# Equity Investment

## Module 49.1: Fixed-Income Instrument Features

### LOS 49.a: Features of a Fixed-Income Security

#### 1. Definition

- Fixed-income instruments represent **debt investments** in which investors lend capital to an issuer in exchange for promised interest (coupon) and repayment of principal (par value).
- Two major forms:
  1. **Loans:** Private, non-tradable debt agreements.
  2. **Bonds:** Standardized, tradable securities.

#### 2. Key Bond Features

#### 3. Coupon Structures

- **Fixed-rate bond:** Constant coupon based on stated rate.
- **Floating-rate note (FRN):** Coupon = Market Reference Rate (MRR) + fixed margin (spread).

  $$\text{Coupon Payment} = (\text{MRR} + \text{Margin}) \times \text{Par Value}$$

  Example: If MRR = 4%, Margin = 50 bps = 0.5%, Coupon = 4.5%.
- **Zero-coupon bond (pure discount):** No coupon; sold below par, pays par at maturity.

  $$\text{Price} = \frac{\text{Par}}{(1+y)^n}$$

  Example: 10-year \$1,000 bond at 7% $\Rightarrow P \approx \$508$.

#### 4. Yield and Price Relationship

$$P \downarrow \Rightarrow Y \uparrow, \quad P \uparrow \Rightarrow Y \downarrow$$

- Fixed cash flows $\Rightarrow$ inverse price-yield relationship.
- Yield reflects expected return based on current price and future cash flows.

Exhibit 1: Core Bond Features and Examples

| Feature | Definition | Example / Notes |
|---|---|---|
| **Issuer** | Entity issuing the bond | Governments (sovereign, local), corporations, supranationals (e.g., IMF, World Bank), or special purpose entities (SPEs). |
| **Maturity / Tenor** | Date or period until final payment. Bonds $\leq 1$ year = **Money Market**; $> 1$ year = **Capital Market**; perpetual bonds = no maturity. | A 10-year corporate bond matures in 2035. A perpetual bond (e.g., UK Consol) never matures. |
| **Principal (Par)** | Amount repaid at maturity; base for coupon calculation. | Typically \$1,000 or €1,000. Mortgage loans may amortize principal over time. |
| **Coupon Rate & Frequency** | Annual interest rate applied to par; defines periodic payments. | 5% annual coupon on \$1,000 = \$50/year. Semiannual = \$25 every 6 months. |
| **Seniority** | Ranking of debt repayment priority in liquidation. | Senior debt repaid before subordinated (junior) debt. |
| **Contingency Provisions** | Embedded options giving rights to issuer or bondholder. | Callable, putable, or convertible bonds. |

**5. Yield Curves**

- Graph plotting yield (%) vs. maturity.
- **Normal curve:** Upward sloping (long-term yields ¿ short-term yields).
- **Inverted curve:** Downward sloping (long-term yields ¡ short-term yields).
- **Benchmark:** Government yield curves used as reference for credit spreads.

**Example: Credit Spread**

$$\text{Corporate Yield} - \text{Gov't Yield} = 6\% - 5\% = 1\% \text{ spread}$$

A higher spread indicates higher credit or liquidity risk.

## LOS 49.b: Bond Indenture and Covenants

**1. Definition**

- The **bond indenture** (or trust deed) is the legal contract between the bond issuer and bondholders.

- Specifies all rights, obligations, and restrictions:
  - Bond features (coupon, maturity, par).
  - Sources of repayment.
  - Collateral and guarantees.
  - Covenants (affirmative and negative).

Exhibit 2: Sources of Bond Repayment by Issuer Type

| **Issuer Type** | **Repayment Source** | **Example** |
|---|---|---|
| **Sovereign Government** | Tax revenues and monetary powers (printing money). | U.S. Treasury or German Bunds. |
| **Local Government** | Local taxes or project revenues (e.g., tolls). | Municipal bonds repaid from property taxes. |
| **Corporations** | Operating cash flows; possibly backed by collateral. | Secured (mortgage-backed) vs. unsecured (debenture). |
| **Special Purpose Entity (SPE)** | Cash flows from underlying assets (ABS/MBS). | Mortgage-backed securities. |

**2. Sources of Repayment**

**3. Secured vs. Unsecured Debt**

- **Secured Bonds:** Have legal claim (lien/pledge) on specific assets (collateral).
- **Unsecured Bonds (Debentures):** Backed only by issuer's overall creditworthiness.

**4. Types of Covenants**

**5. Incurrence Test Example**

$$\text{Restriction: New debt or dividend allowed only if } \frac{\text{Debt}}{\text{EBITDA}} < 3.5$$

Protects bondholders by limiting financial risk.

**6. Covenant Balance**

- **Goal:** Protect bondholders without overly restricting issuer flexibility.
- Too lenient $\rightarrow$ increased default risk.
- Too restrictive $\rightarrow$ limits operational agility.

