) = 2T(n/2) + n for n > 1		
	T(n) = 1 for n = 1		
	Solve using recursion tree method,		

	T(n)		
	T(n/2). T(n/2) n	n	
	T(n/2^2) T(n/2^2) T(n/2^2) T(n/2^2) n/2	n	
	T(n/2^3). T(n/2^3). T(n/2^3). T(n/2^3). T(n/2^3). T (n/2^3). T(n/2^3). T(n/2^3).	n	
	......	n	
	T(n/2^k).....		
		n	
	assume n / 2^k = 1		
	k = logn with base 2		
	T(n) = nk ~ O(nlogn)		
	Using backsubstitution method;		
	T(n) = 2T(n/2) + n		
	T(n/2) = 2T(n/2^2) + n/2		
	T(n) = 2[2T(n/2^2) + n/2] + n		
	T(n) = 2^2T(n/2^2) + n +n		
	T(n) = 2^3T(n/2^3) + 3n		
	....continue for k times		
	T(n) = 2^kT(n/2^k) + kn		
	Asume T(n/2^k) = T(1)		
	k = logn with base 2		
	Thus, T(n) = n +nlogn ~O(nlogn)		
29	**Masters Theorem for dividing functions**		
	T(n) = aT(n/b) + f(n)	loga with b	
	a>=1; b > 1; f(n) = theta(n^k* log^pn)	k	
	case 1: if loga with base b > k then theta(n^(loga with base b))		
	case 2: if loga with base b = k then		

	if p > -1 theta(n^klog^(p+1)n)		
	if p = -1 theta(n^kloglogn))		
	if p < -1 then theta(n^k)		
	case 3: if loga with base b < k		
	then, if p >= 0, theta(n^klog^pn)		
	if p < 0 , theta(n^k)		
	T(n) = 2T(n/2) + 1		
	a = 2		
	b = 2		
	f(n) = theta(n^(0) * log n ^ 0)		
	k = 0; p = 0		
	here, loga with base b > k		
	theta(n^1) where loga with base b is 1		
	T(n) = 4T(n/2) + n		
	log a with base b = 2		
	k = 1		
	p = 0		
	this is an example of case 1		
	theta(n^2)		
	T(n) = 8T(n/2) + n		
	log8 with base 2 = 3 > k = 1		
	theta(n^3)		
	T(n) = 9T(n/3) + 1		
	loga with base b = 2 > k		
	theta(n^2)		

	T(n) = 9T(n/3) + n^2		
	loga with base b = 2 = k	case 2	
	theta(n^2)		
	T(n) = 8T(n/2) + n		
	theta(n^3)		
	T(n) = 2T(n/2) + n		
	loga with base b = k = 1; p = 0		
	case 2		
	theta(nlogn)		
	T(n) = 4T(n/2) + n^2		
	theta(n^2logn)		
	T(n) = 4T(n/2) + n^2logn		
	theta(n^2logn^2)		
	T(n) = 8T(n/2) + n^3		
	theta(n^3logn)		
	T(n) = 2T(n/2) + n/logn		
	log a with base b = k = 1		
	p = -1		
	theta(nloglogn)		
	T(n) = 2T(n/2) + n/logn^2		
	p = -2		
	theta(n)		

	T(n) = 2T(n/2) + n^2		
	loga with base b < k		
	theta(n^2)		
	T(n) = 2T(n/2) + n^2		
	theta(n^2logn)		
	T(n) = 2T(n/2) + n^3		
	loga with base b < k		
	theta(n^3)		
30	T(n) = 2T(n/2) + 1		
	loga with base b = 1		
	k = 0		
	loga with base b > k		
	theta(n^1)		
	T(n) = 4T(n/2) + 1		
	loga with base b = 2		
	k = 0		
	theta(n^2)		
	T(n) = 4T(n/2) + n		
	loga with base b = 2		
	k = 1		
	theta(n^2)		
	T(n) = 8T(n/2) + n^2		
	loga with base b = 3		
	k = 2		

	theta(n^3)		
	T(n) = 16T(n/2) + n^2		
	loga with base b = 4		
	k = 2		
	theta(n^4)		
	T(n) = T(n/2) + n		
	log a with base b = 0		
	k = 1		
	theta(n)		
	T(n) = 2T(n/2) + n^2		
	loga with base b = 1		
	k = 2		
	theta(n^2)		
	T(n) = 2T(n/2) + n^2logn		
	loga with base b = 1		
	k = 2		
	theta(n^2logn)		
	T(n) = 4T(n/2) + n^3log^2n		
	loga with base b = 2		
	k = 3		
	theta(n^3log^2n)		
	T(n) = 2T(n/2) + n^2 / logn		
	log a with base b = 1		
	k = 2		

	theta(n^2)		
	T(n) = T(n/2) + 1		
	log a with base b = 0		
	k = 0		
	theta(logn)		
	T(n) = 2T(n/2) + n		
	log a with base b = 1		
	k = 1		
	p = 0		
	theta(nlogn)		
	T(n) = 2T(n/2) + nlogn		
	log a with base b = 1		
	k = 1		
	p = 1		
	theta(nlog^2n)		
	T(n) = 4T(n/2) + n^2		
	log a with base b = 2		
	k = 2; p = 0		
	theta(n^2logn)		
	T(n) = 4T(n/2) + (nlogn)^2		
	log a with base b = 2		
	k = 2, p = 2		
	theta(n^2 log^3n)		
	T(n) = 2T(n/2) + n/logn		

	log a with base b = 1		
	k = 1; p = -1		
	theta(nloglogn)		
	T(n) = 2T(n/2) + n/log^2n		
	log a with base b = 1		
	k = 1; p = -2		
	theta(n)		
31	**Root function Recurrence relation**		
	T(n) = T(root(n) + 1) for n>2		
	T(n) = 1 for n = 2		
	T(n) = T(root(n)) + 1		
	T(n) = T(n^(1/2)) + 1 ....equation 1		
	using substitution		
	T(n) = T(n^(1/2^2)) + 2.............equation 2		
	T(n) = T(n^(1/2^3)) +3...........equation 3		
	T(n) = T(n^(1/2^k)) + k.............equation 4		
	assume, n = 2^m		
	T(2^m) = T(2^(m/2^k)) + k		
	assume T(2^(m/2^k)) = T(2)		
	thus, m/2^k = 1		
	m = 2^k		
	k = log m with base 2		
	substituting value of n		
	m = logn with base 2		
	therefore, k = loglogn with base 2		
	theta(loglogn with base 2)		

