

<u>S.No.</u>							
	<b>1 Algorithm</b>	<b>Program</b>					
	Design time	Implementation time					
	Domain knowledge	Programmer					
	Any language even English and Maths	Programming language					
	Hardware and software independent	Hardware and operating system dependent					
	Analyze an algorithm	Testing of programs					
	<b>2 Priori Analysis</b>	<b>Posterior Testing</b>					
	Algorithm	Program					
	Independent of language	Language dependent					
	Hardware independent	Hardware dependent					
	Time and space function	watch time and bytes					
	<b>3 Characteristics of algorithm</b>						
	Zero or more inputs						
	Must generate atleast one output						
	Definiteness						
	Finiteness						
	Effectiveness						
	<b>4 How to analyze an algorithm</b>						
	Time						
	Space						
	Network consumption : Data transfer amount						
	Power consumption						
	CPU registers						
	<b>5 Frequency Count Method</b>	Used for time analysis of an algorithm					
	Assign 1 unit of time for each statement						
	For any repetition, calculate the frequency of repetition						
	for(i = 0; i < n; i++) --> condition is checked for n+1 times	2n + 2 units of time ~ n+1 as we see condition i < n only for now					
	any statement within the loop will execute for n times						

	Space complexity depends upon number and kind of variables used					
<b>6</b>	<b>Algorithm : sum(A, n)</b>					
	Single for loop -					
	Time complexity: $O(N)$					
	Space complexity: $O(N)$					
<b>7</b>	<b>Algorithm : Add(A, B, n)</b>	Sum of two square matrices of dimensions $n \times n$				
	Two nested for loops -					
	Time complexity: $O(N^2)$					
	Outer for loop executes for $N+1$ times					
	Inner for loop executes for $N \cdot (N+1)$ times					
	Any statement within inner for loop executes for $(N + 1) \cdot (N + 1)$ times					
	Space complexity: $O(N^2)$					
<b>8</b>	<b>Algorithm : Multiply(A, B, n)</b>					
	Three nested for loops -					
	Time complexity: $O(N^3)$					
	Space complexity: $O(N^2)$					
<b>9</b>	<b>Different algorithm conditions</b>					
	<b>For loops</b>					
	for( $i = n$ ; $i > 0$ ; $i--$ )	$n+1$ times				
	for( $i = 0$ ; $i < n$ ; $i = i + 2$ )	$n/2$ times				
	2 nested for loops where both $i$ and $j$ range from 0 to $n$	$n^2$ times				
	2 nested for loops where $j$ ranges from 0 to $i$	when $i = 0$ ; $j$ loop repeats 0 times; when $i = 1$ ; $j$ loop repeats 1 times; and so on...total number of repetitions: $0 + 1 + 2 + 3 + 4 + \dots + n = O(n^2)$				
	$p = 0$ ; for( $i = 1$ ; $p \leq n$ ; $i++$ ) { $p = p + i$ ; }	$p = k(k+1)/2 \rightarrow$ assuming that the loop exits when $p$ is greater than $n \rightarrow k(k+1) / 2 > n$	$\sim k^2 > n \rightarrow O(\sqrt{n})$			
	for( $i = 1$ ; $i < n$ ; $i = i * 2$ )	will execute for $2^k$ times	$O(\log n)$			
		Assume $i \geq n$ ; $i = 2^k \geq n$				
		$k = \log n$ with base 2				

for(i = n; i >= 1; i = i/2)	i					
	n					
	n/2					
	n/2^2					
	n/2^3					
	.....					
	n/2^k					
	Assume $i < 1 \Rightarrow n / 2^k < 1$		$\sim O(\log n)$ with base 2			
for(i = 0; i * i < n; i++)	$i^2 < n$					
	$i^2 > -n$					
	$i^2 = n \rightarrow i = \text{root}(n)$		$\sim O(\text{root}(n))$			
for(i = 0; i < n; i++) {.....}for(j = 0; j < n; j++){.....}	$O(n)$					
p = 0; for(i = 1; i < n; i*2){.....} for(j = 1; j < p; j*2){.....}	log n times for upper loop; log p times for lower loop		$\sim O(\log(\log n))$			
for(i = 0; i < n; i++) {.....for(j = 0; j < n; j*2){.....}}	Outer loop repeats n times; inner loop repeats logn times		$\sim O(n \log n)$			
for(i = 1; i < n; i = i*3)			$\sim O(\log n)$ with base 3			
<b>While loops</b>						
while vs. do while	do while will execute for minimum one time					
for and while are almost similar	do while will execute as long as the condition is true; for loop will execute until the condition is false					
a = 1;						
while(a < b){ ..... a = a *2;}	1, 2, 2^2, 2^3.....2^k repetitions		$\sim O(\log b)$ with base 2			
	assume $a > b$ ; $2^k > b \Rightarrow k = \log b$ with base 2					
i = n; while(i > 1) {....i = i/2;}			$\sim O(\log n)$ with base 2			
i = 1; k = 1; while(k < n){....k = k + i; i++;}						
	i k					
	1	1				
	2 1 + 1					
	3 2 + 2					
	4 2 + 2 + 3					

	5	$2 + 2 + 3 + 4$					
	.....						
	m	$m(m + 1) / 2$					
	Assume, $k \geq n$	$m(m + 1) / 2 \geq n$	$\sim O(\sqrt{n})$				
	while( $m \neq n$ ) { if( $m > n$ ) $m = m - n$ ; else $n = n - m$ ;		$\sim O(n)$				
	<b>10 Types of time functions</b>						
	$O(1)$ --- constant						
	$O(\log n)$ --- logarithmic						
	$O(n)$ --- linear						
	$O(n^2)$ --- quadratic						
	$O(n^3)$ --- cubic						
	$O(2^n)$ --- exponential						
	<b>11 Order of complexity</b>						
	$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \dots < 2^n < 3^n$ ..... $< n^n$						
	<b>12 Asymptotic Notations</b>						
	Representation of time complexity in simple form which is understandable						
	Big O Notation - works as an upper bound	The function $f(n) = O(g(n))$ iff for all positive constants $c$ and $n_0$ , such that $f(n) \leq c * g(n)$ for all $n \geq n_0$ ; here, $f(n) = O(n)$	e.g. $2n + 3 \leq 10n$ ; All those functions in time order complexity above $n$ become upper bound; below $n$ become lower bound and $n$ is the average bound				

	Big Omega Notation - works as a lower bound	The function $f(n) = \Omega(g(n))$ iff for all positive constants $c$ and $n_0$ , such that $f(n) \geq c \cdot g(n)$ for all $n \geq n_0$ ; here, $f(n) = \Omega(g(n))$	e.g. $2n + 3 \geq 1n$				
	Theta Notation - works as an average bound	The function $f(n) = \theta(g(n))$ iff for all positive constants $c_1$ , $c_2$ and $n_0$ such that $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$	e.g. $f(n) = 2n + 3$ ; $1n \leq 2n + 3 \leq 5n$				
	Most useful is theta notation, then why do we need the other two?	In case we are not able to get the average bound, then we point to its upper or lower bound					
	<b>13 Examples for asymptotic notations</b>						
<b>a</b>	<b><math>f(n) = 2n^2 + 3n + 4</math></b>						
	$2n^2 + 3n + 4 \leq 2n^2 + 3n^2 + 4n^2$ i.e. $9n^2$	$O(n^2)$					
	$2n^2 + 3n + 4 \geq 1n^2$	$\Omega(n^2)$					
	$1n^2 \leq 2n^2 + 3n + 4 \leq 9n^2$	$\Theta(n^2)$					
<b>b</b>	<b><math>f(n) = n^2 \log n + n</math></b>						
	$n^2 \log n \leq n^2 \log n + n \leq 10n^2 \log n$	$O(n^2 \log n)$					
		$\Omega(n^2 \log n)$					
		$\Theta(n^2 \log n)$					
<b>c</b>	<b><math>f(n) = n!</math></b>						
	$1 \leq 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n-1 \cdot n \leq n \cdot n \cdot n \cdot \dots \cdot n$	$O(n^n)$					
		$\Omega(1)$					
		Cannot find theta for $n!$					
<b>d</b>	<b><math>f(n) = \log n!</math></b>						
	$1 \leq \log(1 \cdot 2 \cdot 3 \cdot \dots \cdot n) \leq \log(n \cdot n \cdot n \cdot \dots \cdot n)$	$O(\log n^n)$					
		$\Omega(1)$					
		Cannot find theta for $\log n!$					
	<b>14 Properties of Asymptotic notations</b>						
	General properties -						
	if $f(n)$ is $O(g(n))$ then $a \cdot f(n)$ is $O(g(n))$						
	e.g. $f(n) = 2n^2 + 5$ is $O(n^2)$ , then $7f(n)$ i.e. $14n^2 + 35$ is also $O(n^2)$	This would be true for both $\Omega$ and $\theta$ as well					

Reflexive property -						
If $f(n)$ is given then $f(n)$ is $O(f(n))$						
e.g. $f(n) = n^2$ then $O(n^2)$	A function is an upper bound of itself					
	Similarly, a function is a lower bound of itself					
Transitive property -						
If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ then $f(n)$ is $O(h(n))$						
e.g. $f(n) = n$ ; $g(n) = n^2$ and $h(n) = n^3$	True for all notations					
$n$ is $O(n^2)$ and $n^2$ is $O(n^3)$ then $n$ is $O(n^3)$						
Symmetric property -						
If $f(n)$ is $\theta(g(n))$ then $g(n)$ is $\theta(f(n))$	True for only $\theta(n)$					
e.g. $f(n) = n^2$ $g(n) = n^2$ ; $f(n) = \theta(n^2)$ and $g(n) = \theta(n^2)$						
Transpose symmetric -	True for BigO and Omega notations					
if $f(n) = O(g(n))$ then $g(n)$ is $\Omega(f(n))$						
e.g. $f(n) = n$ and $g(n)$ is $n^2$ then $n$ is $O(n^2)$ and $n^2$ is $\Omega(n)$						
If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ then $g(n) \leq f(n) \leq g(n)$ therefore $f(n) = \theta(g(n))$						
If $f(n) = O(g(n))$ and $d(n) = O(e(n))$ then $f(n) + d(n) = O(\max(g(n), e(n)))$						
e.g. $f(n) = n = O(n)$ , $d(n) = n^2 = O(n^2)$ then $f(n) + d(n) = n + n^2 = O(n^2)$						
If $f(n) = O(g(n))$ and $d(n) = O(e(n))$ then $f(n) * d(n) = O(g(n) * e(n))$						
<b>15 Comparison of functions</b>						
First method is substituting values for $n$ and comparing						
Second method is applying log on both sides						
	Properties of log -					
Example -	$\log ab = \log a + \log b$					
<b><math>f(n) = n^2 \log n</math>; <math>g(n) = n(\log n)^{10}</math></b>	$\log a/b = \log a - \log b$					
Apply log	$\log a^b = b \log a$					

$\log(n^2 \log(n)); \log(n(\log n)^{10})$	$a^{\log_{cb}} = b^{\log_{ca}}$					
$\log(n^2) + \log \log n; \log n + \log \log^2 n$	$a^b = n$ then $b = \log_a n$					
$2 \log n + \log \log n; \log n + 10 \log \log n$						
here; $2 \log n$ is greater than $\log n$ and $\log n$ is a bigger term than $\log \log n$						
so, first term is greater than the second one						
<b><math>f(n) = 3n^{\sqrt{n}}; g(n) = 2^{\sqrt{n} \log_2(n)}</math></b>						
Applying log						
$3n^{\sqrt{n}}; (n^{\sqrt{n}}) \log_2(2)$						
$3n^{\sqrt{n}}; n^{\sqrt{n}}$						
first term is greater than the second one value wise but asymptotically they are equal						
<b><math>f(n) = n^{\log n}; g(n) = 2^{\sqrt{n}}</math></b>						
apply log,						
$\log(n^{\log n}); \log(2^{\sqrt{n}})$						
$\log n \cdot \log n; \sqrt{n} (\log_2(2))$						
$\log^2 n; \sqrt{n}$						
cannot judge, so apply log again						
$2 \log \log n; 1/2 \log n$						
$\log \log n$ is smaller than $\log n$						
thus, second term is greater						
<b><math>f(n) = 2^{\log n}; g(n) = n^{\sqrt{n}}</math></b>						
$\log n \cdot \log_2(2); \sqrt{n} \cdot \log n$						
$\log n; \sqrt{n} \cdot \log n$						
second term is greater						
<b><math>f(n) = 2n; g(n) \text{ is } 3n</math></b>						
both are equal asymptotically						
<b><math>f(n) = 2^n; g(n) = 2^{(2n)}</math></b>						
applying log						
$\log(2^n); \log(2^{2n})$						
$n; 2n$	after applying log, do not cut coefficients					

	second function is greater						
	<b>16 Best, worst and average case analysis</b>						
	Example -						
<b>a</b>	<b>Linear search</b>						
	A = {8, 6, 12, 5, 9, 7, 4, 3, 16, 18} key = 7						
	In linear search, it will start checking for the given key from left hand side						
	total in 6 comparisons, we would get our key						
	Best case - key element is present at first index						
	<b>Best case time - 1 i.e. <math>B(n) = O(1)</math>; <math>\Omega(1)</math>; <math>\Theta(1)</math></b>						
	Worst case - key element is present at the last index						
	<b>Worst case time - n i.e. <math>W(n) = O(n)</math>; <math>\Omega(n)</math>; <math>\Theta(n)</math></b>						
	Average case = all possible case time / no. of cases						
	average case analysis is very difficult for most of the cases						
	Here, average case time = $1 + 2 + 3 + \dots + n/2 = n(n+1)/2n = n+1/2$						
	<b><math>A(n) = n+1/2</math></b>						
<b>b</b>	<b>Binary search tree</b>						
	height = $\log n$						
	time taken for a particular key is $\log n$						
	Best case - element present in the root						
	<b>Best case time - k i.e. <math>B(n) = O(1)</math>; <math>\Omega(1)</math>; <math>\Theta(1)</math></b>						
	Worst case - searching for a leaf element - depends upon the height of the tree						
	<b>Worst case time - <math>\log n</math> i.e. <math>O(\log n)</math></b>						
	min $w(n) = \log n$ ; max $w(n) = n$						
	<b>17 Disjoint sets</b>						
	No common numbers between two sets - intersection is zero						
	Operations - find, union						
	Find - search or check membership						
	Union - Add an edge						
	<b><i>Krisgal algorithm: If you take an edge and both the vertices belong to the same set, then there is a cycle in the graph</i></b>						



