DAA

Tutorial - 4.

Harsha Saini
Sec- F
Class roll no - 08
B.tech (cse)

Practice Problem

1. $T(n) = 3T(\frac{n}{2}) + n^2$

$a = 3 \quad b = 2$

$c = \log_b a = \log_2 3 = 1.5 = n^{1.5}$

$f(n) = n^2$

$f(n) < n^c$

$\therefore O(n^2)$ Ans.

2. $T(n) = 4T(\frac{n}{2}) + n^2$

$a = 4 \quad b = 2$

$c = \log_2 4 = n^2$

$f(n) = n^c$

$\therefore O(n^2 \log n)$ Ans.

3. $T(n) = T(\frac{n}{2}) + 2^n$

$a = 1 \quad b = 2$

$c = \log_2 1 = 0 = n^0 = 1$

$f(n) > n^c$

$\rightarrow O(2^n)$

4. $T(n) = \boxed{2^n} T(\frac{n}{2}) + n^n$

↓

Invalid as a & b should be constant.

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16 \quad b = 4$

$c = \log_4 16 = 2 = n^2$

$n^c > f(n)$

$O(n^2)$ Ans.

6. $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2 \quad b = 2$

$c = \log_2 2 = 1 \Rightarrow n^1$

$f(n) > n^c$

$\therefore O(n\log n)$ Ans.

7. $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2$

$c = \log_2 2 = n^1$

$\therefore n^c > f(n)$

$\therefore O(n)$ Ans

8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4$

$c = \log_4 2 = n^{0.5}$

$f(n) > n^c$

$\therefore O(n^{0.51})$ Ans.

$T(n) = \boxed{0.5}\, T(n/2) + 1/n$

↓

Invalid

$a \geq 1$ & $b \geq 1$

$T(n) = 16\, T(n/4) + n!$

$a = 16 \qquad b = 4$

$c = \log_4 16 = n^2$

$T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4 \qquad b = 2$

$c = \log_2 4 = n^2$

$f(n) < n^c$

$\therefore O(n^2)$ Ans.

2. $T(n) = \boxed{\text{Sqrt}(n)}\, T(n/2) + \log n$

↓

a & b should be constant

a ≥ 1 & a ≥ 0

(13) $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2$

$c = \log_2 3 = n^{1.5}$

$n^c > f(n)$

$O(n^{1.5})$ Ans.

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3 \quad b = 3$

$c = \log_b a = \log_3 3 = 1 = n^1$

$n^c > f(n)$

$\therefore O(n)$ Ans.

(15) $T(n) = 4T(\frac{n}{2}) + cn$

$a = 4 \quad b = 2$

$c = \log_2 4 = n^2$

$n^c > f(n)$

$\therefore O(n^2)$ Ans.

(16) $T(n) = 3T(n/4) + n \log n$

$a = 3 \quad b = 4$

$c = \log_4 3 = 0.79 = n^{0.79}$

$\therefore f(n) > n^c$

$O(n \log n)$ Ans.

17. $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3$

$c = \log_3 3 = n^1$

$n^c > f(n)$

$\therefore O(n)$ Ans.

(18) $T(n) = 6T(\frac{n}{3}) + n^2 \log n$

$a = 6 \quad b = 3$

$c = \log_3 6 = 1.6 = n^{1.6}$

$f(n) > n^c$

$\therefore O(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

$a = 4 \quad b = 2$

$c = \log_2 4 = n^2$

$f(n) < n^c$

$\therefore O(n^2)$ Ans.

(20) $T(n) = 64T(n/8) - n^2 \log n$

$a = 64 \quad b = 8$

$c = \log_8 64 = n^2$ Can't apply.

(21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7 \qquad b = 3$

$c = \log_3 7 = n^{1.77}$

$f(n) > n^c$

$\therefore O(n^2)$ Ans.

(22) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

$a = 1 \qquad b = 2$

$c = \log_2 1 = n^0 = 1$

$f(n) > n^c$

$\therefore O(n(2 - \cos n))$ Ans.