32	**Binary Search Iterative Method**		
	To perform binary search, the prerequisite is that the list must be in sorted order	A = {3, 6, 8, 12, 14, 17, 25, 29, 31, 36, 42, 47, 53, 55, 62}	
	we need two index pointers, one is low at the starting point and the other is high at the end point	l = 1, h = 15 (lowest and highest index); mid = 8	
	mid = low + high / 2 and we take the floor value	key value = 42; A[mid] = 29 --> key > A [mid]	
	the key value is on the right hand side as key value is greater than A[mid]		
	we will change low to mid + 1	l = 9, h = 15; mid = 9 + 15 / 2 = 12	
		A[mid] = 47 > key	
	we will change high to mid - 1 as key < A[mid]		
		h = 11, l = 9, mid = 10; A[mid] = 36	
		A[mid] < key	
	we will change low to mid + 1	l = 11; h = 11; mid = 11; A[mid] = 42	
	we can return the index as we have found the key value	A[mid] = key	
	therefore, binary search looks faster than linear search. It just took 4 comparisons		
	int BinSearch(A, n, key)		
	{		
	l = 1, h = n		
	mid = l + h / 2 - take floor value		
	while(l <= h){		
	if(key == A[mid])		
	{ return index i.e.element is found}		
	else if(key < A[mid])		
	{h= mid-1;}		
	else {		
	l = mid + 1;}		
	}		

	return 0;		
	}		
	Time taken for binary search = logn		
	min time: O(1)		
	max time: O(logn)		
	avg time = add time for each element and divide by number of elements		
33	**Binarysearch Recursive method**		
	Alogirthm RBinarySearch(l,h,key)	T(n)	
	{		
	if(l==h)	1	
	{		
	if(A[low]== key)		
	{		
	return l;		
	}		
	else		
	{		
	return 0;		
	}		
	else		
	{		
	mid = l + h / 2 //taking floor value	1	
	if(key == A[mid])	1	
	{return mid;}		
	if(key < A[mid])	1	
	{		
	return RBinarySearch(l, mid - 1, key)	T(n/2)	
	}		

	else		
	{		
	return RBinarySearch(mid+1, h, key)	T(n/2)	
	}		
	}		
		T(n) = 1; n =1	
		T(n) = T(n/2) + 1 for n > 1	
		theta(logn)	
34	**Heaps**		
a	**Representation of a binary tree using an array**		
	T {A, B, C, D, E, F, G}		
	if a node is at index i;		
	its left child is at node 2*i		
	its right child is at node 2*i + 1		
	its parent is at node i/2		
	if there are missing nodes, we leave a blank in its place in the array		
b	**Full binary tree**		
	In its hieght, it has maximum number of nodes and if we wish to add a node, height would increase		
	Max no. of nodes = 2^h - 1		
c	**Complete binary tree**		
	there is no missing element from first element to the last element in array representation of the binary tree		
	Every full binary tree is also a complete binary tree		
	A complete binary tree is a full binary tree until height h - 1		
	Height of a complete binary tree would be minimum i.e. logn		
d	**Heap**		

	Heap is a complete binary tree		
	Max Heap: every node has value greater than all its descendants {50, 30, 20, 15, 10, 8, 16}		
	Min Heap: every node has value smaller or equal to than all its descendants {10, 30, 20, 35, 40, 32, 25}		
35	**Insert operation in a max heap**		
	Insert 60 in the above given max heap		
	this value should be inserted in the last free space in the array		
	i.e. left child of the left most leaf node		
	Then, adjust the elements to make it as a heap		
	So, compare and move 60 up the levels and in the array check at i/2 indices where initially i would be the last empty index where 60 was inserted		
	Time taken would be equal to the number of swaps		
	this depends upon the height of the tree i.e. logn, hence O (logn)		
	minimum time is of no swaps O(1); max would be O(logn)		
36	**Delete operation in a max heap**		
	From the heap, we need to remove the root / top most element only		
	The last element in the complete binary tree would come in its place		
	Adjust the elements to maintain heap order		
	From the root towards the leaf, adjust		
	Compare the children (2i and 2i +1) and whichever child is greater than compare with the parent		
	Time taken depends upon the height; max could be O(logn)		
	Whenever you delete from max heap, you get the next max element and in case of min heap, it would be the next min element		

37	**HeapSort**		
	For a given set of numbers, create a heap		
	Delete all the elements from the heap		
	Total N elements we have inserted; each element we assume is moved up to the root; so time taken O(NlogN)		
	Then we delete the elements		
	Store deleted elements in the array in free space in the end		
	Deletion also takes O(NlogN) time		
	Thus, heapsort takes O(NlogN)		
38	**Heapify**		
	The process of creating heap but direction is opposite than creating a heap		
	O(N)		
39	**Priority Queue**		
	elements will have priority and they would be inserted and deleted as per the priority order		
	For min heap, smaller the no. higher the priority		
	For max heap, greater the no. higher the priority		
	O(logN) for insertion and/or deletion		
40	**TwoWay MergeSort - Iterative method**	Algorithm Merge(A, B, m, n)	
	merging two sorted lists to get a sorted result	{i = 1, j = 1, k = 1;	
	A = {2, 8, 15, 18} i	while(i <= m && j<=n){	
	B = { 5, 9, 12, 17} j	if(A[i] < B[j])	
	Compare A(i) with B(j) to get C(k) and move to next location	{	
	m + n elements are obtained , thus theta(m + n)	C[k++] = A[i++];	
		}	
		else {	
		C[k++] = B[j++];	

		}	
		for(; i <=m; i++){	
		C[k++]=A[i];	
		}	
		for(;j<= n;j++){	
		C[k++] = B[j];	
		}	
		}	
41	**Merging more than two lists**		
	M-way merging		
	A = {4, 6, 12}		
	B = {3, 5, 9}		
	C = {8, 10, 16}		
	D = {2, 4, 18}		
	One way is that we merge A and B; C and D and then finally merge the two resulting lists --> so we perform merge three times here		
	Another way is that we first merge A and B; then we merge resulting list with C ; and the resulting list with D		
	Two-way mergesort is an iterative process whereas mergeSort is a recursive process		
	A = {9, 3, 7, 5, 6, 4, 8, 2} - given an array and we have to sort them using 2-way mergesort		
Ist pass	We would consider each element as a sorted list and merge	merged n elements in this pass	
	First select two lists 3 and 9; then merge them - 3, 9		
	Similarly, we select two lists 7 and 5 , merge them - 5 and 7		
	Another lists we get are {4, 6} and {2, 8}		
	Now, we have 4 lists with two elements each		
2nd pass	When we merged we kept the resulting 4 lists in another array B; B = {{3, 9}, {5, 7}, {4, 6}, {2, 8}}	merged n elements in this pass	