	<b>Weighted union</b> is used while adding edges and detecting cycle						
	<b>Collapsing find</b> - process of directly linking node to a direct parent of a set is called collapsing find - reduces the time to find						
	<b>18 Divide and conquer - Strategy 1</b>						
	Strategy - an approach for solving a problem						
	If a problem cannot be solved, divide it into sub-problems and find a solution for each sub problem, combine the solutions. <i>One point to note is that each sub problem should be similar to the original problem only.</i>						
	Recursive in nature						
	Should have one method to combine the solutions of each sub problem						
	<b>19 Problems under Divide and Conquer</b>						
	Binary search						
	Finding maximum and minimum						
	MergeSort						
	QuickSort						
	Strassen's matrix multiplication						
	<b>20 Recurrence relation 1: <math>T(n) = T(n-1) + 1</math></b>						
	void test(int n)						
	{						
	if(n > 0){						
	printf("%d",n);						
	test(n-1)						
	}						
	}						
	test(3)						
	3. test(2)						
	2. test(1)						
	1. test(0)						

	each print statement takes constant time 1 and there are n+ 1 calls made to the function. we can ignore the last call when it is not printing					
	f(n) = n + 1 calls ; O(n)					
	T(n) = T(n-1) + 1; if we ignore if condition					
	Let us solve this relation;					
	if we know T(n-1) , we can get T(n)					
	T(n-1) = T(n-2) + 1					
	T(n) = [T(n-2) + 1] + 1					
	T(n) = T(n-3) + 3					
	....continue for k times					
	T(n) = T(n-k) + k					
	We would stop after k substitutions; now we need to find k					
	Assume n - k = 0; therefore n = k					
	T(n) = T(n-n) + n					
	T(n) = T(0) + n					
	<b>T(n) = n + 1 i.e. theta(n)</b>					
21	<b>Recurrence relation 2: T(n) = T(n-1) + n (decreasing function)</b>					
	void test(int n)	T(n)				
	{					
	if(n > 0)		1			
	{					
	for(i = 0; i < n; i++)	n+1				
	{					
	printf("%d", n);	n				
	}					
	test(n-1);	T(n-1)				
	}					
	}					
		T(n) = T(n-1) + 2n + 2 i.e. theta(n)				
	we can also write T(n) = T(n-1) + n for n > 0					
	T(n) = 1 for n = 0					
	T(n)	n time				
	n      T(n-1)	n-1 time				
	n-1.    T(n-2)	n - 2 time				

n-2	T(n-3)	n - 3 time					
.....							
T(2)							
2	T(1)	2 units of time					
1	T(0)	1 unit of time					
for T(0) it does nothing		0 unit of time					
time taken -							
		0 + 1 + 2 + ..... + n-1 + n					
<b><math>\theta(n^2)</math></b>		<b><math>T(n) = n(n+1)/2</math></b>					
$T(n) = T(n-1) + n$							
$T(n-1) = T(n-2) + n-1$							
thus, $T(n) = T(n-2) + (n-1) + n$		**remember, don't add the terms					
$T(n) = T(n-3) + (n-2) + (n-1) + n$							
$T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)) \dots + (n-1) + n$		if we continue for k times					
assume $n - k = 0$ ; $n = k$							
Thus, $T(n) = T(n-n) + (n - n + 1) + (n - n + 2) \dots + (n-1) + n$							
$T(n) = T(0) + n(n+1)/2$							
<b><math>T(n) = 1 + n(n+1)/2</math></b>		<b><math>\theta(n^2)</math></b> ; this extra 1 is owing to the calls					
<b>22 Recurrence relation 3: <math>T(n) = T(n-1) + \log n</math></b>							
void test(int n)		T(n)					
{							
if(n>0)							
{							
for(i = 1; i < n; i = i*2)							
{							
printf("%d", i);		log n times					
}							
test(n-1);		T(n-1)					
}							
}							
$T(n) = T(n-1) + \log n$ for $n > 0$							
$T(n) = 1$ for $n = 0$							
<b>Solve using tree method,</b>							

	$T(n)$					
	$\log n \quad T(n-1)$					
	$\log(n-1) \quad T(n-2)$					
	$\log(n-2) \quad T(n-3)$					
	.....					
	$\log 2 \quad T(1)$					
	$\log 1 \quad T(0)$					
	$\log n + \log(n-1) + \dots + \log 2 + \log 1$					
	<b><math>\log[n(n-1)(n-2)\dots 2.1] = \log(n!)</math></b>	there is no tight bound for this function but there is an upper bound for it				
	<b><math>O(n \log n)</math></b>					
	<b><i>Solving using induction method.</i></b>					
	$T(n) = T(n-1) + \log n$					
	$T(n) = T(n-2) + \log(n-1) + \log(n)$					
	$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$					
	.....					
	$T(n) = T(n-k) + \log n + \log(n-1) + \dots \log 1$					
	Asume $n-k = 0$					
	$T(n) = T(0) + \log n!$					
	<b><math>T(n) = 1 + \log n!</math></b>					
	<b><math>O(n \log n)</math></b>					
	<b>23 How to get the direct answer for a recurrence relation?</b>					
	$T(n) = T(n-1) + 1$	$O(n)$				
	$T(n) = T(n-1) + n$	$O(n^2)$				
	$T(n) = T(n-1) + \log n$	$O(n \log n)$				
	$T(n) = T(n-1) + n^2$	$O(n^3)$				
	$T(n) = T(n-2) + 1$	$O(n/2) \sim O(n)$				
	$T(n) = T(n-100) + n$	$O(n^2)$				
	$T(n) = 2T(n-1) + 1$	???				
	<b>24 Recurrence relation 4: <math>T(n) = 2T(n-1) + 1</math></b>					
	Test(int n)	$T(n)$				
	{					

if(n > 0)		1				
{						
printf("%d", n);		1				
test(n-1);	T(n-1)					
test(n-1);	T(n-1)					
}						
}						
	T(n) = 2T(n-1) + 1					
T(n) = 2T(n-1) + 1 for n > 0						
T(n) = 1 for n = 0						
<b>Solve using recursion tree method</b>						
1 T(n-1) T(n-1)			2			
1 T(n-2) T(n-2)	1 T(n-2) T(n-2)		4			
1 T(n-3) T(n-3) 1 T(n-3) T(n-3)	1 T(n-3) T(n-3) 1 T(n-3) T(n-3)		8			
.....						
T(0). T(0)		2 <sup>k</sup>				
1 + 2 + 2 <sup>2</sup> ..... + 2 <sup>k</sup> = 2 <sup>(k+1)</sup> - 1						
as, a + ar + ar <sup>2</sup> .....ar <sup>k</sup> = a(r <sup>(k+1)</sup> - 1)/(r - 1)						
Assume n - k = 0						
<b>thus, 2<sup>(n+1)</sup> - 1</b>	<b>O(2<sup>n</sup>)</b>					
<b>Back substitution method</b>						
T(n) = 2T(n-1) + 1						
T(n) = 4T(n-2) + 2 + 1						
T(n) = 8T(n-3) + 4 + 2 + 1						
.....						
T(n) = 2 <sup>k</sup> T(n - k) + 2 <sup>k</sup> (k-1) + 2 <sup>k</sup> (k).....2 <sup>3</sup> + 2 <sup>2</sup> + 1						
Assume n - k = 0						
n = k						
T(n) = 2 <sup>n</sup> T(0) + 1 + 2 + 2 <sup>2</sup> .... + 2 <sup>n-1</sup>						
<b>T(n) = 2<sup>n</sup> + 2<sup>n</sup> - 1 i.e. 2<sup>(n+1)</sup> - 1</b>						

<b>25 Master theorem for decreasing function</b>						
$T(n) = T(n-1) + 1$	$O(n)$					
$T(n) = T(n-1) + n$	$O(n^2)$					
$T(n) = T(n-1) + \log n$	$O(n \log n)$					
$T(n) = 2T(n-1) + 1$	$O(2^n)$					
$T(n) = 3T(n-1) + 1$	$O(3^n)$					
$T(n) = 2T(n-1) + n$	$O(n2^n)$					
$T(n) = 2T(n-2) + 1$	$O(2^{n/2})$					
<b><math>T(n) = aT(n-b) + f(n)</math></b>						
$a > 0, b > 0$ and $f(n) = O(n^k)$ where $k \geq 0$						
<b>if <math>a = 1, O(n^{k+1})</math> or <math>O(n \cdot f(n))</math></b>						
<b>if <math>a &gt; 1, O(n^k \cdot a^{n/b})</math></b>						
<b>if <math>a &lt; 1, O(n^k)</math> or <math>O(f(n))</math></b>						
<b>26 Dividing functions</b>						
test(int n)	$T(n)$					
{						
if(n > 1)						
{						
printf("%d", n);		1				
test(n/2)	$T(n/2)$					
}						
}						
$T(n) = T(n/2) + 1$ for $n > 1$						
$T(n) = 1$ for $n = 1$						
$T(n)$						
1 $T(n/2)$						
1 $T(n/2^2)$						
1 $T(n/2^3)$						
.....continue for k times						
1 $T(n/2^k)$						

	assume , $n/2^k = 1$						
	thus, we have taken k steps overall						
	<b>since, <math>n/2^k = 1 \Rightarrow k = \log n</math> with base 2</b>	<b><math>O(\log n)</math></b>					
	<b>Solving by substitution method</b>						
	$T(n) = T(n/2) + 1$						
	$T(n) = T(n/2^2) + 2$						
	$T(n) = T(n/2^3) + 3$						
	....						
	$T(n) = T(n/2^k) + k$						
	assume $n/2^k = 1$						
	thus, $k = \log n$ with base 2						
	<b><math>T(n) = T(1) + \log n</math></b>						
	<b><math>O(\log n)</math></b>						
	<b>27 Recurrence relation: <math>T(n) = T(n/2) + n</math></b>						
	$T(n) = T(n/2) + n$ for $n > 1$						
	$T(n) = 1$ for $n=1$						
	$T(n)$						
	$T(n/2) \quad n$						
	$T(n/2^2) \quad n/2$						
	$T(n/2^3) \quad n/2^2$						
	.....						
	$T(n/2^k). \quad n/2^{(k-1)}$						
	$T(n) = n + n/2 + n/2^2 + n/2^3..... + n/2^k$						
	$T(n) = n[1 + 1/2 + 1/2^2 + 1/2^3 + ..... 1/2^k]$						
	<b><math>T(n) = n \cdot 1 = n</math></b>						
	<b><math>O(n)</math></b>						
	Using substitution method						
	$T(n) = T(n/2) + n$						
	.....						
	$T(n) = T(n/2^2) + n/2 + n$						
	.....						

