Q1. What do you mean by Minimum Spanning Tree? What are the applications of MST?

Ans. Minimum Spanning Tree is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles & with minimum possible edge weighted.

APPLICATIONS →

i) Consider n stations are to be linked using a communication network and lying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.

ii) Designing LAN.

iii) Suppose you want to construct highways or railroads spanning several cities, then we can use concept of MST.

iv) Laying pipelines connecting offshore drilling sites, refineries & consumer markets.

Q2. Analyze time and space complexity of Prim, Kruskal, Dijkstra and Bellman Ford Algorithm.

Ans ⇒ Time Complexity of Prim's Algorithm : $O(|E| \log |V|)$

⇒ Space Complexity of Prim's Algorithm : $O|V|$

⇒ Time Complexity of Krushal's Algorithm : $O|E| \log |E|$

⇒ Space Complexity of Krushal's Algorithm : $O|V|$

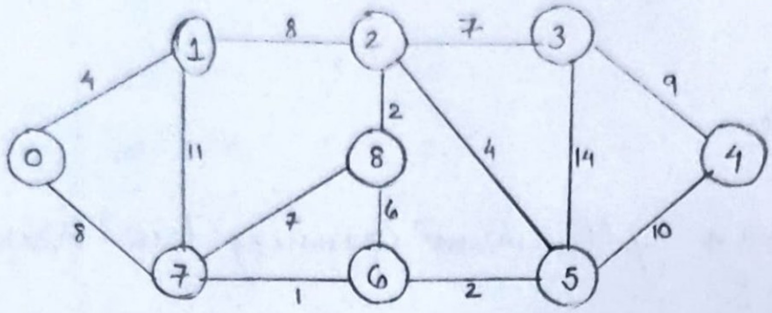
⇒ Time complexity of Dijkstra's Algorithm : $O(V^2)$

⇒ Space Complexity of Dijkstra's Algorithm : $O(V^2)$

⇒ Time Complexity of Bellman Ford's Algorithm : $O(VE)$

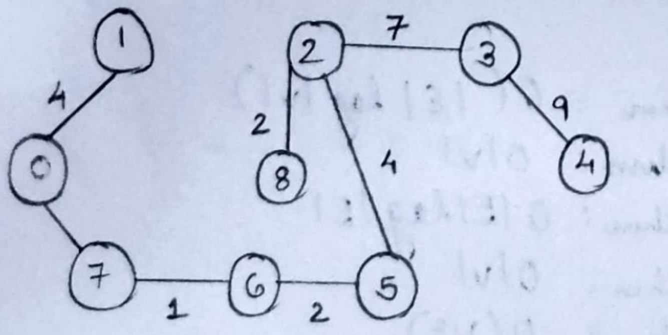
⇒ Space Complexity of Bellman Ford's Algorithm : $O(E)$

Q3) Apply Kruskal and Prim's Algorithm on given graph to compu
MST and its weight.



Ans. Kruskal's Algorithm :-

O	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X



Weight = 1+2+2+4+4+7+8+9
= 37

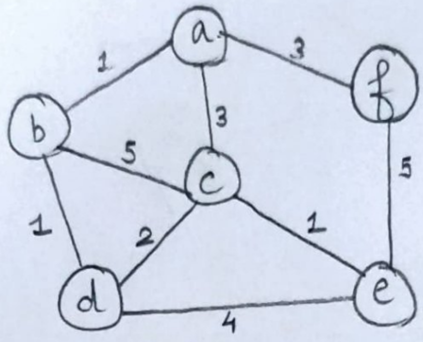
Prim's Algorithm

Weight = 4+8+2+~~4~~ 4+2+7+9+3
= 37

1. Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in following cases:

i) If weight of every edge is increased by 10 units.

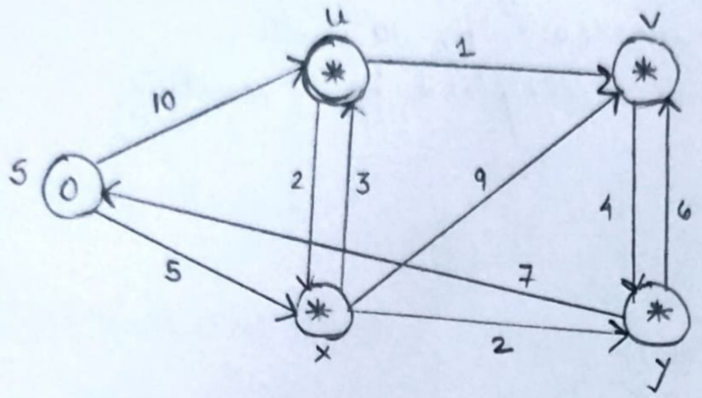
ii) If weight of every edge is multiplied by 10 units.



