| Section VI | Big O | | | | | | | | |
|------------|--|---|---|---------------------------------------|---------------------------|--|--|--|--|
| | Asymptotic notations | | | | | | | | |
| | O(Big O) | Describes an upper bound on time | | on the runtime; similar to a less- | then we also say that X<= | In industry, O and theta have been put together and we have to give the tightest description of runtime | | | |
| | Omega(n) | Describes the lower bound | for example: printing the values in an array is Omega (n) as well as Omega(logn) as well as Omega(1) | | | | | | |
| | Theta(n) | Describes the tight bound on runtime | Theta here means both O and Omega; in this example, it would be Theta(n) | | | | | | |
| | Best Case, Worst Case and Expected Case | | | | | | | | |
| | Best Case: | For example, in Quick Sort, if all the elements are equal, then quick sort will, on average, just traverse through the array once - O(N) time | elements greater than pivot - this gives partial sort. then it recursively sorts the left and | | | | | | |
| | Worst Case: | The pivot could be repeatedly the biggest element in the array. If pivot is the first element in a reversely sorted array. In this cae, our recursion does not divide the array in half and recurse on other half. Instead, it justs shrinks the subarray by 1 element. | Time taken would O(N^2) | | | | | | |
| | | both the above best and worst conditions would rarely happen; thus we can expect a runtime of O(nlogn) | , , | | | | | | |
| | | etween Asymptotic notations Worst Case and Expected | | | | | | | |

| There is no particular relationship between the two concepts | | | | | | | | | | |
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| | 10 15 110 | | | | | | | | | |
| Best Case, Wor | st Case and Expected Case act | ually describe the big O or big | g Theta time for part | ticular scenarios w | hereas these asy | mptotic notations | describe the upp | er, lower and tigh | t bounds for the ru | ıntime |
| | | | | | | | | | | |
| Space complexity | | | | | | | | | | |
| Memory or space required by an algorithm | to create an array - if it is unidimensional, O(N) space complexity; for a 2-D array, O (N^2) | | | | | | | | | |
| Stack space in recursive calls counts too. Each call adds a level tot he stack and takes up actual memory. | However, just because you have N calls does not mean it will take O(N) time: check the example on Page 41 for more details | | | | | | | | | |
| Drop the constants | | | | | | | | | | |
| O(2N) is actually O(N) | | | | | | | | | | |
| Drop the non- dominant terms | | | | | | | | | | |
| O(N^2 + N) becomes O (N^2) | | | | | | | | | | |
| O(N + logN) becomes O(N) | | | | | | | | | | |
| O(5*2^N + 1000N^100) becomes O (2^N) | | | | | | | | | | |
| $O(x!) > O(2^x) > 0$ | $O(x^2) > O(x\log x) > O(x)$ | | | | | | | | | |
| | | | | | | | | | | |
| Multi-Parts algorithms: add versus mutiply | | | | | | | | | | |
| Add: | Non-nested chunk of work A and B | O(A + B) | "DO THIS THEN WHEN YOU ARE ALL DONE, DO THAT" | | | | | | | |

| Multiply | Nested A and B | O(AB) | "DO THIS FOR EACH TIME YOU DO THAT" | | | | |
|--|--|--|--|---|--|--|--|
| Multiply | Nesteu A anu B | O(AB) | TOO DO THAT | | | | |
| Amortized time | | | | | | | |
| | That copying might take additional O(N) time after accounting for initial O(N) time of adding the elements | into a new array does not | that the worst ca every once in a happens it won't | | | | |
| Adding X more space to an array takes additional O(X) time; thus the amortized time for each adding is O(1) | X + X/2 + X/4 + X/8 = 2X | | | | | | |
| logN runtimes | | | | | | | |
| Example: | | | | | | | |
| Binary search. We are looking for an element x in a sorted array. We first compare to the midpoint. If x == middle, then we return else if x < middle, we search on the left side of array | | The total runtime is then a matter of how many steps we can take before it becomes 1 | $2^k = N \Rightarrow k = \log N$ with base 2 | Basically, when you see a problem with logN runtime, the problem space gets halved in each step | | | |
| | | | | | | | |
| Recursive runtimes | | | | | | | |
| Program: | int f(int n){ | | | | | | |
| | if(n <= 1){ | | | | | | |
| | return 1} | | | | | | |
| | return f(n -1) + f(n -1);} | | | | | | |
| | | | | | | | |
| How many calls in the tree? | | | | | | | |
| Do not count and say 2 | | | | | | | |