	$T(n) = T(n/2^3) + n/2^2 + n/2 + n$					
	.....					
	$T(n) = T(n/2^k) + n/2^{k-1} + \dots + n/2^2 + n/2 + n$					
	Assume $n/2^k = 1$					
	$k = \log n$ with base 2					
	$T(n) = T(1) + n[1/2^k + 1/2^{k-1} + \dots + 1/2^2 + \dots + 1]$					
	<b><math>T(n) = 1 + 2n \sim O(n)</math></b>					
	<b>28 Recurrence Relation: <math>T(n) = 2T(n/2) + n</math></b>					
	void test(int n)	$T(n)$				
	{					
	if(n > 1)					
	{					
	for(int i = 0; i < n; i++)					
	{					
	stmt	n				
	}					
	test(n/2);	$T(n/2)$				
	test(n/2);	$T(n/2)$				
	$T(n) = 2T(n/2) + n$ for $n > 1$					
	$T(n) = 1$ for $n = 1$					
	<b>Solve using recursion tree method,</b>					
	$T(n)$					
	$T(n/2). \quad T(n/2) \quad n$	n				
	$T(n/2^2) \quad T(n/2^2) \quad T(n/2^2) \quad T(n/2^2) \quad n/2$	n				
	$T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3). \quad T(n/2^3).$	n				
	.....	n				
	$T(n/2^k).....$					
		n				
	assume $n/2^k = 1$					
	$k = \log n$ with base 2					
	<b><math>T(n) = nk \sim O(n \log n)</math></b>					



	<b>Using backsubstitution method;</b>					
	$T(n) = 2T(n/2) + n$					
	$T(n/2) = 2T(n/2^2) + n/2$					
	$T(n) = 2[2T(n/2^2) + n/2] + n$					
	$T(n) = 2^2T(n/2^2) + n + n$					
	$T(n) = 2^3T(n/2^3) + 3n$					
	....continue for k times					
	$T(n) = 2^kT(n/2^k) + kn$					
	Asume $T(n/2^k) = T(1)$					
	$k = \log n$ with base 2					
	<b>Thus, <math>T(n) = n + n \log n \sim O(n \log n)</math></b>					
<b>29</b>	<b>Masters Theorem for dividing functions</b>					
	$T(n) = aT(n/b) + f(n)$	loga with b				
	$a \geq 1; b > 1; f(n) = \theta(n^k \log^p n)$	k				
	case 1: if loga with base b > k then $\theta(n^{\log_a b})$					
	case 2: if loga with base b = k then					
	if $p > -1$ $\theta(n^k \log^{p+1} n)$					
	if $p = -1$ $\theta(n^k \log \log n)$					
	if $p < -1$ then $\theta(n^k)$					
	case 3: if loga with base b < k					
	then, if $p \geq 0$ , $\theta(n^k \log^p n)$					
	if $p < 0$ , $\theta(n^k)$					
	$T(n) = 2T(n/2) + 1$					
	$a = 2$					
	$b = 2$					
	$f(n) = \theta(n^0 \log^0 n)$					
	$k = 0; p = 0$					
	here, loga with base b > k					
	$\theta(n^1)$ where loga with base b is 1					
	$T(n) = 4T(n/2) + n$					
	log a with base b = 2					

k = 1						
p = 0						
this is an example of case 1						
$\theta(n^2)$						
$T(n) = 8T(n/2) + n$						
log8 with base 2 = 3 > k = 1						
$\theta(n^3)$						
$T(n) = 9T(n/3) + 1$						
loga with base b = 2 > k						
$\theta(n^2)$						
$T(n) = 9T(n/3) + n^2$						
loga with base b = 2 = k	case 2					
$\theta(n^2)$						
$T(n) = 8T(n/2) + n$						
$\theta(n^3)$						
$T(n) = 2T(n/2) + n$						
loga with base b = k = 1; p = 0						
case 2						
$\theta(n \log n)$						
$T(n) = 4T(n/2) + n^2$						
$\theta(n^2 \log n)$						
$T(n) = 4T(n/2) + n^2 \log n$						
$\theta(n^2 \log n^2)$						
$T(n) = 8T(n/2) + n^3$						
$\theta(n^3 \log n)$						
$T(n) = 2T(n/2) + n/\log n$						
log a with base b = k = 1						

	$p = -1$						
	$\theta(n \log \log n)$						
	$T(n) = 2T(n/2) + n/\log n^2$						
	$p = -2$						
	$\theta(n)$						
	$T(n) = 2T(n/2) + n^2$						
	loga with base $b < k$						
	$\theta(n^2)$						
	$T(n) = 2T(n/2) + n^2$						
	$\theta(n^2 \log n)$						
	$T(n) = 2T(n/2) + n^3$						
	loga with base $b < k$						
	$\theta(n^3)$						
<b>30</b>	$T(n) = 2T(n/2) + 1$						
	loga with base $b = 1$						
	$k = 0$						
	loga with base $b > k$						
	$\theta(n^1)$						
	$T(n) = 4T(n/2) + 1$						
	loga with base $b = 2$						
	$k = 0$						
	$\theta(n^2)$						
	$T(n) = 4T(n/2) + n$						
	loga with base $b = 2$						
	$k = 1$						
	$\theta(n^2)$						
	$T(n) = 8T(n/2) + n^2$						
	loga with base $b = 3$						

k = 2						
$\theta(n^3)$						
$T(n) = 16T(n/2) + n^2$						
log a with base b = 4						
k = 2						
$\theta(n^4)$						
$T(n) = T(n/2) + n$						
log a with base b = 0						
k = 1						
$\theta(n)$						
$T(n) = 2T(n/2) + n^2$						
log a with base b = 1						
k = 2						
$\theta(n^2)$						
$T(n) = 2T(n/2) + n^2 \log n$						
log a with base b = 1						
k = 2						
$\theta(n^2 \log n)$						
$T(n) = 4T(n/2) + n^3 \log^2 n$						
log a with base b = 2						
k = 3						
$\theta(n^3 \log^2 n)$						
$T(n) = 2T(n/2) + n^2 / \log n$						
log a with base b = 1						
k = 2						
$\theta(n^2)$						
$T(n) = T(n/2) + 1$						
log a with base b = 0						
k = 0						

	$\theta(\log n)$					
	$T(n) = 2T(n/2) + n$					
	log a with base b = 1					
	k = 1					
	p = 0					
	$\theta(n \log n)$					
	$T(n) = 2T(n/2) + n \log n$					
	log a with base b = 1					
	k = 1					
	p = 1					
	$\theta(n \log^2 n)$					
	$T(n) = 4T(n/2) + n^2$					
	log a with base b = 2					
	k = 2; p = 0					
	$\theta(n^2 \log n)$					
	$T(n) = 4T(n/2) + (n \log n)^2$					
	log a with base b = 2					
	k = 2, p = 2					
	$\theta(n^2 \log^3 n)$					
	$T(n) = 2T(n/2) + n/\log n$					
	log a with base b = 1					
	k = 1; p = -1					
	$\theta(n \log \log n)$					
	$T(n) = 2T(n/2) + n/\log^2 n$					
	log a with base b = 1					
	k = 1; p = -2					
	$\theta(n)$					
<b>31</b>	<b>Root function Recurrence relation</b>					
	$T(n) = T(\sqrt{n} + 1)$ for $n > 2$					

	$T(n) = 1$ for $n = 2$					
	$T(n) = T(\text{root}(n)) + 1$					
	$T(n) = T(n^{(1/2)}) + 1$ .....equation 1					
	using substitution					
	$T(n) = T(n^{(1/2^2)}) + 2$ .....equation 2					
	$T(n) = T(n^{(1/2^3)}) + 3$ .....equation 3					
	$T(n) = T(n^{(1/2^k)}) + k$ .....equation 4					
	assume, $n = 2^m$					
	$T(2^m) = T(2^{(m/2^k)}) + k$					
	assume $T(2^{(m/2^k)}) = T(2)$					
	thus, $m/2^k = 1$					
	$m = 2^k$					
	$k = \log m$ with base 2					
	substituting value of $n$					
	$m = \log n$ with base 2					
	therefore, $k = \log \log n$ with base 2					
	$\theta(\log \log n \text{ with base } 2)$					
	<b>32 Binary Search Iterative Method</b>					
	To perform binary search, the prerequisite is that the list must be in sorted order	$A = \{3, 6, 8, 12, 14, 17, 25, 29, 31, 36, 42, 47, 53, 55, 62\}$				
	we need two index pointers, one is low at the starting point and the other is high at the end point	$l = 1, h = 15$ (lowest and highest index); $mid = 8$				
	$mid = \text{low} + \text{high} / 2$ and we take the floor value	key value = 42; $A[mid] = 29 \rightarrow \text{key} > A[mid]$				
	the key value is on the right hand side as key value is greater than $A[mid]$					
	we will change low to $mid + 1$	$l = 9, h = 15; mid = 9 + 15 / 2 = 12$				
		$A[mid] = 47 > \text{key}$				
	we will change high to $mid - 1$ as $\text{key} < A[mid]$					
		$h = 11, l = 9, mid = 10; A[mid] = 36$				
		$A[mid] < \text{key}$				
	we will change low to $mid + 1$	$l = 11; h = 11; mid = 11; A[mid] = 42$				
	we can return the index as we have found the key value	$A[mid] = \text{key}$				

	therefore, binary search looks faster than linear search. It just took 4 comparisons					
	int BinSearch(A, n, key)					
	{					
	l = 1, h = n					
	mid = l + h / 2 - take floor value					
	while(l <= h){					
	if(key == A[mid])					
	{ return index i.e.element is found}					
	else if(key < A[mid])					
	{h= mid-1;}					
	else {					
	l = mid + 1;}					
	}					
	return 0;					
	}					
	Time taken for binary search = logn					
	min time: O(1)					
	max time: O(logn)					
	avg time = add time for each element and divide by number of elements					
	<b>33 Binarysearch Recursive method</b>					
	Alogirthm RBinarySearch(l,h,key)	T(n)				
	{					
	if(l==h)		1			
	{					
	if(A[l]== key)					
	{					
	return l;					
	}					
	else					
	{					
	return 0;					





	Every full binary tree is also a complete binary tree					
	A complete binary tree is a full binary tree until height $h - 1$					
	Height of a complete binary tree would be minimum i.e. $\log n$					
<b>d</b>	<b>Heap</b>					
	Heap is a complete binary tree					
	Max Heap: every node has value greater than all its descendants {50, 30, 20, 15, 10, 8, 16}					
	Min Heap: every node has value smaller or equal to than all its descendants {10, 30, 20, 35, 40, 32, 25}					
	<b>35 Insert operation in a max heap</b>					
	Insert 60 in the above given max heap					
	this value should be inserted in the last free space in the array					
	i.e. left child of the left most leaf node					
	Then, adjust the elements to make it as a heap					
	So, compare and move 60 up the levels and in the array check at $i/2$ indices where initially $i$ would be the last empty index where 60 was inserted					
	Time taken would be equal to the number of swaps					
	this depends upon the height of the tree i.e. $\log n$ , hence $O(\log n)$					
	minimum time is of no swaps $O(1)$ ; max would be $O(\log n)$					
	<b>36 Delete operation in a max heap</b>					
	From the heap, we need to remove the root / top most element only					
	The last element in the complete binary tree would come in its place					
	Adjust the elements to maintain heap order					
	From the root towards the leaf, adjust					
	Compare the children ( $2i$ and $2i + 1$ ) and whichever child is greater than compare with the parent					
	Time taken depends upon the height; max could be $O(\log n)$					
	Whenever you delete from max heap, you get the next max element and in case of min heap, it would be the next min element					
	<b>37 HeapSort</b>					

	For a given set of numbers, create a heap					
	Delete all the elements from the heap					
	Total N elements we have inserted; each element we assume is moved up to the root; so time taken $O(N\log N)$					
	Then we delete the elements					
	Store deleted elements in the array in free space in the end					
	Deletion also takes $O(N\log N)$ time					
	Thus, heapsort takes $O(N\log N)$					
	<b>38 Heapify</b>					
	The process of creating heap but direction is opposite than creating a heap					
	$O(N)$					
	<b>39 Priority Queue</b>					
	elements will have priority and they would be inserted and deleted as per the priority order					
	For min heap, smaller the no. higher the priority					
	For max heap, greater the no. higher the priority					
	$O(\log N)$ for insertion and/or deletion					
	<b>40 TwoWay MergeSort - Iterative method</b>	Algorithm Merge(A, B, m, n)				
	merging two sorted lists to get a sorted result	{i = 1, j = 1, k = 1;				
	A = {2, 8, 15, 18} i	while(i <= m && j <= n){				
	B = { 5, 9, 12, 17} j	if(A[i] < B[j])				
	Compare A(i) with B(j) to get C(k) and move to next location	{				
	m + n elements are obtained , thus theta(m + n)	C[k++] = A[i++];				
		}				
		else {				
		C[k++] = B[j++];				
		}				
		for(; i <= m; i++){				
		C[k++] = A[i];				
		}				
		for(; j <= n; j++){				
		C[k++] = B[j];				
		}				