Ans i) The shortest path may change. The reason is that there may be different no. of edges in diffrent paths from 's' to 't'. For eg:- Let the shortest path of weight 15 and has edges 5. Let there we another path with 2 edges and total weight 25. The weight of shortest path is increased by 5*10 and becomes 15+50. Weight of other path is increased by 2*10 & becomes 25 + 20. So, the shortest path changes to other path with weight as 45.

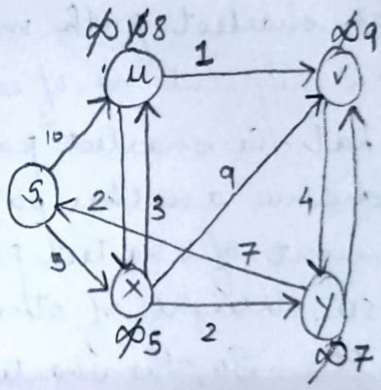
ii) If we multiply all edges weight by 10, the shortest path doesnot change. The reason is that weights of all path from 's' to 't' gets multiplied by same unit. The number of edges or path doesn't matter.

Q5. Apply Dijkstra & Bellman Ford algorithm on graph given right side to compute shortest path to all nodes from node S.



Ans. Dijkstra's Algorithm :-

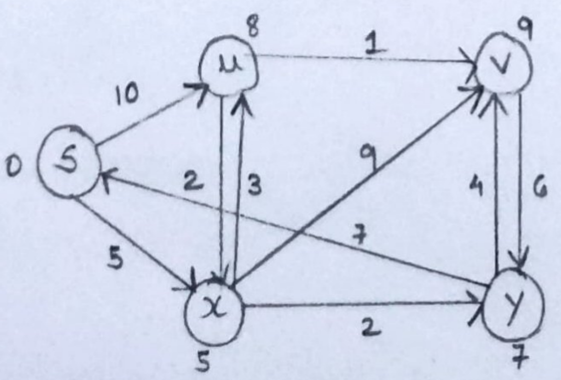
NODE	SHORTEST DIST FROM SOURCE NODE
u	8
x	5
v	9
y	7



Bellman Ford Algorithm →

	S	u	v	x	y
1st →	0	~~10~~ ∞	∞	~~∞~~ 5	∞
2nd →	0	10	~~∞~~ 11	5	∞
3rd →	0	~~10~~ 8	~~11~~ 9	5	~~∞~~ 7
4th →	0	8	9	5	7

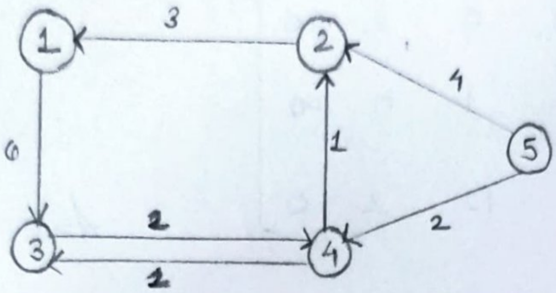
graph does not have negative cycle.



→ Final Graph

Q6) Apply all pair shortest path algorithm - Floyd Warshall on below mentioned graph. Also analyze space & time complexity of it.

Ans.



$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & \infty & \infty & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & \infty & 1 & 1 & 0 & \infty \\
5 & \infty & 4 & \infty & 2 & 0
\end{array}
$$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & \infty & 1 & 1 & 0 & \infty \\
5 & \infty & 4 & \infty & 2 & 0
\end{array}
$$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & 3 & 1 & 1 & 0 & \infty \\
5 & 6 & 4 & 12 & 2 & 0
\end{array}
$$

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
3 & 1 & 1 & 0 & \infty \\
6 & 4 & 12 & 2 & 0
\end{bmatrix}
\end{array}
$$

Ans.

Time Complexity → $O(|V|^3)$

Space Complexity → $O(|V|^2)$