| It will have recursive calls with a depth N and 2^N nodes at the bottom most level | More genrically, 2^0 + 2^1 + 2^2 +2^n-1 = 2^n - 1 nodes | | The space complexity would still be O (n) - even though we have O(2^n) nodes in tree total, only O(n) exists at a time | | | | |
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| | | | | | | | |
| Examples and I | | | | | | | |
| Example 1 | O(n) time as we iterate through array once in each loop which are non-nested | | | | | | |
| Example 2 | O(N^2) time as we have two nested loops | | | | | | |
| Example 3 | j basically runs for N-1 steps in first iteration, N-2 steps in second iteration and so on. | 1 + 2 + 3 ++ N-1 = N(N - 1)/2 ~ N^2 | O(N^2) | | | | |
| Example 4 | For each element of array A, the inner loop goes through b iterations where b is the length of array B. Thus, time complexity is O(ab) | | | | | | |
| Example 5 | Similar to example 4, 100,000 units of work is still constant; so the run time is O(ab) | | | | | | |
| Example 6 | O(N) time as the array is iterated even if half of it (constant 1/2 can be ignored) | | | | | | |
| Example 7 | There is no established relationship between N and M , thus all but the last one are equivalent to O(N) | | | | | | |
| Example 8 | s = length of the longest string Sorting each string would take elements of the array; thus O(a array would take O(s * aloga) a would take O(s) time in additio take O(aloga); thus adding the aloga) = O(a *s (log a + log s) | O(slogs) and we do this for a a * slogs). Now, sorting the as each string comparison in to array sorting that would | | | | | |
| Example 9 | Approach 1: for summing up the nodes in a BST, each node is exactly traversed once, thus O(N) time complexity | | | | | | |
| | Approach 2: The number of recursive calls is 2 and the depth is logN in a BST> O (branches ^ depth) = O(2 ^ logN) where the base of logN is also 2 => time complexity = O(N) after simpllifying | | | | | | |

| |) as the for loop stant time and runs N)) time | | | |
|--|---|---|--|--|
| process c | ne recursive alls from N to N-1 to o on until 1 | | | |
| that we br time. this string. So path with | h 1: We make a tree for an example string say 'abc anch 4 times at the root , then 3 times, then 2 time gives us 4*3*2*1 leaf nodes. We could say n! leaf r , total nodes would be n * n! as each leaf node is a n nodes. Also, string concatenation will also take C me complexity in worst case would be O(n * n * n!) | es, and then 1 nodes for n length attached to a D(n) time. Thus, | | |
| nodes; at total node 1/n!). Nov n! * e who Thus, the | n 2: At level 6, we have 6! / 0! nodes; at level 5 we level 4 we have 6!/2! nodesat level 0, we have 6! s in the tree in terms of n can be: n!(1/0! + 1/1 | !/6! nodes. so, the 1/2! + 1/3!+ of Euler's number: Iropped further. | | |
| pattern for | We can also get tighter runtime as O(1.6^N) if we consider that there might be just one call instead of 2 at the bottom of call stack of chepth) = O(2^N). | | | |
| time. And, 2^2 + 2^3 | vious example, we deduce that fib(n) taken 2^n we have fib(1) + fib(2) + fib(3) +fib(n) = 2^1 ++ 2^n = 2^n (n+1) - 2, thus we can say that the s approx. 2^n | | | |
| are doing which the reduces to and fib(i - array at ea we are do amount of | e in this program we memoization due to amount of work b looking up fib(i - 1) 2) values in memo ach call fib(i). Thus, ing a constant f work n times in n ce time complexity: | | | |
| times we until we guntil we gunte case 1. As number of | ne is the number of can divide n by 2 et down to the base s, we know the firmes we can ntil we get 1 is O | | | |
| | | | | |
| Additional Problems | | | | |
| | op iterates through ne complexity is O | | | |
| through b | sive call iterates calls as it subtracts iteration, thus time y is O(b) | | | |