		}					
<b>41</b>	<b>Merging more than two lists</b>						
	M-way merging						
	A = {4, 6, 12}						
	B = {3, 5, 9}						
	C = {8, 10, 16}						
	D = {2, 4, 18}						
	One way is that we merge A and B; C and D and then finally merge the two resulting lists --> so we perform merge three times here						
	Another way is that we first merge A and B; then we merge resulting list with C ; and the resulting list with D						
	Two-way mergesort is an iterative process whereas mergeSort is a recursive process						
	A = {9, 3, 7, 5, 6, 4, 8, 2} - given an array and we have to sort them using 2-way mergesort						
<b>1st pass</b>	We would consider each element as a sorted list and merge	merged n elements in this pass					
	First select two lists 3 and 9; then merge them - 3, 9						
	Similarly, we select two lists 7 and 5 , merge them - 5 and 7						
	Another lists we get are {4, 6} and {2, 8}						
	Now, we have 4 lists with two elements each						
<b>2nd pass</b>	When we merged we kept the resulting 4 lists in another array B; B = {{3, 9}, {5, 7}, {4, 6}, {2, 8}}	merged n elements in this pass					
	We merge two lists each						
<b>3rd pass</b>	C = {{3, 5, 7, 9}, {2, 4, 6, 8}}	merged n elements in this pass					
	we merge the above two lists to get a single sorted list						
	D = {2, 3, 4, 5, 6, 7, 8, 9}						
	log(no of elements) = no. of passes						
	<b>Time complexity: O(n(logn))</b>						
<b>42</b>	<b>MergeSort</b>						
	A = {9, 3, 7, 5, 6, 4, 8, 2}	Algorithm MergeSort(l, h){	T(n)				
	If there is a single element, we can consider it as a base or small problem {Divide and conquer}						
		if(l < h){					

		mid = (l + h) / 2;	1				
		MergeSort(l, mid);	T(n/2)				
		MergeSort(mid + 1, h);	T(n/2)				
		Merge(l, mid, h);	n				
		}	T(n) = 2T (n/2) + n for n > 1				
		}	T(n) = 1 for n = 1				
	<b>time complexity: theta(nlogn)</b>		using master's theorem, a = 2, b = 2, k = 1				
	merging is done in post order traversal		loga with base b = 1 = k				
			thus, it is case 2				
			<b>theta(nlogn)</b>				
<b>43 Pros of MergeSort</b>		<b>Cons of MergeSort</b>					
	works great for Large size lists	Extra space (not inplace sort)					
	suitable for Linked List	no small problem					
	supports external sorting	recursive and uses a stack (need n + logn space) i.e. space complexity: O(n + logn) where n is the extra space and logn is the stack space					
	stable: the order of duplicates is maintained						
		insertion sort (O(n^2))					
		mergesort O(nlogn)					
		for small problems, n <= 15; insertionsort works better --> use insertion sort					
<b>43 QuickSort</b>							
	students arranging themselves in increasing order of heights						
	<b>10</b> 80 90 60 30 20						
	5 6 3 4 2 1 <b>9</b>						
	4 6 7 <b>10</b> 16 12 13 14						
	A = {10, 16, 8, 12, 15, 6, 3, 9, 5, INFINITY}	partition(l, h){					
	select first element as a pivot	pivot = A[l];					

	pivot = 10	i = l; j = h;					
	we need to find the sorted position for 10	while(i<j){do					
	i starting from pivot and j starting from infinity	{					
	i would check for elements greater than 10; j would check for elements smaller than pivot	i++;					
	we are using the partitioning procedure	} while(A[i]<= pivot);					
	increment i until next value is greater than 10 and decrement j until next value is smaller than pivot; stop and swap	do					
	{10, 5, 8, 9, 3, 6, 15, 12, 16}	{					
	send pivot element at j position	j--;					
	now, we can sort the two lists around the partitioning position by performing quicksort recursively	}while(A[j] > pivot);					
		if(i<j){					
	QuickSort(l, h)	swap(A[i], A[j]);					
	{	}					
	if(l < h)	swap(A[l], A[j]);					
	{	return j;					
	j = partition(l, h);	}					
	QuickSort(l, j);						
	QuickSort(j+ 1, h);						
	}						
	}						
	<b>44 QuickSort Analysis</b>						
	suppose it is partitioning in the middle of 1 and 15th index						
	then, two partitions: [1, 7] ; [9, 15]						
	further partitions: [1, 3]; [5, 7]; [9, 11]; [13, 15]						
	at each level , n elements are being handled						
	and there are logn levels						
	thus time complexity for best case: O(nlogn)						
	median : middle element of a sorted list						
	best case of quicksort is that the partitioning occurs exactly at the middle						
	worstcase: if we have an already sorted list						
	time complexity for worstcase: O(n^2)						
	to handle this, try taking middle element as a pivot						

2. select random element as a pivot						
<b>45 Strassen's matrix multiplication</b>						
A = [a11 a12						
a21. a22]						
B = [b11 b12						
b21 b22]						
Cij = Summing up Aik*Bkj						
for(i = 0; i < n ; i++){						
for(j = 0; i < n ; j++){						
C[i,j]= 0;						
for(k=0;k<n;k++){						
C[i,j] += A[i, k]*B[k, i];						
}						
}						
}						
C11 = a11*b11 + a12*b21						
C21 = a11*b12 + a12*b22	A = [a11]					
C21 = a21*b11 + a22*b21	B = [b11]					
c22 = a21*b12 + a22*b22	C = [a11*b11]					
for [2*2] matrix, we would use above formula	for [1*1] matrix, use above formula					
we assume that the matrix has dimensions of power of 2	Algorithm MM(A, B, n)					
	{					
	if(n <= 2					
8 times the function is calling itself	{					
T(n) = 8T(n/2) + n^2 for n > 1	C = 4 formula stated above;					
a = 8, b = 2, log a with base b = 3	}					
k = 2	else					
it is case 1 of master's theorem	{					
theta(n^3)	mid = n/2					
	MM(A11, B11, n/2) + MM(A12, B21, n/2);					
	MM(A11, B12, n/2) + MM(A12, B22, n/2);					
	MM(A21, B11, n/2) + MM(A22, B21, n/2);					
	MM(A22, B22, n/2) + MM(A21, B12, n/2);					

		}					
		}					
	Strassen's approach -						
	has given 4 different formulas with 7 multiplications	$P = (A_{11} + A_{22})(B_{11} + B_{22})$					
	$C_{11} = A_{11} \cdot B_{11} + A_{12} \cdot B_{21}$	$Q = (A_{21} + A_{22}) B_{11}$					
	$C_{21} = A_{11} \cdot B_{12} + A_{12} \cdot B_{22}$	$R = A_{11}(B_{12} - B_{22})$					
	$C_{21} = A_{21} \cdot B_{11} + A_{22} \cdot B_{21}$	$S = A_{22}(B_{21} - B_{11})$					
	$C_{22} = A_{21} \cdot B_{12} + A_{22} \cdot B_{22}$	$T = (A_{11} + A_{12})B_{22}$					
		$U = (A_{21} - A_{11})(B_{11} + B_{12})$					
		$V = (A_{12} - A_{22})(B_{21} + B_{22})$					
		$C_{11} = P + S - T + V$					
		$C_{12} = R + T$					
		$C_{13} = Q + S$					
		$C_{22} = P + R - Q + U$					
		$T(n) = 7T(n/2) + n^2$ for $n > 2$					
		$T(n) = 1$ for $n \leq 2$					
		using master's theorem,					
		$O(n^{\log_7 \text{ with base } 2}) = O(n^{2.81})$					
	<b>Strategies used for solving optimization problems - Greedy Method, Dynamic programming, branch and bound</b>						
	<b>46 Greedy method</b>						
	Design which we can adopt to solve similar problems		Greedy method says that each problem should be solved in stages - each stage we give an input, check if the solution is feasible then we pick it up and move to next stage				

	Solving optimization problems		Algorithm Greedy(a, n) a = {a1, a2, a3, a4, a5}; n = 5				
	<b>Optimization problem:</b> Problems which require either minimum or maximum result		{				
	Suppose we have a problem P where we need to travel from source A to destination B, we can have several solutions such as walking on foot, travel by an airplane, ride on a bike, travel by a bus, drive on a car, go by a train and so on..... Now, we notice that we also have some constraints. The solutions that satisfy the conditions given in a problem are called <b>feasible solutions</b>	Minimum cost journey - " <b>Minimization problem</b> "; then feasible solutions giving minimum cost are called <b>optimal solutions</b> . <b>There can be many feasible solutions but only one optimal solution</b>	for i = 1 to n do {x = select(a):				
	Example: selecting a car to purchase	Example: Hire a person for your company	if feasible(x) then				
	Method 1: Looking at all the models available in the city	Method 2: Conduct an assessment center to filter people at each stage and get the best person	{				
	Method 2: Checking for the features of the cars and filtering and selecting based on your preferences - greedy method	So, the person may not be the best but the approach is greedy here as we are using our criteria and constraints to choose the best person	solution = solution + x;				
	<b>47 Knapsack problem</b>		}				
	n = 7; m = 15	Bag capacity is 15 kgs and we have been given 7 objects. we have to fill this bag with these objects. Profit is the gain we get by transferring this object. Problem is a container loading problem. Problem is filling the container with the objects as the capacity of container is limited	}				
<b>Objects</b>	{0 1 2 3 4 5 6 7 }	Optimization and maximization problem	}				
<b>Profits</b>	{P 10 5. 15 7 6 18 3 }	Constraints : Bag weight limit					
<b>Weights</b>	{W 2 3 5 7 1 4 1 }						
<b>Profit by weight</b>	{P/W 5 1.3 3 1 6 4.5 3}						
<b>0&lt;=x&lt;= 1</b>	Objects are divisible i.e. we can take just half kg of object 1 and may be 2 kgs of object 2 and so on						
<b>x</b>	()						
	x1 x2 x3 x4 x5 x6 x7						
<b>Method 1</b>	Take the thing that have maximum profit						



<b>Method 2</b>	Take things with smaller weight so that you can put in more things						
<b>Method 3</b>	Take things that have highest profit by weight						
	Let's use method 3						
	First, I include object 5 that has maximum profit by weight. Then I check remaining weight I can put in. We can still put in 14 kgs. Then we select all the quantity of object 1. Remaining weight limit 12 kgs. Add all of object 6. Remaining weight limit 8 kgs. Add all of object 3. Remaining weight limit 3 kgs. Add all of object 7. Remaining weight limit is 2 kgs. Add 2/3 of object 2 as we have only 2 kgs limit remaining.						
<b>x</b>	( 1    2/3    1    0    1    1    1 )						
	Calculate total profit and verify weight						
	<b>Total weight</b> = $1*2 + 2/3*3 + 1*5 + 0*7 + 1*1 + 1*4 + 1*1 = 15$	//Multiplying x elements by Weight w for each object					
	<b>Total profits</b> = $1*10 + 2/3*5 + 1*15 + 1*6 + 1*18 + 1*3 = 54.6$	//Multiplying x elements by Profit P for each object					
<b>48</b>	<b>0/1 Knapsack problem</b>						
	Objects are indivisible and fractions are not allowed i.e. either you include the whole thing or you do not include it at all						
<b>49</b>	<b>Job sequencing with deadlines</b>	n = 5 (tasks)					
<b>Jobs</b>	J1    J2    J3    J4    J5						
<b>Profits</b>	20    15    10    5    1						
<b>Deadlines</b>	2    2    1    3    3						
	Assume that there is a machine, on which each job has to be processed and each job takes 1 unit of time ( hour) for completion						
	Set of the jobs which can be completed within their deadlines such that profit is maximized	Constraints: deadlines must be met					
<b>deadlines</b>	0-----1-----2-----3	maximum 3 slots / jobs					
<b>time slots</b>	9am-----10am-----11am-----12am						
<b>Jobs chosen</b>	J2            J1            J4						
<b>Profits</b>	$15 + 20 + 5 = 40$						
<b>Sequence</b>	J1 --> J2 ---> J4    J2 --> J1 --> J4						



Example:												
List	x1	x2	x3	x4	x5							
Sizes	20	30	10	5	30							
Increasing order of sizes	5	10	20	30	30							
Lists	x4	x3	x1	x2	x5							
	First x4 and x3 are merged, cost = 15; then result is merged with x1; cost = 35; x2 and x5 are merged with cost = 60; the two resulting lists are merged with cost = 95											
Total cost	15 + 35 + 60 + 95 = 205											
	3*5 + 3*10 + 2*20 + 2*30 + 2*30 = 205					//multiplying distance of each node and size of each node						
52	Huffman Coding											
	Compression technique used to reduce the size of data or message											
Message	BCCABBDDEAECBBAEDDCC											
	Length = 20											
	it has to be sent using ASCII codes (8- bit)											
	A 65 01000001					Size = 8*20 = 160 bits						
	B 66 01000010											
	C 67											
	D 68											
	E 69											
	Can we use our own codes instead of ASCII codes?											
	Fixed size method											
Character	A	B	C	D	E							
Count	3	5	6	4	2	Total count = 20						
Code	000	001	010	011	100							
message	BCCABBDDEAECBBAEDDCC											
bit code	001010....					size = 20*3 = 60 bits						
						5*8 = 40 bits for ASCII code translations						
						5*3 = 15 bits --> our assigned codes						

		40 + 15 = 55 bits					
		message: 60 bits					
		chart: 55 bits					
		total message size: 115 bits					
		so, the message size reduced from 160 bits to 115 bits					
		thus, 40% reduction in size with fixed sized code					
	Huffman coding - variable sized code	element that appears more / often should have a smaller sized code					
<b>character</b>	A    B    C    D    E						
<b>count</b>	3    5    6    4    2						
<b>code</b>							
	first, arrange the letters with increasing count / frequency						
<b>character</b>	E    A    D    B    C						
<b>count</b>	2    3    4    5    6						
<b>code</b>	000    001    01    10    11	Merge two smaller ones, we get 5, then combine with D, we get 9. Combine B and C, we get 11. Finally, combine two resulting lists, we get 20.					
<b>bit count</b>	6    9    8    10    12	On left side paths, mark as 0 and on right side mark as 1					
<b>total bits for message</b>	45 bits	Bit count for message can also be obtained from the tree, by counting number of edges for a letter and multiplying by the number of occurrences for that letter in the message i.e. summation of distance and frequency of a letter					
<b>ASCII codes for chart</b>	5*8 = 40 bits						
<b>assigned codes</b>	12 bits						
<b>total bits for tree/table</b>	52 bits						
<b>Size of total msg</b>	52 + 45 = <b>97 bits</b>						
<b>Message transferred</b>	001111101101111001011000111110100010100000110	A tree or a table would be needed along with it					
<b>Decoding</b>	BCCD...						