	We merge two lists each		
3rd pass	C = {{3, 5, 7, 9}, {2, 4, 6, 8}}	merged n elements in this pass	
	we merge the above two lists to get a single sorted list		
	D = {2, 3, 4, 5, 6, 7, 8, 9}		
	log(no of elements) = no. of passes		
	Time complexity: O(n(logn))		
42	**MergeSort**		
	A = {9, 3, 7, 5, 6, 4, 8, 2}	Algorithm MergeSort(l, h){	T(n)
	If there is a single element, we can consider it as a base or small problem {Divide and conquer}		
		if(l < h){	
		mid = (l + h) / 2;	1
		MergeSort(l, mid);	T(n/2)
		MergeSort(mid + 1, h);	T(n/2)
		Merge(l, mid, h);	n
		}	T(n) = 2T(n/2) + n for n > 1
		}	T(n) = 1 for n = 1
	time complexity: theta(nlogn)		using master's theorem, a = 2, b = 2, k = 1
	merging is done in post order traversal		loga with base b = 1 = k
			thus, it is case 2
			theta(nlogn)
43	**Pros of MergeSort**	**Cons of MergeSort**	
	works great for Large size lists	Extra space (not inplace sort)	
	suitable for Linked List	no small problem	
	supports external sorting	recursive and uses a stack (need n + logn space) i.e. space complexity: O(n + logn) where n is the extra space and logn is the stack space	
	stable: the order of duplicates is maintained		
		insertion sort (O(n^2))	

		mergesort O(nlogn)	
		for small problems, n <= 15; insertionsort works better --> use insertion sort	
43	**QuickSort**		
	students arranging themselves in increasing order of heights		
	10 80 90 60 30 20		
	5 6 3 4 2 1 **9**		
	4 6 7 **10** 16 12 13 14		
	A = {10, 16, 8, 12, 15, 6, 3, 9, 5, INFINITY}	partition(l, h){	
	select first element as a pivot	pivot = A[l];	
	pivot = 10	i = l; j = h;	
	we need to find the sorted position for 10	while(i<j){do	
	i starting form pivot and j starting from infinity	{	
	i would check for elements greater than 10; j would heck for elements smaller than pivot	i++;	
	we are using the partitioning procedure	} while(A[i]<= pivot);	
	increment i until next vlue is greater than 10 and decrement j until next value is smaller than pivot; stop and swap	do	
	{10, 5, 8, 9, 3, 6, 15, 12, 16}	{	
	send pivot element at j position	j--;	
	now, we can sort the two lists around the partitioning position by performing quicksort recursively	}while(A[j] > pivot);	
		if(i<j){	
	QuickSort(l, h)	swap(A[i], A[j]);	
	{	}	
	if(l < h)	swap(A[l], A[j]);	
	{	return j;	
	j = partition(l, h);	}	
	QuickSort(l, j);		
	QuickSort(j+ 1, h);		

	}		
	}		
44	**QuickSort Analysis**		
	suppose it is partitioning in the middle of 1 and 15th index		
	then, two partitions: [1, 7] ; [9, 15]		
	further partitions: [1, 3]; [5, 7]; [9, 11]; [13, 15]		
	at each level , n elements are being handled		
	and there are logn levels		
	thus time complexity for best case: O(nlogn)		
	median : middle element of a sorted list		
	best case of quicksort is that the partitioning occurs exactly at the middle		
	worstcase: if we have an already sorted list		
	time complexity for worstcase: O(n^2)		
	to handle this, try taking middle element as a pivot		
	2. select random element as a pivot		
45	**Strassen's matrix multiplication**		
	A = [a11 a12		
	a21. a22]		
	B = [b11 b12		
	b21 b22]		
	Cij = Summing up Aik*Bkj		
	for(i = 0; i < n ; i++){		
	for(j = 0; i < n ; j++){		
	C[i,j]= 0;		
	for(k=0;k<n;k++){		
	C[i,j] += A[i, k]*B[k, i];		

	}		
	}		
	}		
	C11 = a11*b11 + a12*b21		
	C21 = a11*b12 + a12*b22	A = [a11]	
	C21 = a21*b11 + a22*b21	B = [b11]	
	c22 = a21*b12 + a22*b22	C = [a11*b11]	
	for [2*2] matrix, we would use above formula	for [1*1] matrix, use above formula	
	we assume that the matrix has dimensions of power of 2	Algorithm MM(A, B, n)	
		{	
		if(n <= 2	
	8 times the function is calling itself	{	
	T(n) = 8T(n/2) + n^2 for n > 1	C = 4 formula stated above;	
	a = 8, b = 2, log a with base b = 3	}	
	k = 2	else	
	it is case 1 of master's theorem	{	
	theta(n^3)	mid = n/2	
		MM(A11, B11, n/2) + MM(A12, B21, n/2);	
		MM(A11, B12, n/2) + MM(A12, B22, n/2);	
		MM(A21, B11, n/2) + MM(A22, B21, n/2);	
		MM(A22, B22, n/2) + MM(A21, B12, n/2);	
		}	
		}	
	Strassen's approach -		
	has given 4 different formulas with 7 multiplications	P = (A11 + A22)(B11 + B22)	
	C11 = A11*B11 + A12*B21	Q = (A21 + A22) B11	
	C21 = A11*B12 + A12*B22	R = A11(B12 - B22)	
	C21 = A21*B11 + A22*B21	S = A22(B21 - B11)	

	C22 = A21*B12 + A22*B22	T = (A11 + A12)B22	
		U = (A21 - A11)(B11 + B12)	
		V = (A12 - A22)(B21 + B22)	
		C11 = P + S - T + V	
		C12 = R + T	
		C13 = Q + S	
		C22 = P + R- Q + U	
		T(n) = 7T(n/2) + n^2 for n > 2	
		T(n) = 1 for n<= 2	
		using master's theorm,	
		O(n^(log7 with base 2)) = O(n^2.81)	
	Strategies used for solving optimization problems - Greedy Method, Dymanic programming, branch and bound		
46	**Greedy method**		
	Design wich we can adopt to solve similar problems		Greedy method says that each problem should be solved in stages - each stage we give an input, check if the solution is feasible then we pick it up and move to next stage
	Solving optimization problems		Algorithm Greedy(a, n) a = {a1, a2, a3, a4, a5} ; n = 5
	Optimization problem: Problems which require either minimum or maximum result		{
	Suppose we have a problem P where we need to travel from source A to destination B, we can have several solutions such as walking on foot, travel by an airplane, ride on a bike, travel by a bus, drive on a car, go by a train and so on..... Now, we notice that we also have some constraints. The solutions that satisfy the conditions given in a problem are called **feasible solutions**	Minimum cost journey - "**Minimization problem**"; then feasible solutions giving minimum cost are called **optimal solutions**. ***There can be many feasible solutions but only one optimal solution***	for i = 1 to n do {x = select(a):
	Example: selecting a car to purchase	Example: Hire a person for your company	if feasible(x) then