| 3 | It does constant amount of work, thus time complexity is O(1) | | | |
|----|--|--|--|--|
| 4 | The variable count will eventually equal a/b. The while loop iterates through count times. Thus time complexity is O(a/b) | | | |
| 5 | The algorithm is actually doing a binary search to find the square root. Thus the runtime is O(logN) | | | |
| 6 | This is straightforward loop that stops when guess*guess > n or guess> sqrt(n); hence time complexity is O(sqrt(n)) | | | |
| 7 | Ifa binary tree is not balanced, the max time to find an element would be the depth of the tree. The tree since it is imbalanced could be a straight list downwards and ave depth n. Thus, runtime is O(n) | | | |
| 8 | Without any ordering or sorting in a binary tree, we might need to traverse through all the nodes in the tree , thus the time complexity is O(n) where n is the number of nodes in the tree. | | | |
| 9 | The first call to appendToNew takes 1 copy. The second call takes 2 copies. The third call takes 3 copies. And so on. The total time will be sum through 1 to n, which is O(n^2) | | | |
| 10 | The runtime would be O(d) where d is the number of digits in the given number. A number with d digits can have a value upto 10^d. If n = 10^d, then d = log n. Thus, time complexity is O(log n) where the log is with base 10. | | | |
| 11 | If the length of the string is k, then to check if the string is inOrder or sorted , takes $O(k)$ time. Also, suppose the length of the string is c characters. Now, to get strings of c characters and k length would be $O(c^{\Lambda}k)$. for example, you wish to construct astring length 3 with just two characters a and b, thus the number of strings possible would be $2^{\Lambda}3$. Similarly here , that runtime would be $O(c^{\Lambda}k)$. Thus, overall runtime to get all the strings of k length with c characters and check if they are sorted would be $O(kc^{\Lambda}k)$. | | | |

| | First of all the runtime for mergeSort would be O(blogb) . then , for each element in a , we are doing binary search of b - runtime would be O(a * log b). hence overall runtime is O 12 (b log b + a log b). | | | | | | | |
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| Section IX | Interview Ques | tions | | | | | | |
| Chapter 1 | Arrays and Stri | | | | | | | |
| | Hash Table | | | | | | | |
| | | a data structure that maps keys cient lookup. | | | | | | |
| | Hash Table Imp | olementation | | | | | | |
| | Approach 1 | We use an array of Linked List | s and a hash code function. | | | | | |
| | | To insert a key (a string or any we follow the following steps: | other datatype) and value | | | | | |
| | Compute the key's hashcode or long. Two different keys coul as there may be numerous key | | ld have the same hashcode, | | | | | |
| | | 2. Then, we map the hash cod This could be done with somet array_length. Two different has to the same index. | hing like hash (key) % | | | | | |
| | | 3. At this index, there is a linke Store the key and value in the List to tackle collisions: you co with same hashcode or two dif index | index. We must use a Linked uld have two different keys | | | | | |
| | | | If it is the worst case, collisions are very high, runtime would be O(N) where N is the number of keys. And, if it is the best case, collisions are minimum, look up time would be O(1) | | | | | |
| | We can implement a look up system with a balanced b search tree. This gives us O(logN) lookup time. The advantage of this approach is potentially less space, s we no longer allocate a large array. We can also iterate through keys in order. | | ogN) lookup time. The potentially less space, since | | | | | |
| | ArrayList & Res | sizable Arrays | | | | | | |
| | A typical implementation is that when the array is full, the array-like datastructure with dynamic resizing, you should use an ArrayList. A typical implementation is that when the array is full, the array doubles in size (in Java, the size might instead increase to get an array of size N/8 ++ 2+ 1 = N. Therefore, elements takes O(N) worktotal. insertion on an average takes O though some insertions take O(worst case. | | | each capacity e N: N/2 + N/4 + e, inserting N I. Thus, each O(1), even | | | | |
| | StringBuilder | | | | | | | |

| | Normally, concatenating n strings of x characters each would take O (xn^2). | | | | | | | |
|-----------|--|---|---|---|--|--|--|--|
| | | n reduce this complexity as it cr ring them to one string only if ne | | | | | | |
| Chapter 2 | Linked Lists | | | | | | | |
| | LinkedList is a datastructure representing a sequence of nodes. | Singly Linked List> there is a pointer to the next node | Doubly Linked List> there is a pointer to the next and previous nodes | | | | | |
| | Unlike an array, LinkedList does not provide constant time access to any element of the list. It takes iterating through K elements to get the Kth element | | Benefit of a Linked List is that one can add or remove items from the beginning of the list in constant time | | | | | |
| Chapter 3 | Stacks and Que | eues | | | | | | |
| | Stack uses LIFO | Operations of a stack: pop(), push(item), peek(), isEmpty() | A stack does not offer constant-time access to the ith item. However, it allows constant time adds and removes as it does not require shifting elements around. | most useful case: recursive algorithms - one needs to push temporary data oto a stack as one recurses, but then remove them as one backtracks | | | | |