<b>53</b>	<b>Minimum Cost Spanning Tree</b>						
	$G = (V, E)$						
	$V = \{1, 2, 3, 4, 5, 6\}$	$ V  = n = 6$					
	$E = \{(1,2), (2,3), (3,4), (4,5), (5, 6), (6,1)\}$	$ V  - 1 = 5 \text{ edges}$					
	the tree should not have a cycle						
	S is a subset of G, WHERE IN $S = (V', E')$	$V' = V;  E'  =  V  - 1$					
	Number of edges in graph = 6 out of which I have to select 5 edges for spanning tree - thus i can select in <b>6C5 ways</b>	Suppose we have 7 edges, out of which the seventh edge (3,5) divides the graph into two cycles of less tha 6 vertices, then we can select 5 edges for spanning tree in <b>7C5 - 2 ways</b>					
<b>General formula</b>	$ E C( V -1) - \text{no. of cycles}$						
	Now, if we have a weighted graph, I wish to know the number of possible spanning tree						
	Vertices = 4						
	Edges = 3						
	cost = 14						
	similarly , depending upon the edges we select, cost may vary each time						
	Can I found the minimum cost spanning tree?						
<b>Method 1</b>	Try all possible spanning trees and get the minimum cost spanning tree						
<b>Method 2</b>	Prim's algorithm (Greedy method)						
<b>Method 3</b>	Krskal's algorithm (Greedy method)						
<b>Method 2:</b>	<b>Prim's algorithm</b>						
	Select the minimum cost edge from the graph first	(6,1) ; w = 10					
	Then, following this select minimum cost edge but make sure it is connected to previously chosen vertices	(5, 6); w = 25					
		(5, 4); w = 22					
		(4, 3); w = 12					
		(3, 2); w = 16					
		(2, 7); w = 14					
	Now, if we add costs of all the chosen edges, total cost = 99						

	For non connected graphs we cannot find the minimum cost spanning tree or spanning tree						
<b>Method 3</b>	<b>Kruskal's method</b>						
	Always select smallest cost edge						
	(1,6); w = 10						
	(3, 4); w = 12						
	(2, 7); w = 14						
	(2, 3); w = 16						
	(4, 5); w = 22						
	(5, 6); w = 25						
	total cost = 99						
	vertices count :  V	To get a minimum cost edge each time, min heap can be used					
	edges count:  V  - 1	theta(nlogn)					
	theta( V  E )						
	theta(n.e) = theta(n^2)						
	for non-connected graphs, spanning tree cannot be found						
	Kruskal algo may give spanning tree for those non connected componr=ents but bot for the graph as a whole						
	if in a certain graph, certain edges' weights are missing, then use the given weights of remaining edges to guess the weight						
<b>54</b>	<b>Dijkstra algorithm</b>						
	Single source shortest path to all the vertices						
	find the shortest path to a vertex annd update it to other vertices. this updation is called relaxation						
	<b>Relaxation</b>						
	if(d[u] +c(u,v) < d[v]){ d[v] = d[u] + c(u,v)}						
	no of vertices =  V						
	at most no. of vertices relaxing =  V						

	worst case time of Dijkstra algorithm: $\theta(n^2)$										
	Example - starting vertex is 1										
<b>selected vertex</b>	2	3	4	5	6						
<b>4</b>	50	45	<b>10</b>	infinity	infinity						
<b>5</b>	50	45	<b>10</b>	<b>25</b>	infinity						
<b>2</b>	45	45	<b>10</b>	<b>25</b>	infinity						
<b>3</b>	45	45	<b>10</b>	<b>25</b>	infinity						
<b>6</b>	45	45	<b>10</b>	<b>25</b>	infinity						
	Another example - starting vertex is 1										
<b>selected vertex</b>	{2,	3,	4}								
<b>2</b>	{3,	infinity,	5}								
<b>4</b>	{3,	infinity,	5}								
<b>3</b>	{3,	7,	5}								
	{3,	7,	5}								
	Another example - starting vertex is 1										
<b>selected vertex</b>	{2,	3,	4}								
<b>2</b>	{3,	infinity,	5}								
<b>4</b>	{3,	infinity,	5}								
<b>3</b>	{3,	7,	5}								
	{-3,	7,	5}								
	Dijkstra algorithm might work or might not work in case of an edge having negative weightage										
<b>55</b>	<b>Dynamic programming</b>										
	<b>Dynamic programming vs greedy method</b>										

	In Greedy method, we try to follow a predefined procedure that gives us the best / optimal result. The procedure is already known for optimization. But, in dynamic programming, we try to get all the solutions and then decide the best solution. Mostly dynamic programming questions are solved using recursive procedures. They follow a principle of optimality. In greedy method, decision is taken just once and followed through whereas in dynamic programming, decision is taken at each step					
	<b>Example:</b>					
	<b>Fibonacci series</b>					
	fib(n) = 0 if n = 0	T(n) = 2T(n-1) + 1{Approximating T(n-2) ~ T(n-1) here}				
	fib(n) = 1 if n = 1	Time taken would be O(2^n) by using Master's theorem				
	fib(n) = fib(n-2) + fib(n-1) if n > 1	Why can't we reduce the function calls to reduce the time taken?				
		For this, we would take a global array and initially fill it with -1				
	int fib(n) {	F = {-1,-1,-1,-1,-1}				
	if(n <= 1){	Then, as the function calls f(1), mark it as 1				
	return n;}	Then f(0) is marked as 1				
	return fib(n-2) + fib(n-1);	Then use the stored result to get the rest.				
	}	Finally, F would get updated as we solve: F = {0, 1, 1, 2, 3, 5}				
		Total 6 calls are made then i.e. n+1 calls i.e. O(n)				
		This is called result of memorization				
	From the above example, we can see reduction in number of calls from O(2^n) to O(n) using memorization. It follows top down approach. The same problem can be solved using tabular method (iterative process) as shown below:					
	int fib(int n) {					
	if(n <= 1) {					
	return n;}					
	F[0] = 0; F[1] = 1;					
	for(int i = 2; i <= n; i++){					
	F[i] = F[i-2] + F[i-1];					
	}					
	return F[n];					
	}					



	F= {0, 1, 1, 2, 3, 5}																	
	This is a bottom - up approach i.e. starting from F[0] and moving to F[n]																	
	<b>56 Multistage Graph</b>																	
	<i>A multistage graph is a directed weighted graph. The vertices are divided into stages such that the edges are connecting vertices from one stage to next stage only. First stage and last stage will have only one vertex to represent start and end point. This is usually used to represent resource allocation.</i>																	
	The objective of the problem is that I have to select a path which gives me minimum cost.												//it is a minimization or optimization problem					
	Dynamic programming works on principle of optimality. Principle of optimality says that a problem can be solved in a sequence of decisions.																	
	From first stage I have to select one optimal vertex that leads to minimum cost and I have to take this decision at each stage. Thus, I can apply dynamic programming here.																	
<b>V</b>	1	2	3	4	5	6	7	8	9	10	11	12						
<b>Cost</b>	16	7	9	18	15	7	5	7	4	2	5	0	cost(5, 12) = 0 ; here 5 is the stage and 12 is the vertex					
<b>d</b>	2/3	7	6	8	8	10	10	10	12	12	12	12	cost(4, 9) = 4					
													cost(4, 10) = 2					
	Formula for multistage graph:												cost(4, 11) = 5					
	<b>cost(lth stage, jth vertex no.) = cost(i, j) = min{C(j, i) + cost(i+1, i)}</b>												cost(3, 6) = min{ C(6, 9) + cost(4, 9) , C(6, 10) + cost(4, 10)} = min{6 + 4, 5 + 2} = 7					
													Similarly, cost(3, 7) = min{8, 5} = 5					
	Now, we will solve it by going in forward direction and taking decisions based on above data;												cost(3, 8) = min{7, 11} = 7					
	d(1,1) = 2												Similarly, cost(2, 2) = min{C(2, 6) + cost(3, 6), C(2, 7) + cost(3, 7), C(2, 8) + cost(3, 8)} = min{11, 7, 8} = 7					
	d(2,2) = 7												cost(2, 3) = min{9, 12} = 9					
	d(3, 7) = 10												cost(2, 4) = min{18} = 18					
	d(4, 10) = 12												cost(2, 5) = min{16, 15} = 15					
	<b>Path: 2---&gt; 7---&gt; 10----&gt; 12</b>												cost(1,1) = min{16, 16, 21, 17} = 16					
	d(1, 1) = 3																	
	d(2, 3) = 6																	
	d(3, 6) = 10																	
	d(4, 10) = 12																	
	<b>Path: 3---&gt; 6---&gt; 10----&gt; 12</b>																	

	So, we have two paths with same cost.														
<b>57</b>	<b>Multistage Graph (Program)</b>														
	<b>Cost adjacency Matrix</b>									main(){					
	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	int stages = 4, min;					
<b>0</b>	0	0	0	0	0	0	0	0	0	int n = 8;					
<b>1</b>	0	0	2	1	3	0	0	0	0	int cost[9], d[9], path[9];					
<b>2</b>	0	0	0	0	0	2	3	0	0	int c[9][9] = {{0,0,0,0,0,0,0,0,0}, {0,0,2,1,3,0,0,0,0}, {0,0,0,0,0,2,3,0,0}, .....}					
<b>3</b>	0	0	0	0	0	6	7	0	0	cost[n] = 0;					
<b>4</b>	0	0	0	0	0	6	8	9	0	for(int i = n-1; i >=1; i--){					
<b>5</b>	0	0	0	0	0	0	0	0	6	min = 32767;					
<b>6</b>	0	0	0	0	0	0	0	0	4	for(int k = i + 1; k <= n; k++){					
<b>7</b>	0	0	0	0	0	0	0	0	5	if(C[i][k] != 0 && C[i][k] + C[k] < min){					
<b>8</b>	0	0	0	0	0	0	0	0	0	min = C[i][k] + C[k] ;					
										d[i] = k;					
	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>						
<b>cost</b>	--	9	7	11	12	6	4	5	0	}					
<b>d</b>	--	2	6	6	5	8	8	8	--	}					
<b>path</b>	--	1	2	6	8					cost[i] = min;					
										}					
	Path is calculated using the following formula: p[i] d[p[i-1]] ; p [2] = d[p[2-1]] = d[1] = 2									p[1] = 1; p[stages] = n;					
	<b>time complexity: O(n^2)</b>									for(i = 2; i < stages; i++){					
										p[i] = p[d[i-1]];					
<b>58</b>	<b>All Pairs Shortest Path</b>														
	If I use Dijkstra algorithm on each vertex to find the shortest path of all, the time complexity would be O(n^3).														
<b>A0 =</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>											
	<b>1</b>	0	3	INF	7										
	<b>2</b>	8	0	2	INF										
	<b>3</b>	5	INF	0	1										
	<b>4</b>	2	INF	INF	0										
	Considering vertex 1 as intermediate vertex									A0[2,3]	A0[2,1] + A0[1,3]				
<b>A1 =</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>						2	< 8 + INF				