	Method 1: Looking at all the models available in the city	Method 2: Conduct an assessment center to filter people at each stage and get the best person	{
	Method 2: Checking for the features of the cars and filtering and selecting based on your preferences - greedy method	So, the person may not be the best but the approach is greedy here as we are using our criteria and constraints to choose the best person	solution = solution + x;
47	**Knapsack problem**		}
	n = 7; m = 15	Bag capacity is 15 kgs and we have been given 7 objects. we have to fill this bag with these objects. Profit is the gain we get by transferring this object. Problem is a container loading problem. Problem is filling the container with the objects as the capacity of container is limited	}
Objects	{0 1 2 3 4 5 6 7 }	Optimization and maximization problem	}
Profits	{P 10 5. 15 7 6 18 3 }	Constraints : Bag weight limit	
Weights	{W 2 3 5 7 1 4 1 }		
Profit by weight	{P/W 5 1.3 3 1 6 4.5 3}		
0<=x<= 1	Objects are divisible i.e. we can take just half kg of object 1 and may be 2 kgs of object 2 and so on		
x	()		
	x1 x2 x3 x4 x5 x6 x7		
Method 1	Take the thing that have maximum profit		
Method 2	Take things with smaller weight so that you can put in more things		
Method 3	Take things that have highest profit by weight		
	Let's use method 3		

	First, I include object 5 that has maximum profit by weight. Then I check remaining weight I can put in. We can still put in 14 kgs. Then we select all the quantity of object 1. Remaining weight limit 12 kgs. Add all of object 6. Remaining weight limit 8 kgs. Add all of object 3. Remaining weight limit 3 kgs. Add all of object 7. Remaining weight limit is 2 kgs. Add 2/3 of object 2 as we have only 2 kgs limit remaining.		
x	(1 2/3 1 0 1 1 1)		
	Calculate total profit and verify weight		
	Total weight = 1*2 + 2/3*3 + 1*5 + 0*7 + 1*1 + 1*4 + 1*1 = 15	//Multiplying x elements by Weight w for each object	
	Total profits = 1*10 + 2/3*5 +1*15 + 1*6 + 1*18 + 1*3 = 54.6	//Multiplying x elements by Profit P for each object	
48	**0/1 Knapsack problem**		
	Objects are indivisble and fractions are not allowed i.e. either you include the whole thing or you do not include it at all		
49	**Job sequencing with deadlines**	n = 5 (tasks)	
Jobs	J1 J2 J3 J4 J5		
Profits	20 15 10 5 1		
Deadlines	2 2 1 3 3		
	Assume that there is a machine, on which each job has to be processed and each job takes 1 unit of time (hour) for completion		
	Set of the jobs which can be completed within their deadlines such that profit is maximized	Constraints: deadlines must be met	
deadlines	0---------1-----------2-----------3	maximum 3 slots / jobs	
time slots	9am------10am-----11am-----12am		
Jobs chosen	J2 J1 J4		
Profits	15 + 20 + 5 = 40		
Sequence	J1 --> J2 ---> J4 || J2 --> J1 --> J4		

Job consider	**Slot assign**	**Solution**	**Profit**
J1	[1,2]	J1	20
J2	[0,1][1,2]	J1J2	20 + 15
J3	[0,1][1,2]	J1J2	20 + 15
J4	[0,1][1,2][2,3]	J1J2J4	20 + 15 + 5
J5	[0,1][1,2][2,3]	J1J2J4	20 + 15 + 5
50	**Job sequencing with deadlines another example**	n = 7 (jobs)	
Jobs	J1 J2 J3 J4 J5 J6 J7		
Profits	35 30 25 20 15 12 5		
Deadlines	3 4 4 2 3 1 2		
deadlines	0..........1...........2.............3............4	4 SLOTS AVAILABLE	
Jobs chosen	J4 J3 J1 J2		
Profits	20 25 35 30	110	
51	**Optimal Merge Patern**		
	A = {3, 8, 12, 20}		
	B = {5, 9, 11, 16}		
	C = {3, 5, 8, 9, 11, 12, 16 20}	How merging works for two sorted lists ; time = theta(m + n)	
	what happens if we have 4 lists?		
List	A B C D		
Sizes	6 5 2 3		
Choice 1	We can merge at a time two lists - first A and B; the merge it with C and finally with D. total cost= 11 + 13 + 16 = 40		

Choice 2	Merge A anb; C and D that gives cost 11 and 5 respectively. Merge resulting two lists which will cost 16. Total cost would be 11 + 5 + 16 = 32		
Choice 3	Merge Cand D, resulting list is merged with B and then finally with A; total cost = 5 + 10 + 16 = 31		
optimal method	Always merge two small sized lists , then combined time would be reduced		
Example:			
List	x1 x2 x3 x4 x5		
Sizes	20 30 10 5 30		
Increasing order of sizes	5 10 20 30 30		
Lists	x4 x3 x1 x2 x5		
	First x4 and x3 are merged, cost = 15; then result is merged with x1; cost = 35; x2 and x5 are merged with cost = 60; the two resulting lists are merged with cost = 95		
Total cost	15 + 35 + 60 + 95 = 205		
	3*5 + 3*10 + 2*20 + 2*30 + 2*30 = 205	//multiplying distance of each node and size of each node	
52	**Huffman Coding**		
	Compression technique used to reduce the size of data or message		
Message	BCCABBDDAECCBBAEDDCC		
	Length = 20		
	it has to be sent using ASCII codes (8- bit)		
	A 65 01000001	Size = 8*20 = 160 bits	
	B 66 01000010		

	C 67		
	D 68		
	E 69		
	Can we use our own codes instead of ASCII codes?		
	Fixed size method		
Character	A B C D E		
Count	3 5 6 4 2	Total count = 20	
Code	000 001 010 011 100		
message	BCCABBDDAECCBBAEDDCC		
bit code	001010....	size = 20*3 = 60 bits	
		5*8 = 40 bits for ASCII code translations	
		5*3 = 15 bits --> our assigned codes	
		40 + 15 = 55 bits	
		message: 60 bits	
		chart: 55 bits	
		total message size: 115 bits	
		so, the message size reduced from 160 bits to 115 bits	
		thus, 40% reduction in size with fixed sized code	
	Huffman coding - variable sized code	element that appears more / often should have a smaller sized code	
character	A B C D E		
count	3 5 6 4 2		
code			
	first, arrange the letters with increasing count / frequency		
character	E A D B C		
count	2 3 4 5 6		

