

Q7.

```

void fun (int n)
{
    int j = 1, i = 0;
    while (i < n)
    {
        i = j;
        j++;
    }
}

```

$$j = 1$$

$$i = 1$$

$$j = 2$$

$$i = 1 + 2$$

$$j = 3$$

$$i = 1 + 2 + 3$$

$$i = 1 + 2 + 3 + \dots + m < n$$

$$\Rightarrow \frac{m(m+1)}{2} < n$$

$$\frac{m^2 + m}{2} < n \Rightarrow m^2 < \sqrt{n}$$

$$\Rightarrow m \approx \sqrt{n}$$

By summation method

$$\sum_{i=1}^m 1 \Rightarrow 1 + 1 + 1 + \dots + m = 1 + 1 + \dots + \sqrt{n}$$

$$= \sqrt{n}$$

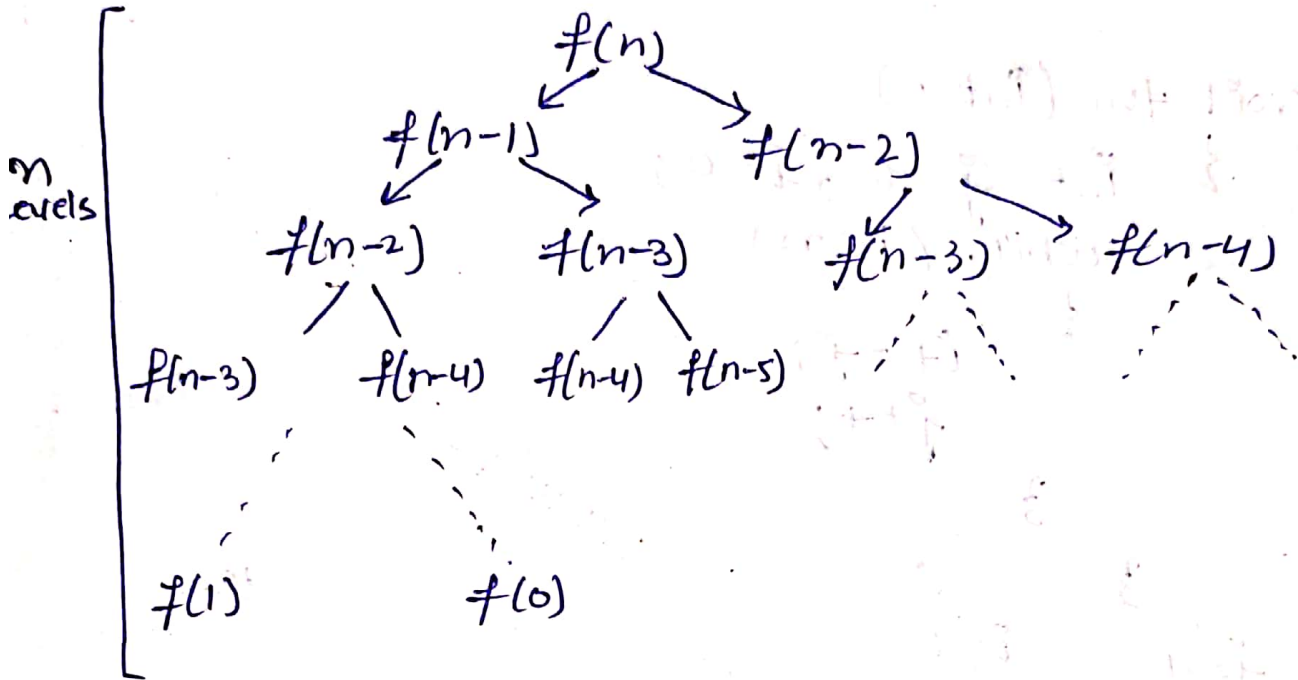
$$T(n) = \sqrt{n} \text{ Ans}$$

Insd. for fibonacci series

$$f(n) = f(n-1) + f(n-2)$$

$$f(0) = 0 \quad f(1) = 1$$

forming a tree



At every function call, we get 2 function calls for  $n$  levels,  $2 \times 2 \dots n$  times  $= 2^n$

$$T(n) = O(2^n) \quad \underline{\underline{\text{Ans}}}$$

Maximum space :- Space complexity depends on the maximum depth of the tree so

Space Complexity =  $O(n)$

Ans 3.

$$T(n) = O(n^3)$$

### Multiplication of two square matrix

```
for (i=0; i<81; i++)
```

```
{ for(j=0; j<C1; j++)
```

```
for(k=0; k<C1; k++)
```

$$\mathbb{E}[\mu(\sigma_j^2)] = a \sigma_j^2 + b \sigma_j^2$$
$$\{ \quad \}$$

$n \log n$

```
void quickSort (int arr[], int low, int high)
```

```
{  
    if (low < high)
```

```
{  
        int pi = partition(arr, low, high);
```

```
        quickSort(arr, low, pi-1);
```

```
        quickSort(arr, pi+1, high);  
    }
```

```
}
```

```
}
```

```
int partition (int arr[], int low, int high)
```

```
{  
    int pivot = arr[high];
```

```
    int i = (low-1);
```

```
    for (int j = low; j <= high-1; j++)
```

```
{  
        if (arr[j] < pivot)
```

```
{  
            i++;
```

```
            swap(arr[i], arr[j]);  
        }
```

```
}
```

```
}
```

```
    swap(arr[i+1], arr[high]);
```

```
    return (i+1);  
}
```

```
}
```