	For generating a single matrix C after single multiplication of 2 matrices of order (5X4) and (5X3) , we would need to do 60 multiplications									
	((A1.A2).A3).A4 or (A1.A2).(A3.A4) or ....there could be several ways then, how to choose the right way?									
	T(n) = 2nCn/n+1 trees are possible thus, with 3 nodes ; T(3) = 5									
	Using tabular approach (bottom up approach),									
<b>m</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	in m[1,1] i.e. A1 , nothing is multiplied, hence it can be taken as zero					
	<b>1</b>	0	120	88	158	m[1,2] = A1 . A2				
	<b>2</b>	-	0	48	104	(5X4) (4X6)				
	<b>3</b>	-	-	0	84	Total cost of multiplication = 5 * 4 * 6 = 120				
	<b>4</b>	-	-	-	0					
<b>s</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	m[1,3] = A1.A2.A3					
	<b>1</b>	-	1	1	3	Two possibilities: A1.(A2.A3) or (A1.A2).A3				
						(5X4) (4X6) (6X2)				
	<b>2</b>	-	-	2	3	for A1.(A2.A3) --> m[1,1] + m[2,3] + (5*4*2)	for (A1.A2).A3 --> m[1,2] + m[3,3] + (5*6*2)			
	<b>3</b>	-	-	-	3	0 + 48 + 40	120 + 0 + 60			
	<b>4</b>	-	-	-	-	88	180			
					Similarly, m[2,4]					
	formula:				A2.(A3.A4) (A2.A3).A4					
	m[i,j] = m[i,k] + m[k+1, j] + di-1 * dk * dj				(4X6) (6X2)(2X7) (4X6) (6X2)(2X7)					
					for A2.(A3.A4) --> m[2,2] + m[3,4] + (4*6*7)	for (A2.A3).A4 --> m[2,3] + m[4,4] + (4*2*7)				
	<b>we are generating n(n -1)/2 elements</b>				0 + 84 + 168	48 + 0 + 56				
	<b>time complexity = O(n^3)</b>				252	104				
					m[1,4]					

		$\min\{m[1,1] + m[2,4] + (5*4*7), m[1,2] + m[3,4] + (5*6*7), m[1,3] + m[4,4] + (5*2*7)\}$					
		$\min\{0+104+140, 120 + 84+210, 88+70\}$					
		$\min\{244, 414, 158\}$					
60	<b>Matrix chain multiplication - A few pointers</b>						
	<b>Condition of the multiplication:</b> The number of columns in the first matrix involved in the multiplication must be equal to the number of rows in the second matrix						
<b>A=</b>	a11   a12   a13	2X3 dimension					
	a21   a22   a23						
<b>B=</b>	b11   b12	3X2 dimension					
	b21   b22						
	b31   b32						
<b>A*B =</b>	$a_{11}*b_{11} + a_{12}*b_{21} + a_{13}*b_{31}$ $a_{11}*b_{12} + a_{12}*b_{22} + a_{13}*b_{32}$	12 multiplications ( $2*3*2$ )					
	$a_{21}*b_{11} + a_{22}*b_{21} + a_{23}*b_{31}$ $a_{21}*b_{12} + a_{22}*b_{22} + a_{31}*b_{32}$	2X2 dimensions of the resultant matrix					
	<b>A1 X A2 X A3 {Multiplication of more than two matrices}</b>						
	2X3   3X4   4X2						
	d0 d1. d1 d2   d2 d3						
	Same answer by the two following methods (Associative property)						
	<b>Method 1:</b> (A1 X A2) X A3	<b>Method 2:</b> A1 X (A2 X A3)					
	2X3 3X4 i.e ( $2*3*4$ ) = 24 multiplications for A1 X A2	2X3   3X4   4X2					
	Now, (A1 X A2) X A3 would require ( $2*4*2$ ) = 16 multiplications	A2 X A3 requires ( $3*4*2$ ) = 24 multiplications					
	Thus, altogether 40 multiplications are required	A1 X (A2 X A3) requires ( $2*3*2$ ) = 12 multiplications					
		Altogether, 36 multiplications are needed here.					
	Now, dynamic programming asks us to find all the possible methods for matrix multiplication and check which one costs the minimum --> this implies that for 10 matrices, there would be numerous methods and we would have to check for all before proceeding with any one of them. Thus, we need a formula to check all that						
	We need to find C[1,3]						
	<b>Method 1:</b> (A1 X A2) X A3	<b>Method 2:</b> A1 X (A2 X A3)					
	C[1,2] = 24;   C[3,3] = 0	C[1,1] = 0; C[2,3] = 24					

	$C[1,2] + C[3,3] + d_0 \cdot d_2 \cdot d_3 = 40$	$C[1,1] + C[2,3] + d_0 \cdot d_1 \cdot d_3 = 36$					
	<b><math>C[i,j] = C[i, k] + C[k+1, j] + d_{i-1} \cdot d_k \cdot d_j</math></b>						
	After generalization,						
	$C[i,j] = \min \{ C[i,k] + C[k+1,j] + d_{i-1} \cdot d_k \cdot d_j \}$						
	where $i \leq k \leq j$						
	$A_1 \times A_2 \times A_3 \times A_4$						
	$d_0 \ d_1 \ d_1 \ d_2 \ d_2 \ d_3 \ d_3 \ d_4$						
	Check which method works the best for the above matrix chain multiplication	$2n \ C \ n / n + 1$ multiplications are possible where $n = \text{no. of matrices} - 1$					
	1. $A_1 (A_2 (A_3 A_4))$	Modified formula: $2(n-1) \ C \ (n-1) / n$					
	2. $A_1 ((A_2 A_3) A_4)$	Now, for $n = 4$ , $2 \cdot 3 \cdot 3 / 4 = 6 \cdot 5 \cdot 4 / 3 \cdot 2 \cdot 1 / 4 = 5$					
	3. $(A_1 A_2) (A_3 A_4)$	$n = 5$ , 14 multiplications					
	4. $(A_1 (A_2 A_3)) A_4$						
	5. $((A_1 A_2) A_3) A_4$						
	Applying the formula:						
<b>4-1 = 3 values</b>	$C[1,4] = \min \{ k = 1; C[1,1] + C[2,4] + d_0 \cdot d_1 \cdot d_4,$						
	$k = 2; C[1,2] + C[3,4] + d_0 \cdot d_2 \cdot d_4,$						
	$k = 3; C[1,3] + C[4,4] + d_0 \cdot d_3 \cdot d_4 \}$						
	$1 \leq k \leq 4$						
	1. $A_1 (A_2 A_3 A_4)$						
	2. $(A_1 A_2) (A_3 A_4)$						
	3. $(A_1 A_2 A_3) A_4$						
	here, $C[1,1] = 0; C[4,4] = 0$						
<b>4-2 = 2 values</b>	$C[2,4] = \min \{ k = 2; C[2,2] + C[3,4] + d_1 \cdot d_2 \cdot d_4$						
	$k = 3; C[2,3] + C[4,4] + d_1 \cdot d_3 \cdot d_4 \}$						
	$2 \leq k \leq 4$						

<b>4-3 = 1 value</b>	$C[3,4] = C[3,3] + C[4,4] + d_2 \cdot d_3 \cdot d_4$								
	A1 X A2 X A3 X A4								
	3X2   2X4   4X2   2X5								
	d0 d1   d1 d2 d2   d3   d3 d4								
	As we notice that there is a repetition of the values needed , such as C[3,4] or C[4,4], there is unnecessary calculation, we are repeating, thus we should use a table (4X4)								
<b>C table</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	$C[1,2] = \min\{k=1; C[1,1] + C[2,2] + d_0 \cdot d_1 \cdot d_2\}$				
	<b>1</b>	0	24	28	58	Thus, $C[1,2] = 3 \cdot 2 \cdot 4 = 24$			
	<b>2</b>	-	0	16	36	$C[2,3] = \min\{k = 2; C[2,2] + C[3,3] + d_1 \cdot d_2 \cdot d_3\}$			
	<b>3</b>	-	-	0	40	Thus, $C[2,3] = 2 \cdot 4 \cdot 2 = 16$			
	<b>4</b>	-	-	-	0	$C[3,4] = d_2 \cdot d_3 \cdot d_4 = 4 \cdot 2 \cdot 5 = 40$			
						$C[1,3] = \min\{k=1; C[1,1] + C[2,3] + d_0 \cdot d_1 \cdot d_3\}$			
<b>k table</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>		$k = 2; C[1,2] + C[3,3] + d_0 \cdot d_2 \cdot d_3\}$			
	<b>1</b>	-	1	1	3	$C[1,3] = \min\{16 + 3 \cdot 2 \cdot 2, 24 + 3 \cdot 4 \cdot 2\}$			
	<b>2</b>	-	-	2	3	$C[1,3] = \min\{28, 48\} = 28$			
	<b>3</b>	-	-	-	3				
	<b>4</b>	-	-	-	-	$C[2,4] = \min\{C[2,2] + C[3,4] + d_1 \cdot d_2 \cdot d_4, C[2,3] + C[4,4] + d_1 \cdot d_3 \cdot d_4\}$			
						$C[2,4] = \min\{40 + 2 \cdot 4 \cdot 5, 16 + 2 \cdot 2 \cdot 5\}$			
						The result shows that we need to do minimum of 58 multiplications to get the result of A1 X A2 X A3 X A4			
						The k table will give the paranthesization			
						$((A1) (A2 A3))(A4)$			
						How much time it has taken ? $1+2+3+4 = 4(5) / 2$ i.e. $n(n+1)/2 \sim n^2$			
						we also tried n posisble values of k to compute this value, thus time taken is $n^2 \cdot n$ i.e. $O(n^3)$			
						$k = 2; C[1,2] + C[3,4] + d_0 \cdot d_2 \cdot d_4,$			
						$k=3; C[1,3] + C[4,4] + d_0 \cdot d_3 \cdot d_4\}$			
						$C[1,4] = \min\{36 + 3 \cdot 2 \cdot 5, 24 + 40 + 3 \cdot 4 \cdot 5, 28 + 3 \cdot 2 \cdot 5\}$			
						$C[1,4] = \min\{66, 124, 58\} = 58$			
<b>61 Matrix chain multiplication Program</b>									
						main{			
						int n = 5;			

		int P[] = {5, 4, 6, 2, 7};				
	P = { 5,4,6,2,7}	int m[5][5] = {0};				
		int s[5][5] = {0};				
		int j, min, q;				
		for(int d = 1; d < n - 1; d++)				
		{				
		for(int i = 1; i < n - d; i++)				
		{				
		j = i + d;				
		min = 32767;				
		for(int k = 1; k <= j - 1; k++)				
		{				
		q = m[i][k] + m[k + 1][j] + P[i				
		- 1] * P[k] * P[j];				
		if(q < min)				
		{				
		min = q;				
		s[i][j] = k;				
		}				
		}				
		m[i][j] = min;				
		}				
		cout << m[1][n - 1];				
		}				
	<b>62 Single source shortest path (Bellman Ford Algorithm)</b>	Example of Bellman Ford algorithm:				
	For doing this, we already have Dijkstra algorithm but it may not work correctly if we have negative weights, thus we need some other method that works with negative weights	edges --> (3,2)(4,3)(1,4)(1,2)				
	Bellman Ford algorithm says that we should relax the edges N-1 times where the number of vertices is equal to N	mark source vertex 1 as 0 and rest all as infinity				
	V  = N = 7	there are 4 vertices, so we should relax all the edges for 3 times				
	So, we should relax them for  V  - 1 times	First iteration:				
	so, we would cover all possible paths even the longest path	for (3,2) --> infinity - 10 is infinity only, so no change				



Relaxation means between a pair of vertices u and v if there is an edge, then check if:	for (4,3) --> infinity + 3 < infinity only, so no change					
if( $d[u] + C(u,v) < d[v]$ ) {	for (1,4), $0 + 5 < \text{infinity}$ , thus vertex 4 is updated to 5					
$d[v] = d[u] + C(u,v)$	for (1,2) $0 + 4 < \text{infinity}$ , thus vertex 2 is updated to 4					
edgeList --> (1,2)(1,3)(1,4)(2,5)(3,2)(3,5)(4,3)(4,6)(5,7)(6,7)	Second iteration:					
Now, I have to relax these edges for $ V  - 1$ i.e. 6 times	for (3,2), vertex 2 is already 4, which is less than $d[u] + C(u,v)$ in this case					
Initially, mark the distance for source vertex as 0 and for the rest of the vertices as infinity	for (4,3) vertex 3 is updated to $5 + 3 = 8$					
Now, let's relax edge (1,2)	for (1,4) --> no change					
here, $d[u] = 0$ ; $d[v] = \text{infinity}$ ; $C(u,v) = 6$	for(1,2) --> no change					
$0 + 6 < \text{infinity}$ ; thus $d[v] = 6$	Third iteration					
for vertex 2, distance is 6;	for(3,2), $8 - 10 = -2 < d[v]$ which is 4 right now; thus updated for vertex 2 as -2					
similarly, relaxing (1,3); thus its distance is updated to 5 from infinity	for the rest of the edges there won't be any change					
in similar way, (1,4) is relaxed, following that the distance of vertex 4 is updated to 5 from infinity	results obtained :					
Now, relaxing (2,5); $d[u] = 6$ ; $C(u,v) = -1$ ; $d[v] = \text{infinity}$	vertex 1 --> 0					
the distance (2,5) is updated to $6 - 1 = 5$	vertex 2 --> -2					
similarly, relaxing (3,2); $d[u] = 5$ , $C(u,v) = -2$ , $d[v] = 6$	vertex 3 --> 8					
$5 - 2 = 3 < 6$ thus, $d[v]$ is updated here to 3 i.e. at vertex 2, distance is updated to 3	vertex 4 --> 5					
Now, relaxing (3,5), $d[u] = 5$ , $C(u,v) = 1$ , $d[v] = 5$ here $d[u] + C(u,v)$ is not smaller than $d[v]$ hence the distance of vertex 5 is not modified	Now, if I relax one more time extra, there's no change					
Moving to (4,3), relaxing it --> $d[u] = 5$ , $C(u,v) = -2$ ; $d[v] = 5$ , $d[u] + C(u,v) = 3 < d[v]$ hence distance of vertex 3 is updated to 3	<b>Drawback of Bellman Ford algorithm:</b>					
following this (4,6) is relaxed again and checked, $d[u] = 5$ , $C(u,v) = -1$ ; thus distance of vertex 6 is updated to $5 - 1 = 4$	let us an edge(2,4) in the above example					
Now, relaxing (5,7), $d[u] = 5$ ; $C(u,v) = 3$ ; $d[v] = \text{infinity}$	we see even after N-1 iterations, there is one vertex changing, we note that there's a problem					
thus, $d[v]$ is updated to 8 for edge (5,7)	the reason is that there is a cycle of edges where total weight of edges is negative i.e. $5 + 3 + (-10) = -2$ , thus graph cannot be solved					
moving to (6,7), $d[u] = 4$ , $C(u,v) = 3$ ; $d[v] = 8$	hence, for a negative weighted cycle, the bellman ford algorithm fails					