code	000 001 01 10 11	Merge two smaller ones, we get 5, then combine with D, we get 9. Combine B and C , we get 11. Finally, combine two resulting lists , we get 20.	
bit count	6 9 8 10 12	On left side paths, mark as 0 and on right side mark as 1	
total bits for message	45 bits	Bit count for message can also be obtained from the tree, by counting number of edges for a letter and multiplying by the number of occurences for that letter in the message i.e. summation of distance and frequency of a letter	
ASCII codes for chart	5*8 = 40 bits		
assigned codes	12 bits		
total bits for tree/table	52 bits		
Size of total msg	52 + 45 = **97 bits**		
Message transferred	001111101101111001011000111110100010100000110	A tree or a table would be needed along with it	
Decoding	BCCD...		
53	**Minimum Cost Spanning Tree**		
	G = (V, E)		
	V = {1, 2, 3, 4, 5, 6}	|V| = n = 6	
	E= {(1,2), (2,3), (3,4), (4,5), (5, 6), (6,1)}	|V| - 1 = 5 edges	
	the tree should not have a cycle		
	S is a subset of G, WHERE IN S = (V', E')	V' = V; |E'| = |V| - 1	
	Number of edges in graph = 6 out of whch I have to select 5 edges for spanning tree - thus i can select in **6C5 ways**	Suppose we have 7 edges, out of which the seventh edge (3,5) divides the graph into two cycles of less tha 6 vertices, then we can select 5 edges for spanning tree in **7C5 - 2 ways**	

General formula	|E|C(|V|-1) - no. of cycles		
	Now, if we have a weighted graph, I wish to know the number of possible spanning tree		
	Vertices = 4		
	Edges = 3		
	cost = 14		
	similarly , depending upon the edges we select, cost may vary each time		
	Can I found the minimum cost spanning tree?		
Method 1	Try all possible spanning trees and get the minimum cost spanning tree		
Method 2	Prim's algorithm (Greedy method)		
Method 3	Kriskal's algorithm (Greedy method)		
Method 2:	**Prim's algorithm**		
	Select the minimum cost edge from the graph first	(6,1) ; w = 10	
	Then, following this select minimum cost edge but make sure it is connected to previously chosen vertices	(5, 6); w = 25	
		(5, 4); w = 22	
		(4, 3); w = 12	
		(3, 2); w = 16	
		(2, 7); w = 14	
	Now, if we add costs of all the chosen edges, total cost = 99		
	For non connected graphs we cannot find the minimum cost spanning tree or spanning tree		
Method 3	**Kruskal's method**		
	Always select smallest cost edge		
	(1,6); w = 10		

	(3, 4); w = 12		
	(2, 7); w = 14		
	(2, 3); w = 16		
	(4, 5); w = 22		
	(5, 6); w = 25		
	total cost = 99		
	vertices count : |V|	To get a minimum cost edge each time, min heap can be used	
	edges count: |V| - 1	theta(nlogn)	
	theta(|V||E|)		
	theta(n.e) = theta(n^2)		
	for non-connected graphs, spanning tree cannot be found		
	Kruskal algo may give spanning tree for those non connected componr=ents but bot for the graph as a whole		
	if in a certain graph, certain edges' weights are missing, then use the given weights of remaining edges to guess the weight		
54	**Dijkstra algorithm**		
	Single source shortest path to all the vertices		
	find the shortest path to a vertex annd update it to other vertices. this updation is called relaxation		
	Relaxation		
	if(d[u] +c(u,v) < d[v]){ d[v] = d[u] + c(u,v)}		
	no of vertices = |V|		
	at most no. of vertices relaxing = |V|		
	worst case time of Dijkstra algorithm: theta(n*n)		

	Example - starting vertex is 1		
selected vertex	2 3 4 5 6		
4	50 45 **10** infinity infinity		
5	50 45 **10** **25** infinity		
2	45 45 **10** **25** infinity		
3	45 45 **10** **25** infinity		
6	45 45 **10** **25** infinity		
	Another example - starting vertex is 1		
selected vertex	{2, 3, 4}		
2	{3, infinity, 5}		
4	{3, infinity, 5}		
3	{3, 7, 5}		
	{3, 7, 5}		
	Another example - starting vertex is 1		
selected vertex	{2, 3, 4}		
2	{3, infinity, 5}		
4	{3, infinity, 5}		
3	{3, 7, 5}		
	{-3, 7, 5}		
	Dijkstra algorithm might work or might not work in case of en edge having negative weightage		
55	**Dynamic programming**		
	Dynamic programming vs greedy method		

	In Greedy method, we try to follow a predefined procedure that gives us the best / optimal result. The procedure is already known for optimization. But, in dynamic programming, we try to get all the solutions and then decide the best solution. Mostly dynamic programming questions are solved using recursive procedures. They follow a prnciple of optimality. In greedy method, decision is taken just once and followed through whereas in dynamic programming, decision is taken at each step		
	Example:		
	Fibonacci series		
	fib(n) = 0 if n = 0	T(n) = 2T(n-1) + 1{Approximating T(n-2) ~ T (n-1) here}	
	fib(n) = 1 if n = 1	Time taken would be O(2^n) by using Master's theorem	
	fib(n) = fib(n-2) + fib(n-1) if n > 1	Why can't we reduce the function calls to reduce the time taken?	
		For this, we would take a global array and initially fill it with -1	
	int fib(n) {	F = {-1,-1,-1,-1,-1}	
	if(n<= 1){	Then, as the function calls f(1), mark it as 1	
	return n;}	Then f(0) is marked as 1	
	return fib(n-2) + fib(n-1);	Then use the stored result to get the rest.	
	}	Finally, F would get updated as we solve: F = {0, 1, 1, 2, 3, 5}	
		Total 6 calls are made then i.e. n+1 calls i.e. O(n)	
		This is called result of memorization	
	From the above example, we can see reduction in number of calls from O(2^n) to O(n) using memorization. It follows top down approach. The same problem can be solved using tabular method (iterative process) as shown below:		
	int fib(int n) {		
	if(n <= 1) {		
	return n;}		
	F[0] = 0; F[1] = 1;		