$\log(\log n)$

```
for (i=2; i < n; i = i*i)
```

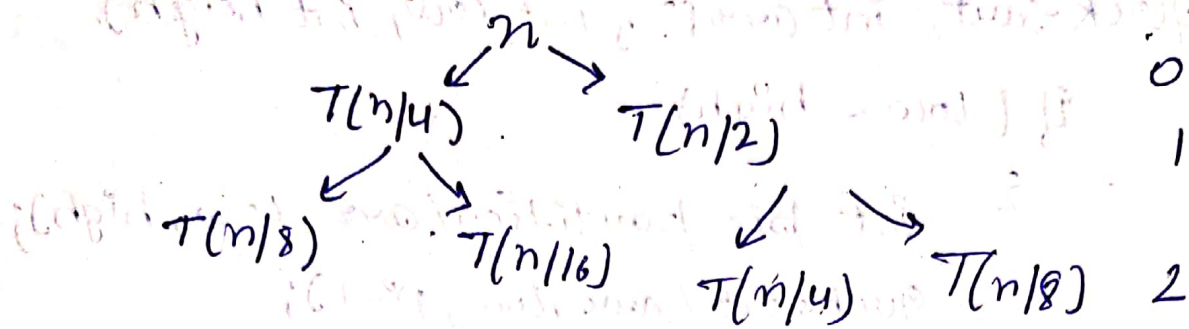
```
{
```

```
    count++;  
}
```

```
}
```



Ans 4.  $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + cn^2$



At Level 0  $\rightarrow cn^2$

$$1 \rightarrow \frac{n^4}{4^2} + \frac{n^2}{2^2} = c \frac{5n^4}{16}$$

$$2 \rightarrow \frac{n^2}{8^2} + \frac{n^4}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{2^2} = \left(\frac{5}{16}\right)^2 n^2 c$$

max levels =  $\frac{n}{2^k} = 1$  taking log both side

$$k = \log n$$

$$T(n) = c\left(n^2 + \frac{5}{16}n^2 + \left(\frac{5}{16}\right)^2 n^2 + \dots\right)$$

$$cn^2 \left[ 1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^{\log n} \right]$$

$$= cn^2 \times 1 \times \left( \frac{1 - \left(\frac{5}{16}\right)^{\log n}}{1 - \frac{5}{16}} \right)$$

$$= cn^2 \frac{16}{11} \left( 1 - \left(\frac{5}{16}\right)^{\log n} \right)$$

$$= O(n^2 c)$$

Ans.

```
int fun (int n)
{
    for (i=1; i<=n; i++)
    {
        for (j=1; j<n; j+=i)
            // do(1)
    }
}
```

|   |       |
|---|-------|
| i | j     |
| 1 | 1+3+5 |
| 2 | 1+4+7 |
| 3 | 1+5+9 |
| ⋮ |       |

$$\sum_{i=1}^n \frac{n-1}{i}$$

$$T(n) = \frac{n-1}{1} + \frac{n-1}{2} + \dots + \frac{n-1}{n}$$

$$= n \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right) - 1 \left[ 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right]$$

$$\therefore n \log n - \log n$$

$$= O(n \log n)$$

Ans 6.

$$\text{for } (i = 2; i \leq n; i = \text{Pow}(i, k))$$
$$\{$$
$$\quad \text{do } O(1)$$

Where  $k$  is constant

$$TC = 2, 2^k, 2^{k^2}, 2^{k^3} \dots 2^{k \log k (\log n)}$$

$$2^{k \log k (\log n)} = 2^{\log n} = n$$

So there are total

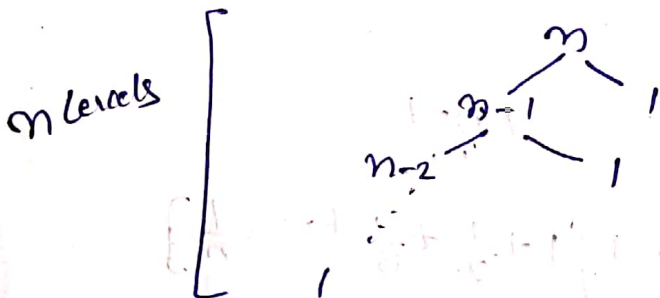
$\log_k (\log n)$  iterations

$$T(n) = O(\log_k \log n)$$

Ans 7.

Given algo divides array in 99% & 1% part

$$T(n) = T(n-1) + O(1)$$



$$T(n) = T(n-1) + T(n-2) + \dots + T(1) + O(1)$$

$$= n$$
$$T(n) = O(n)$$

Lowest height = 2

highest height =  $n$

diff =  $n-2$        $n \geq 1$

the given algorithm provides linear result.

Ans 8

Considering large values of  $n$

a)  $100 < \log \log n < \log n < (\log n)^2 < \sqrt{n} < n < n \log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

b)  $1 < \log \log n < \sqrt{\log n} < \log n < \log 2n < 2 \log n < n < n \log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

c)  $96 < \log_8 n < \log 2n < 5n < n \log_6 n < n \log_2 n < \log(n!) < 6n^2 < 7n^3 < n! < 8^{2^n}$