d[v] is updated to 7 here for edge (6,7)						
let us continue second time;						
there won't be any change in (1,2), (1,3), (1,4)						
when we check for (2,5); d[u] = 3 C(u,v) = -1 d[v] = 5; d[v] for edge (2,5) is updated to 2						
similarly, for edge (3,2), the value is change to 3 - 2 = 1; earlier it was 3						
(4,3) and (4,6), there's no change						
for (5,7) d[u] has changed from 5 to 2; thus d[v] changes to 2 + 3 = 5						
for (6,7) there won't be any change						
let us continue third time;						
there won't be any change in (1,2), (1,3), (1,4),						
when we check for (2,5); d[u] = 1 C(u,v) = -1 d[v] = 2; d[v] for edge (2,5) is updated to 0						
for (3,2) there own't be any change						
for (3,5) again thee won't be any change						
for (4,3), (4,6) --> no change						
for (5,7) d[v] gets updated to 0 + 3 = 3						
for (6,7) --> no change						
let us check for fourth time,						
we notice for all edges --> no change						
results obtained:						
vertex 1 --> 0						
vertex 2 --> 1						
vertex 3 --> 3						
vertex 4 --> 5						
vertex 5 --> 0						
vertex 6 --> 4						
vertex 7 --> 3						
so, finally these are the shortest paths						
<b>time complexity: <math>O( E ( V  - 1)) \sim O( V  E ) \sim O(N^2)</math></b>						
If it is a complete graph, that is between every two vertex there is an edge, then number of edges is $N(N - 1) / 2$						
i.e. $ E  = N(N - 1) / 2$						
then time complexity = $O( E  V ) O(N((N - 1) / 2)(N - 1)) \sim O(N^3)$						

63	<b>0/1 Knapsack Problem</b>									
	m = 8; n = 4 where m is the capacity of the bag									
	P = {1, 2, 5, 6}									
	w = (2, 3, 4, 5)									
	Have to give a solution of what all should be included to maximize profit									
	x can be either 0 or 1; we cannot take fraction of an object									
	x would be like {1, 0, 1....}									
	sum of profits is maximized i.e. $\max\{\sum(P_i x_i)\}$									
	$\sum(w_i x_i) \leq m$									
	2^4 solutions possible i.e. n^4 solutions									
	<b>let us use tabulation method</b>									
	as the capacity is 8 so we would take 0-8 columns and 0-5 rows									
<b>V</b>		<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
<b>P</b>	<b>W</b>	<b>0</b>	0	0	0	0	0	0	0	0
<b>1</b>	<b>2</b>	<b>1</b>	0	0	1	1	1	1	1	1
<b>2</b>	<b>3</b>	<b>2</b>	0	0	1	2	2	3	3	3
<b>5</b>	<b>4</b>	<b>3</b>	0	0	1	2	5	5	6	7
<b>6</b>	<b>5</b>	<b>4</b>	0	0	1	2	5	6	6	7
			V[i,w] = $\max\{V[i-1, w], V[i-1, w-w[i]] + P[i]\}$							
			V[4,1] = $\max\{V[3,1], V[3, 1-5] + 6\} = \max\{V[3,1], V[3,-4] + 6\}$							
			V[3,-4] is an undefined location; thus, V[4,1] is equal to V[3,1]							
			FROM THE ABOVE FORMULA WE DEDUCE THAT UPTO 5TH WEIGHT, WE WOULD BE GETTING A NEGATIVE NUMBER							
			so, we fill these values as it is from previous row until column 4							
			for 5th column, V[4,5] = $\max\{V[3,5], V[3,0] + 6\} = \max\{5, 0 + 6\} = 6$							
			V[4,6] = $\max\{V[3,6], V[3, 1] + 6\} = \max\{6, 6\} \rightarrow$ so take 6 only							
			V[4,7] = $\max\{V[3,7], V[3,2] + 6\} = \max\{7, 7\} = 7$							
			V[4,8] = $\max\{V[3,8], V[3,3] + 6\} = \max\{7, 2 + 6\} = \max\{7, 8\} = 8$							

	Now, we need to write down $x_1$ , $x_2$ , $x_3$ and $x_4$ values						
	let us come with maximum profit i.e. 8 which we got only by including 4th object						
	thus $x_4 = 1$ ; remaining profit = $8 - 6 = 2$						
	now, check for object 3, if there is a value 2; check if the value 2 is there in for object 2 as well at the same place, if yes, then object 3 was not taken i.e. $x_3 = 0$ ; if the same 2 is there for object 1 then $x_2 = 0$ else $x_2 = 1$						
	here, $x_3 = 0$						
	$x_2 = 1$						
	$x_1 = 0$ as the remaining profit is 0						
	$x = \{0 \ 1 \ 0 \ 1\}$ for maximizing the profit						
	<b>let us use sets method</b>						
	we will prepare sets of P and w						
	$s_0 = \{(0,0)\}$						
	$s_0(1) = \{(1,2)\}$ here, we added first object to elements of set $s_0$						
	$s_1 = \{(0,0),(1,2)\}$ merged the above two sets to get $s_1$						
	$s_1(1) = \{(2,3), (3,5)\}$ , added second object to elements of $s_1$						
	$s_2 = \{(0,0),(1,2),(2,3),(3,5)\}$ merged the above two sets to get $s_2$						
	$s_2(1) = \{(5,4),(6,6),(7,7),(8,9)\}$						
	$s_3 = \{(0,0),(1,2),(2,3),(3,5),(5,4),(6,6),(7,7)\}$ removed (8,9) as it exceeded the permitted limit						
	in the above set, we notice that as profit increases , weight increases, but at (5,4) profit has increased from (3,5) whereas weight has decreased						
	thus, we discard (3,5) with lesser profit {dominance rule}						
	so, $s_3 = \{(0,0),(1,2),(2,3),(5,4),(6,6),(7,7)\}$						
	now, considering the fourth object,						
	$s_3(1) = \{(6,5),(7,7),(8,8),(11,9),(12,11),(13,12)\}$						
	$s_4 = \{(0,0),(1,2),(2,3),(5,4),(6,6), (7,7), (8,8)\}$ merged above two sets and removed elements using dominance rule and those exceeding the weight limits						
	time taken is almost $(2^n)$						
	maximum order is (8,8)						
	(8,8) belongs to $s_4$						

	check whether it belongs to s3 ----> it does not, hence object 4 is included					
	now (8,8) - (6,5) = (2,3)					
	(2,3) --> check if it belongs to s3 --> yes, check whether it belongs to s2 -> yes, thus object 3 is not included --> now, check if it belongs to s1 --> no, that means object 2 is included and object 1 is not					
	2-2, 3-3 = (0,0)					
	(0,0) belongs to set 1 and set 0 as well therefore object 1 is not included					
	x = {0 1 0 1}					
<b>64</b>	<b>0/1 Knapsack Dynamic Programming</b>					
		main()				
	P = {0, 1, 2, 5, 6}	{				
	wt = {0, 2, 3, 4, 5}	int P[5] = {0, 1, 2, 5, 6}				
		int wt[5] = {0, 2, 3, 4, 5}				
	n = 4, m = 8	int m = 8, n = 4;				
		int k[5][9];				
	i = n ; j = m;					
	while(i > 0 && j > 0) {	for(int i = 0; i <= n; i++)				
	if(k[i][j] == k[i-1][j])	{				
	{	for(int w = 0; w <= m; w++)				
	cout << i << "=0" << endl; i--;	{				
	}	if(i==0    w == 0)				
	else {	{				
	cout << i << "=1" << endl; i--; j = j - wt[i];	k[i][w] = 0;				
	}	} else if (wt[i] <= w)				
	}	{				
		k[i][w] = max{P[i] + k				
		[i-1][w - wt[i]], k[i-1][w];				
		}				
		else				
		{				
		k[i][w] = k[i-1][w];				
		}				
		}				
		cout<<k[n][w];				

					}														
	65 Optimal Binary Search Tree																		
	keys --> 10, 20, 30, 40, 50, 60, 70																		
	time taken for searching a particular key in a BST is logn where n is the number of nodes and logn is the minimum height of the tree																		
	if the target key is not in the tree, the search would be unsuccessful																		
	2n C n / n + 1 binary searches are possible for n keys																		
	cost is dependent upon number of comparisons needed																		
	A balanced binary search tree would require the minimum number of comparisons																		
	In an optimal binary search tree problem, in addition to the number of comparisons we also consider the frequency of search of those keys																		
	1	2	3	4		C[0,2]													
keys	10	20	30	40		10 20													
frequency	4	2	6	3		4 2													
	j																		
	0	1	2	3	4														
i	0	0	4	8 <sub>1</sub>	20 <sub>3</sub>	26 <sub>3</sub>	I = j - i = 0												
	1	0	0	2	10 <sub>3</sub>	16 <sub>3</sub>	0 - 0 = 0												
	2	0	0	0	6	12 <sub>3</sub>	1 - 1 = 0												
	3	0	0	0	0	3	2 - 2 = 0												
	4	0	0	0	0	0	3 - 3 = 0												
							4 - 4 = 0												
	w(0,4) = sum(f(i)) where 0<=i <= 4						set up cost for all diagonala values as 0;												
	calculating For first case, C[0,2] = C[0,0] + C[1,2] + w[0,2]						I = j - i = 1												
	C[0,2] = 0 + 2 + 6						C[0,1] i.e. considering only key 1												
	C[0,2] = 8						cost is 4												
	Similarly, calculating cost C[0,2] in the second case = C[0,1] + C[2,2] + w[0,2]						similarly, C[1,2] i.e. considering only second key												
	here, C[0,2] = 4 + 0 + 6 = 10						cost is 2												
							Similarly, C[2,3] = 6 and C[3,4] = 3												

C[1,3] (CASE 1 when 20 is the root and 30 is it's right node) = $2*1 + 6*2 = 14$	Now, $l = j - i = 2$ i.e. we would consider 2 keys at a time					
C[1,3] (CASE 2 when 30 is the root and 20 is its left node) = $6*1 + 2*2 = 10$	i.e C[0,2] , C[1,3] and C[2,4]					
	C[0,2]: key 10 and 20 with frequency 4 and 2 respectively					
C[2,4] (CASE 1 when 30 is the root and 40 is its right node) = $6*1 + 3*2 = 12$	Two possibilities: 10 in the root and 20 it's right node OR 20 in the root and 10 it's left node					
C[2,4] (CASE 2 when 40 is the root and 30 is on the left) = $3*1 + 6*2 = 15$	for first case: Cost = $4*1 + 2*2 = 8$					
	for second case: Cost = $2*1 + 4*2 = 10$					
$l = j - i = 3$	minimum cost is 8 and the root is 1					
C[0,3] , C[1,4] for C[0,3], $w[0,3] = 12$						
For C[0,3] , (CASE 1 : 10 as a root and 20 it's right node, 30 is the right node of 20) = $C[0,0] + C[1,3] + w[0,3] = 0 + 10 + 12 = 22$						
(CASE 2, 10 as a root and 30 it's right node, 20 is the left node of 30)						
(CASE 3 when 20 is the root, 10 it's left child and 30 it's right child) = $C[0,1] + C[2,3] + 12 = 4 + 6 + 12 = 22$						
(CASE 4 when 30 is the root, 20 is the left child and 10 is its left child) = $C[0,2] + C[3,3] + 12 = 8 + 0 + 12 = 20$						
(CASE 5 when 30 is the root, 10 is its left child and 20 is the right child of 10)						
C[1,4] ; $w[1,4] = 11$						
$C[1,4] = \min\{C[1,1] + C[2,4] + 11; C[1,2] + C[3,4] + 11; C[1,3] + C[4,4] + 11\}$						
$C[1,4] = \min\{0 + 12 + 11; 2 + 3 + 11; 10 + 0 + 11\} = \min\{23, 16, 21\} = 16$						
$l = j - i = 4 = 4 - 0$ ; $w[0,4] = 15$						
$C[0,4] = \min\{C[0,0] + C[1,4] + 15; C[0,1] + C[2,4] + 15; C[0,2] + C[3,4] + 15; C[0,3] + C[4,4] + 15\} = \min\{0 + 16 + 15; 4 + 12 + 15; 8 + 3 + 15; 20 + 0 + 15\} = \min\{31, 31, 26, 35\} = 26$						