	for(int i = 2; i <= n; i++){		
	F[i] = F[i-2] + F[i - 1];		
	}		
	return F[n];		
	}		
	F= {0, 1, 1, 2, 3, 5}		
	This is a bottom - up approach i.e. starting from F[0] and moving to F[n]		
56	**Multistage Graph**		
	A multistage graph is a directed weighted graph. The vertices are divided into stages such that the edges are connecting vertices from one stage to next stage only. First stage and last stage will have only one vertex to represent start and end point. This is usually used to represent resource allocation.		
	The objective of the problem is that I have to select a path which gives me minimum cost.	//it is a minimization or optimization problem	
	Dymanic programming works on principle of optimality. Principle of optimality says that a problem can be solved in a squence of decisions.		
	From first stage I have to select one optimal vertex that leads to minimum cost and I have to take this decision at each stage. Thus, I can apply dynamic programming here.		
V	1 2 3 4 5 6 7 8 9 10 11 12		
Cost	16 7 9 18 15 7 5 7 4 2 5 0	cost(5, 12) = 0 ; here 5 is the stage and 12 is the vertex	
d	2/3 7 6 8 8 10 10 10 12 12 12 12	cost(4, 9) = 4	
		cost(4, 10) = 2	
	Formula for multistage graph:	cost(4, 11) = 5	
	cost(lth stage, jth vertex no.) = cost(i, j) = min{C(j, l) + cost(i+1, l)}	cost(3, 6) = min{ C(6, 9) + cost(4, 9) , C(6, 10) + cost(4, 10)} = min{6 + 4, 5 + 2} = 7	
		Similarly, cost(3, 7) = min{8, 5} = 5	
	Now, we will solve it by going in forward direction and taking decisions based on above data;	cost(3, 8) = min{7, 11} = 7	

	d(1,1) = 2	Similarly, cost(2, 2) = min{C(2, 6) + cost(3, 6), C(2, 7) + cost(3, 7), C(2, 8) + cost(3, 8)} = min{11, 7, 8} = 7	
	d(2,2) = 7	cost(2, 3) = min{9, 12} = 9	
	d(3, 7) = 10	cost(2, 4) = min{18} = 18	
	d(4, 10) = 12	cost(2, 5) = min{16, 15} = 15	
	Path: 2---> 7---> 10----> 12	cost(1,1) = min{16, 16, 21, 17} = 16	
	d(1, 1) = 3		
	d(2, 3) = 6		
	d(3, 6) = 10		
	d(4, 10) = 12		
	Path: 3---> 6---> 10----> 12		
	So, we have two paths with same cost.		
57	**Multistage Graph (Program)**		
	Cost adjacency Matrix	main(){	
	0 1 2 3 4 5 6 7 8	int stages = 4, min;	
0	0 0 0 0 0 0 0 0 0	int n = 8;	
1	0 0 2 1 3 0 0 0 0	int cost[9], d[9], path[9];	
2	0 0 0 0 0 2 3 0 0	int c[9][9] = {{0,0,0,0,0,0,0,0,0}, {0,0,2,1,3,0,0,0,0}, {0,0,0,0,0,2,3,0,0}, ...}	
3	0 0 0 0 0 6 7 0 0	cost[n] = 0;	
4	0 0 0 0 0 6 8 9 0	for(int i = n-1; i >=1; i--){	
5	0 0 0 0 0 0 0 0 6	min = 32767;	
6	0 0 0 0 0 0 0 0 4	for(int k = i + 1; k <= n; k++){	
7	0 0 0 0 0 0 0 0 5	if(C[i][k] != 0 && C[i][k] +C[k] < min){	
8	0 0 0 0 0 0 0 0 0	min = C[i][k] +C[k] ;	
		d[i] = k;	
	0 1 2 3 4 5 6 7 8		
cost	-- 9 7 11 12 6 4 5 0	}	
d	-- 2 6 6 5 8 8 8 --	}	

path	-- 1 2 6 8	cost[i] = min;	
		}	
	Path is calculated using the following formula: p[i] d[p[i-1]] ; p [2] = d[p[2-1]] = d[1] = 2	p[1] = 1; p[stages] = n;	
	time complexity: O(n^2)	for(i = 2; i < stages; i++){	
		p[i] = p[d[i-1]];	
58	**All Pairs Shortest Path**		
	If I use Dijkstra algorithm on each vertex to find the shortest path of all, the time complexity would be O(n^3).		
A0 =	**1 2 3 4**		
1	0 3 INF 7		
2	8 0 2 INF		
3	5 INF 0 1		
4	2 INF INF 0		
	Considering vertex 1 as intermediate vertex	A0[2,3] A0[2,1] + A0[1,3]	
A1 =	**1 2 3 4**	2 < 8 + INF	
1	0 3 INF 7		
2	8 0 2 15	A0[2,4] A0[2,1] + A0[1,4]	
3	5 8 0 1	INF > 8 + 7 = 15	
4	2 8 INF 0		
		A0[3,2] A0[3,1] + A0[1,2]	
	Considering vertex 2 as intermediate vertex	INF > 5 + 3 = 8	
A2 =	**1 2 3 4**		
1	0 3 5 7	A[1,3] A[1,2] + A[2,3]	
2	8 0 2 15	INF < 3 + 2 = 5	
3	5 8 0 1		
4	2 5 7 0	for(k = 1; k <= n; k++){	
		for(i = 1; i <= n; i++){	
	Considering vertex 3 as intermediate matrix	for(j= 1; j <= n; j++){	

A3 =	**1 2 3 4**	A[i,j] = min(A[i,j], A[i,k] + A[k,j])	
1	0 3 5 6	}	
2	7 0 2 3	}	
3	5 8 0 1	}	
4	2 5 7 0		
	Considering vertex 4 as intermediate matrix		
A4 =	**1 2 3 4**		
1	0 3 5 6		
2	5 0 2 3		
3	3 6 0 1		
4	2 5 7 0		
	Formula used to make the above matrices:		
	A[i,j] = min{A[i,j], A[i,k] + A[k, j]}		
	time complexity = O(n^3)		
59	**Matrix chain multiplication**		
	A1 . A2 . A3 . A4		
	(5X4) (4X6) (6X2) (2X7)		
	For generating a single matrix C after single multiplication of 2 matrices of order (5X4) and (5X3) , we would need to do 60 multiplications		
	((A1.A2).A3).A4		
	or (A1.A2).(A3.A4)		
	or ....there could be several ways		
	then, how to choose theright way?		
	T(n) = 2nCn/n+1 trees are possible		

	thus, with 3 nodes ; T(3) = 5		
	Using tabular approach (bottom up apprach),		
m	**1 2 3 4**	in m[1,1] i.e. A1 , nothing is multiplied, hence it can be taken as zero	
1	0 120 88 158	m[1,2] = A1. A2	
2	- 0 48 104	(5X4) (4X6)	
3	- - 0 84	Total cost of multiplication = 5 *4* 6 = 120	
4	- - - 0		
		m[1,3] = A1.A2.A3	
s	**1 2 3 4**	Two possibilities: A1.(A2.A3) or (A1.A2).A3	
1	- 1 1 3	(5X4) (4X6) (6X2)	
2	- - 2 3	for A1.(A2.A3) --> m[1,1] + m[2,3] + (5*4*2)	for (A1.A2).A3 --> m[1,2] + m[3,3] + (5*6*2)
3	- - - 3	0 + 48 + 40	120 + 0 + 60
4	- - - -	88	180
		Similarly, m[2,4]	
	formula:	A2.(A3.A4) (A2.A3).A4	
	m[i,j] = m[i,k] + m[k+1, j] + di-1 * dk * dj	(4X6) (6X2)(2X7) (4X6) (6X2)(2X7)	
	we are generating n(n -1)/2 elements	for A2.(A3.A4) --> m[2,2] + m[3,4] + (4*6*7)	for (A2.A3).A4 --> m[2,3] + m[4,4] + (4*2*7)
	time complexity = O(n^3)	0 + 84 + 168	48 + 0 + 56
		252	104
		m[1,4]	
		min{m[1,1] + m[2,4] + (5*4*7), m[1,2] + m[3,4] + (5*6*7), m[1,3] + m[4,4] + (5*2*7)}	
		min{0+104+140, 120 + 84+210, 88+70}	
		min{244, 414, 158}	
60	**Matrix chain multiplication - A few pointers**		
	Condition of the multiplication: The number of columns in the first matrix involved in the multiplication must be equal to the number of rows in the second matrix		
A=	a11 a12 a13	2X3 dimension	

	a21 a22 a23		
B=	b11 b12	3X2 dimension	
	b21 b22		
	b31 b32		
A*B =	a11*b11 + a12*b21 + a13*b31 a11*b12 + a12*b22 + a13*b32	12 multiplications (2*3*2)	
	a21*b11 + a22*b21 + a23*b31 a21*b12 + a22*b22 + a31*b32	2X2 dimensions of the resultant matrix	
	A1 X A2 X A3 {Multiplication of more than two matrices}		
	2X3 3X4 4X2		
	d0 d1. d1 d2 d2 d3		
	Same answer by the two following methods (Associative property)		
	Method 1: (A1 X A2) X A3	**Method 2:** A1 X (A2 X A3)	
	2X3 3X4 i.e (2*3*4) = 24 multiplications for A1 X A2	2X3 3X4 4X2	
	Now, (A1 X A2) X A3 would require (2*4*2) = 16 multiplications	A2 X A3 requires (3*4*2) = 24 multiplications	
	Thus, altogether 40 multiplications are required	A1 X (A2 X A3) requires (2*3*2) = 12 multiplications	
		Altogether, 36 multiplications are needed here.	
	Now, dynamic programming asks us to find all the possible methods for matrix multiplication and check which one costs the minimum --> thsi implies that for 10 matrices, there would be numerous methods and we would have to check for all before proceeding with any one of them. Thus, we need a formula to check all that		
	We need to find C[1,3]		
	Method 1: (A1 X A2) X A3	**Method 2:** A1 X (A2 X A3)	
	C[1,2] = 24; C[3,3] = 0	C[1,1] = 0; C[2,3] = 24	
	C[1,2] + C[3,3] + d0*d2*d3 = 40	C[1,1] + C[2,3] + d0*d1*d3 = 36	
	C[i,j] = C[i, k] + C[k+1, j] + di-1 * dk * dj		
	After generalization,		

	C[i,j] = min { C[i,k] + C[k+1,j] + di-1 * dk * dj}		
	where i<=k<j		
	A1 X A2 X A3 X A4		
	d0 d1 d1 d2 d2 d3 d3 d4		
	Check which method works the best for the above matrix chain multiplication	2n C n / n + 1 multiplications are possible where n = no. of matrices - 1	
	1. A1 (A2 (A3A4))	Modified formula: 2(n-1) C (n-1) / n	
	2. A1 ((A2A3)A4)	Now, for n = 4, 2*3C3 / 4 = 6*5*4/3*2*1 / 4 = 5	
	3. (A1A2)(A3A4)	n = 5, 14 multiplications	
	4. (A1(A2A3))A4		
	5. ((A1A2)A3)A4		
	Applying the formula:		
4-1 = 3 values	C[1,4] = min { k = 1; C[1,1] + C[2,4] + d0*d1*d4,		
	k = 2; C[1,2] + C[3,4] + d0*d2*d4,		
	k = 3; C[1,3] + C[4,4] + d0*d3*d4}		
	1<= k < 4		
	1. A1 (A2A3A4)		
	2. (A1 A2) (A3 A4)		
	3. (A1A2A3) A4		
	here, C[1,1] = 0; C[4,4] = 0		
4-2 = 2 values	C[2,4] = min{k = 2; C[2,2] + C[3,4] + d1*d2*d4		
	k = 3; C[2,3] + C[4,4] + d1*d3*d4}		
	2<=k<4		

4-3 = 1 value	C[3,4] = C[3,3] + C[4,4] + d2*d3*d4		
	A1 X A2 X A3 X A4		
	3X2 2X4 4X2 2X5		
	d0 d1 d1 d2 d2 d3 d3 d4		
	As we notice that there is a repetition of the values needed , such as C[3,4] or C[4,4], there is unneccessary calculation, we are repeating, thus we should use a table (4X4)		
C table	**1** **2** **3** **4**	C[1,2] = min{k=1; C[1,1] + C[2,2] + d0*d1*d2	
1	0 24 28 58	Thus, C[1,2] = 3*2*4 = 24	
2	- 0 16 36	C[2,3] = min{k = 2; C[2,2] + C[3,3] + d1*d2*d3	
3	- - 0 40	Thus, C[2,3] = 2*4*2 = 16	
4	- - - 0	C[3,4] = d2*d3*d4 = 4*2*5 = 40	
		C[1,3] = min{k=1; C[1,1] + C[2,3] + d0*d1*d3	
k table	**1** **2** **3** **4**	k = 2; C[1,2] + C[3,3] + d0*d2*d3}	
1	- 1 1 3	C[1,3] = min{16 + 3*2*2, 24 + 3*4*2}	
2	- - 2 3	C[1,3] = min{28, 48] = 28	
3	- - - 3		
4	- - - -	C[2,4] = min{C[2,2] + C[3,4] + d1*d2*d4,	
		C[2,3] + C[4,4] + d1*d3*d4}	
	The result shows that we need to do minimum of 58 multiplications to get the result of A1 X A2 X A3 X A4	C[2,4] = min{40 + 2*4*5 , 16 + 2*2*5}	
	The k table will give the paranthesization	C[2,4] = min{80, 36} = 36	
	((A1) (A2 A3))(A4)		
	How much time it has taken ? 1+2+3+4 = 4(5) /2 i.e. n(n+1)/2 ~ n^2	C[1,4] = min{k = 1; C[1,1] + C[2, 4] + d0*d1*d4,	
	we also tried n posisble values of k to compute this value, thus time taken is n^2 * n i.e. O(n^3)	k = 2; C[1,2] + C[3,4] + d0*d2*d4,	
		k=3; C[1,3] + C[4,4] + d0*d3*d4}	
		C[1,4] = min{36 + 3*2*5, 24 + 40 + 3*4*5, 28 + 3*2*5}	