	root[0,4] = 3					
	then left child is root[0,2] and right child is root[3,4]					
	root[0,2] is 1 and root[3,4] is 4; root[3,4] is further subdivided into root[3,3] and root[4,4]					
	root[0,2] is further divided into root[0,0] and root[1,2]					
	root[1,2] is the second key and its further divided into root[1,1] and root[2,2]					
	$C[i,j] = \min\{C[i,k-1] + C[k,j]\} + w(i,j)$ where $i < k \leq j$					
66	<b>Optimal Binary Search Tree Successful And Unsuccessful Probability</b>					
	Unsuccessful nodes can be represented by dummy nodes					
	If there are n keys, there could be n+1 square dummy nodes					
	Successful search probability represents probability of getting a given key in the lot whereas unsuccessful search probability represents a range of values of the key					
	$\text{cost}[0,n] = P_i * \text{level}(a_i) + Q_i * (\text{level}(e_i) - 1)$					
	this cost value is calculated over $1 \leq i \leq n$ for successful searches ( $P_i$ ) and $0 \leq i \leq n$ for unsuccessful searches ( $Q_i$ )					
	the above cost is minimized for optimal binary search tree					
	Is it possible that we find out the best tree without calculating the cost of all the trees					
	Let's use dynamic programming to find out the minimum cost arrangement without actually computing cost for each binary search tree possible					
	$C[i,j] = \min\{C[i,k-1] + C[k,j]\} + w(i,j)$ where $i < k \leq j$					
	Let us consider it for just three nodes					
	i.e. $C[0,3] = \min\{C[0,0] + C[1,3] + w[0,3], C[0,1] + C[2,3] + w[0,3], C[0,2] + C[3,3] + w[0,3]\}$ where $0 < k \leq 3$					
	$C[0,0] = C[3,3] = 0$					
	$C[1,3] = \min\{C[1,1] + C[2,3], C[1,2] + C[3,3]\} + w[1,3]$ where $1 < k \leq 3$					



	Here, $C[1,1] = C[3,3] = 0$										
	Values we need, $C[0,0]$ , $C[1,1]$ , $C[2,2]$ , $C[3,3]$ , $C[0,1]$ , $C[1,2]$ , $C[2,3]$ , $C[0,2]$ , $C[1,3]$ , $C[0,3]$										
	$i, j$										
$j - i = 0$	$C[0,0]$	$C[1,1]$	$C[2,2]$	$C[3,3]$		$w[0,2] = q_0 + p_1 + q_1 + p_2 + q_2$					
$j - i = 1$	$C[0,1]$	$C[1,2]$	$C[2,3]$			$w[0,3] = q_0 + p_1 + q_1 + p_2 + q_2 + p_3 + q_3$					
$j - i = 2$	$C[0,2]$	$C[1,3]$				thus, $w[0,3] = w[0,2] + p_3 + q_3$					
$j - i = 3$	$C[0,3]$					$w[i, j] = w[i, j-1] + p_i + q_j$					
	we basically need to find the cost, weight and the root at each stage										
	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>j --&gt;</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>keys</b>	10	20	30	40		$j - i = 0$	$w_{00} = q_0 = 2$ $C_{00} = 0$ $r_{00} = 0$	$w_{11} = q_1 = 3$ $C_{11} = 0$ $r_{11} = 0$	$w_{22} = q_2 = 1$ $C_{22} = 0$ $r_{22} = 0$	$w_{33} = q_3 = 1$ $C_{33} = 0$ $r_{33} = 0$	$w_{44} = q_4 = 1$ $C_{44} = 0$ $r_{44} = 0$
<b>p<sub>i</sub></b>	3	3	1	1		$j - i = 1$	$w_{01} = w[0,0] + p_1 + q_1 = 8$ $C_{01} = 8$ $r_{01} = 1$	$w_{12} = 7$ $C_{12} = 7$ $r_{12} = 2$	$w_{23} = 3$ $C_{23} = 3$ $r_{23} = 3$	$w_{34} = 3$ $C_{34} = 3$ $r_{34} = 4$	
<b>q<sub>i</sub></b>	2	3	1	1	1	$j - i = 2$	$w_{02} = 12$ $C_{02} = 19$ $r_{02} = 1$	$w_{12} = 9$ $C_{12} = 12$ $r_{12} = 2$	$w_{24} = 5$ $C_{24} = 8$ $r_{24} = 3$		
						$j - i = 3$	$w_{03} = 14$ $C_{03} = 25$ $r_{03} = 2$	$w_{14} = 11$ $C_{14} = 19$ $r_{14} = 2$			
						$j - i = 4$	$w_{04} = 16$ $C_{04} = 32$ $r_{04} = 2$				
	Finally , the tree looks like the below tree:										
	$r[0,4] = 20$										
$r[0,1] = 10$						$r[2,4] = 30$					
						$r[3,4] = 40$					
<b>67</b>	<b>Travelling Salesman Problem Using Dynamic Programming</b>										
	A directed weighted graph is given and every edge is having weight										
	we have to start from some vertex and travel all the vertices and return back to the starting vertex and the cost of travelling should be minimum										

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>						
<b>1</b>	0	10	15	20						
<b>2</b>	5	0	9	10						
<b>3</b>	6	13	0	12						
<b>4</b>	8	8	9	0						
	$g(i,s) = \min \{ C_{ik} + g(k, s - \{k\}) \}$ where $k$ belongs to $s$									
	$g(1, \{2,3,4\}) = \min \{ C_{1k} + g(k, \{2,3,4\} - \{k\}) \}$ where $k$ belongs to $\{2,3,4\}$				Now, this is a recursive formula for which result can be obtained by generating recursive tree					
	vertex 1									
<b>here, costs can be taken from given table</b>	vertex 2 $g(1,2) = \min \{ C_{12} + g(2, \{3,4\}) \}$ ; $C_{12} = 10$				vertex 3 $g(1,3) = \min \{ C_{13} + g(3, \{2,4\}) \}$ ; $C_{13} = 15$		vertex 4 $g(1,4) = C_{14} + g(4, \{2,3\})$ ; $C_{14} = 20$			
	vertex 3 $C_{23} + g(3, \{4\})$    vertex 4 $C_{24} + g(4, \{3\})$ ; $C_{23} = 9, C_{24} = 10$				vertex 2 $C_{32} + g(2, \{4\})$    vertex 4 $C_{34} + g(4, \{2\})$ ; $C_{32} = 13, C_{34} = 12$		vertex 2 $C_{42} + g(2, \{3\})$    vertex 3 $C_{43} + g(3, \{2\})$ ; $C_{42} = 8, C_{43} = 9$			
	$g(3,\{4\}) = C_{34} + g(4, \text{phi i.e. nothing})$ ; $g(4, \{3\}) = C_{43} + g(3, \text{phi})$ ; $C_{34} = 12, C_{43} = 9$ ; $g(4, \text{phi}) \sim g(4,1) = 8$ ; similarly, $g(3, \text{phi}) = 6$				$g(2, \{4\}) = C_{24} + g(4, \text{phi})$ ; $g(4,\{2\}) = C_{42} + g(2, \text{phi})$ ; $C_{24} = 10, C_{42} = 8, g(4, \text{phi}) = 8, g(2, \text{phi}) = 5$		$g(2,\{3\}) = C_{23} + g(3, \text{phi})$ ; $g(3, \{2\}) = C_{32} + g(2, \text{phi})$ ; $C_{23} = 9, C_{32} = 13$ ; $g(3, \text{phi}) = 6, g(2, \text{phi}) = 5$			
	we use tabular method to solve these from bottom to top actually,									
	$g(2,\text{phi}) = 5$									
	$g(3, \text{phi}) = 6$									
	$g(4, \text{phi}) = 8$									
	$g(2,\{3\}) = 15$									
	$g(2,\{4\}) = 18$									
	$g(3,\{2\}) = 18$									
	$g(3,\{4\}) = 20$									
	$g(4,\{2\}) = 13$									

	$g(4, \{3\}) = 15$									
	$g(2, \{3, 4\}) = \text{minimum of the options} = 25$									
	$g(3, \{2, 4\}) = 25$									
	$g(4, \{2, 3\}) = 23$									
	$g(1, \{2, 3, 4\}) = 35$									
	<b>68 Reliability Design Problem</b>									
	we have to set up a system									
	system consists of certain devices									
	Reliability here means what is the probability of a device working perfectly / good				Example:					
DEVICES	D1	D2	D3	D4	$D_i$	$C_i$	$r_i$	$u_i$		
COST	C1	C2	C3	C4	D1	30	0.9	2		
RELIABILITY	$r_1$	$r_2$	$r_3$	$r_4$	D2	15	0.8	3		
	0.9	0.9	0.9	0.9	D3	20	0.5	3		
					$C = 105$					
	Reliability of the entire system = product of individual reliability of the devices = $0.9^4 = 0.6561$ i.e. 35% chance that the system might experience a failure				At least one copy of each device must be taken, so $C_1 + C_2 + C_3 = 30 + 15 + 20 = 65$					
	In case of failure, system is designed to have parallel copies of devices as a back up				Remaining amount = $C - \sum(C_i) = 105 - 65 = 40$					
	here, $r_1 = 0.9$				if I spend the entire remaining amount on D1, I would be able to purchase 1 D1 device					
	$1 - r_1 = 1 - 0.9 = 0.1$				<b><math>u_i = [C - \sum(C_i) / C_i] + 1</math> copies of any device <math>D_i</math></b>					
	$(1 - r_1)^3 = 0.1^3 = 0.001$				for D2, $40/15 = 2 + 1 = 3$ copies					
	probability that three copies are working perfectly = $1 - (1 - r_1)^3 = 0.999$				for D3, $40/20 = 2 + 1 = 3$ copies					
	Reliability has improved by using a parallel system				Solving this is similar to the set method of 0/1 knapsack problem, let's delve deeper					
	How many devices should I buy within the given cost such that the system's reliability is maximized				$(R, C)$					
					$s_0 = \{(1, 0)\}$					
					consider D1, $s_{11} = \{(0.9, 30)\}$					
					if we take 2 copies of the first device D1, $s_{12} = \{(1(1-0.9)^2)\} = 1 - 0.1^2 = 0.99$					
					thus, $s_{12} = \{(0.99, 60)\}$					

		$s_1 = \{(0.9, 30), (0.99, 60)\}$					
		consider D2 one copy, $s_{2_1} = \{(0.72, 45), (0.792, 75)\}$					
		Taking 2 copies of D2, $s_{2_2} = 1 - (1 - r_2)^2 = 1 - (1 - 0.8)^2 = 1 - 0.04 = 0.96$					
		$\{(0.864, 60), (0.9504, 90)\}$					
		in the last case, remaining cost = $105 - 90 = 15$ ; we cannot purchase D3 as its cost is 20 which is more than our remaining amount; so we would discard that set option as it is unfeasible					
		consider 3 copies of D2, reliability = $1 - (1 - r_2)^3 = 1 - (1 - 0.8)^3 = 1 - 0.008 = 0.992$					
		$s_{2_3} = \{(0.8928, 75), (\dots, 105)\}$ --> second case is discarded as it is unfeasible					
		$s_2 = \{(0.72, 45), (0.792, 75), (0.864, 60), (0.8928, 75)\}$					
		in $s_2$ , the third case has cost decreased but reliability is increased, thus we would use dominance rule to remove / discard the third case					
		Now, consider one copy of D3, $s_{3_1} = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$					
		$r_3 = 0.5$ ; $1 - (1 - r_3)^2 = 1 - 0.25 = 0.75$					
		$s_{3_2} = \{(0.54, 85), (0.648, 100)\}$ the third case has cost exceeding 105 thus it has been discarded					
		Consider three copies; $1 - (1 - r_3)^3 = 0.875$					
		$s_{3_3} = \{(0.63, 105)\}$					
		$s_3 = \{(0.36, 65), (0.432, 80), (0.4464, 95), (0.54, 85), (0.648, 100), (0.63, 105)\}$					
		Using dominance rule, $s_3 = \{(0.36, 65), (0.432, 80), (0.54, 85), (0.648, 100)\}$					
		Maximum reliability = 0.648 and the cost would be 100					
		for getting the number of copies of devices, go backward from 0.648, 100 ordered pair					
		which shows 1 copy of D1, 2 copies of D2, 2 copies of D3					
69	Longest Common Subsequence (LCS)						