		C[1,4] = min{66, 124, 58} = 58	
61	**Matrix chain multiplication Program**		
	A1 X A2 X A3 XA4	main{	
	5X4 4X6 6X2 2X7	int n = 5;	
		int P[] = {5, 4, 6, 2, 7};	
	P = { 5,4,6,2,7}	int m[5][5] = {0};	
		int s[5][5] = {0};	
		int j, min, q;	
		for(int d = 1; d < n -1; d++)	
		{	
		for(int i = 1; i < n - d; i++)	
		{	
		j = i + d;	
		min = 32767;	
		for(int k = 1; k < = j - 1; k++)	
		{	
		q = m[i][k] + m[k + 1][j] + P[i - 1] * P[k] * P[j];	
		if(q < min)	
		{	
		min = q;	
		s[i][j] = k;	
		}	
		}	
		m[i][j] = min;	
		}	
		}	
		count << m[1][n -1];	
		}	

62	**Single source shortest path (Bellman Ford Algorithm)**	Example of Bellman Ford algorithm:	
	For doing this, we already have Dijkstra algorithm but it may not eork correctly if we have negative weights, thus we need some other method that works with negative weights	edges --> (3,2)(4,3)(1,4)(1,2)	
	Bellman Ford algorithm says that we should relax the edges N-1 times where the number of vertices is equal to N	mark source vertex 1 as 0 and rest all as infinity	
	|V| = N = 7	there are 4 vertices, so we should relax allt the edges for 3 times	
	So, we should relax them for |V| - 1 times	First iteration:	
	so, we would cover all possible paths even the longest path	for (3,2) --> infinity - 10 is infinity only, so no change	
	Relaxation means between a pair of vertices u and v if there is an edge, then check if:	for (4,3) --> infinity + 3 s infinity only, so no change	
	if(d[u] + C(u,v) < d[v]){	for (1,4), 0 +5 < infinity , thus vertex 4 is updated to 5	
	d[v] = d[u] + C(u,v)}	for (1,2) 0 + 4 < infinity, thus vertex 2 is updated to 4	
	edgeList --> (1,2)(1,3)(1,4)(2,5)(3,2)(3,5)(4,3)(4,6)(5,7)(6,7)	Second iteration:	
	Now, I have to relax these edges for |V| - 1 i.e. 6 times	for (3,2), vertex 2 is already 4, which is less than d[u] + C(u,v) in this case	
	Initially, mark the distance for source vertex as 0 and for the rest of the vertices as infinity	for (4,3) vertex 3 is updated to 5+3 = 8	
	Now, let's relax edge (1,2)	for (1,4)--> no change	
	here, d[u] = 0; d[v] = infinity; C(u,v) = 6	for(1,2) --> no change	
	0 + 6 < infinity ; thus d[v] = 6	Third iteration	
	for vertex 2, distance is 6;	for(3,2), 8 - 10 = -2 < d[v] which is 4 right now; thus updated for vertex 2 as -2	
	similarly, relaxing (1,3); thus its distance is updated to 5 from infinity	for the rest of the edges there won't be any change	
	in similar way, (1,4) is relaxed, following that the distance of vertex 4 is updated to 5 from infinity	results obtained :	
	Now, relaxing (2,5); d[u] = 6 ; C(u,v) = -1; d[v] = infinity	vertex 1 --> 0	
	the distance (2,5) is updated to 6 - 1 = 5	vertex 2 --> -2	
	similarly, relaxing (3,2) ; d[u] = 5, C(u,v) = -2, d[v] = 6	vertex 3 --> 8	

	5 - 2 = 3 < 6 thus, d[v] is updated here to 3 i.e. at vertex 2 , distance is updated to 3	vertex 4 --> 5	
	Now, relaxing (3,5), d[u] = 5 , C(u,v) = 1, d[v] = 5 here d[u] + C(u,v) is not smaller than d[v] hence the distance of vertex 5 is not modified	Now, if I relax one more time extra, there's no change	
	Moving to (4,3), relaxing it --> d[u] = 5, C(u,v) = -2; d[v] = 5, d [u] + C(u,v) = 3 < d[v] hence distance of vertex 3 is updated to 3	**Drawback of Bellman Ford algorithm:**	
	following this (4,6) is relaxed again and checked, d[u] = 5, C (u,v) = -1; thus distance of vertex 6 is updated to 5-1 = 4	let us an edge(2,4) in the above example	
	Now, relaxing (5,7), d[u] = 5; C(u,v) = 3; d[v] = infinity	we see even after N-1 iterations, there is one vertex changing, we note that there's a problem	
	thus, d[v] is updated to 8 for edge (5,7)	the reason is that there is a cycle of edges where total weight of edges is negative i.e. 5 + 3+ (-10) = -2 , thus graph cannot be solved	
	moving to (6,7), d[u] = 4, C(u,v) = 3; d[v] = 8	hence, for a negative weighted cycle , the bellman ford algorithm fails	
	d[v] is updated to 7 here for edge (6,7)		
	let us continue second time;		
	there won't be any change in (1,2), (1,3), (1,4)		
	when we check for (2,5); d[u] = 3 C(u,v) = -1 d[v] = 5; d[v] for edge (2,5) is updated to 2		
	similarly, for edge (3,2), the value is change to 3 - 2 = 1; earlier it was 3		
	(4,3) and (4,6), there's no change		
	for (5,7) d[u] has changed from 5 to 2; thus d[v] changes to 2 + 3 = 5		
	for (6,7) there won't be any change		
	let us continue third time;		
	there won't be any change in (1,2), (1,3), (1,4),		
	when we check for (2,5); d[u] = 1 C(u,v) = -1 d[v] = 2; d[v] for edge (2,5) is updated to 0		
	for (3,2) there own't be any change		
	for (3,5) again thee won't be any change		

	for (4,3), (4,6) --> no change		
	for (5,7) d[v] gets updated to 0 + 3 = 3		
	for (6,7) --> no change		
	let us check for fourth time,		
	we notice for all edges --> no change		
	results obtained:		
	vertex 1 --> 0		
	vertex 2 --> 1		
	vertex 3 --> 3		
	vertex 4 --> 5		
	vertex 5 --> 0		
	vertex 6 --> 4		
	vertex 7 --> 3		
	so, finally these are the shortest paths		
	time complexity: O(|E|(|V| - 1)) ~ O(|V||E|) ~ O(N^2)		
	If it is a complete graph, that is between every two vertex there is an edge, then number of edges is N(N - 1) / 2		
	i.e. |E| = N(N - 1) /2		
	then time complexity = O(|E||V|) O(N((N -1)/2)(N - 1)) ~ O (N^3)		
63	**0/1 Knapsack Problem**		