Exhibit 3: Bond Covenant Comparison

| Covenant Type | Definition / Purpose | Examples |
|---|---|---|
| **Affirmative (Positive)** | Specify actions issuer **must perform**. | • Provide audited financial statements.<br>• Maintain insurance and compliance with laws.<br>• Use proceeds for declared purpose.<br>• **Cross-default clause:** Default on any other debt = default here.<br>• **Pari passu clause:** Equal ranking with other senior debt. |
| **Negative (Restrictive)** | Restrict actions that could increase credit risk. | • Limitations on additional debt.<br>• Restrictions on asset sales, leasebacks, and collateral pledges.<br>• **Negative pledge:** Cannot issue more senior secured debt.<br>• Dividend or share buyback limits (based on leverage or coverage ratios). |

## LOS Integration: Yield Curve & Indenture Context

### 1. Yield Curve Insights

- Provides benchmark yields across maturities.
- Corporate spreads over government curve reflect perceived credit risk.

### 2. Credit Structure Summary

## Example Summary Calculations

### 1. Fixed Coupon Example

$$\text{Coupon Payment} = 0.05 \times \$1{,}000 = \$50 \text{ per year}$$

Exhibit 4: Bond Hierarchy and Claim Priority

| **Type** | **Priority / Risk Level** |
|---|---|
| Senior Secured Debt | First claim on specific assets (lowest credit risk). |
| Senior Unsecured Debt | Claim on issuer's general assets. |
| Subordinated (Junior) Debt | Paid after senior claims; higher yield required. |
| Equity (Common / Preferred) | Residual claim only; highest risk, highest expected return. |

Semiannual = \$25 every 6 months.

**2. Zero-Coupon Example**

$$P_0 = \frac{1{,}000}{(1.07)^{10}} = \$508$$

Investor earns entire yield from price appreciation at maturity.

**3. Floating-Rate Example**

$$\text{Coupon Rate} = \text{MRR} + 1.25\%$$

If MRR = 3.75% → Coupon = 5.00% → Payment = 0.05 × \$1,000 = \$50.

## Key Takeaways Summary

Exhibit 5: Module 49.1 Summary Overview

| **LOS** | **Core Insights** |
|---|---|
| 49.a | Fixed-income securities define issuer, maturity, par, coupon, seniority, and options. Prices and yields move inversely. Yield curves plot yield vs. maturity. |
| 49.b | The bond indenture defines repayment sources, collateral, and covenants. Affirmative covenants require issuer actions; negative covenants restrict risk-increasing actions. Proper covenant balance protects bondholders. |

**Conceptual Summary:**

- Bonds = legally binding debt with predictable cash flows.
- Issuer's credit quality, maturity, and covenants define risk-return tradeoff.
- Yield curves and spreads reflect market perception of credit risk.
- The bond indenture legally protects investors through covenants and collateral provisions.

# Module 50.1: Fixed-Income Cash Flows and Types

## LOS 50.a: Common Cash Flow Structures and Contingency Provisions

### 1. Overview

- Fixed-income instruments differ by **cash flow timing, composition, and embedded options**.
- Main structures:
  1. Bullet (single repayment at maturity)
  2. Amortizing (principal repaid over time)
  3. Partially amortizing (balloon payment)
  4. Sinking fund or structured (ABS/MBS waterfalls)

### 2. Bullet Structure

- Interest (**coupon**) paid periodically; principal repaid fully at maturity.
- Example: \$1,000 par, 5% annual coupon, 5-year maturity.

$$\text{Coupon Payment} = 0.05 \times 1{,}000 = \$50 \text{ annually}$$

Exhibit 1: Bullet Bond Cash Flow Schedule

| Year | Coupon (\$) | Principal (\$) |
|---|---|---|
| 1–4 | 50 | 0 |
| 5 | 50 | 1,000 |

### 3. Fully Amortizing Loan

- Each payment includes both interest and principal.
- Final payment reduces principal to zero.
- Example parameters: $N = 5, I/Y = 5\%, PV = 1,000, FV = 0$

$$\text{PMT} = 230.97$$

Exhibit 2: Fully Amortizing Bond Example ($1,000, 5%, 5 years)

| Year | Payment | Interest | Principal Repay. |
|---|---|---|---|
| 1 | 230.97 | 50.00 | 180.97 |
| 2 | 230.97 | 40.95 | 190.02 |
| 3 | 230.97 | 31.45 | 199.52 |
| 4 | 230.97 | 21.48 | 209.49 |
| 5 | 230.97 | 10.47 | 220.50 |

**4. Partially Amortizing (Balloon Payment)**

- Partial principal repayment with final "balloon" at maturity.
- Example parameters: $N = 5$, $I/Y = 5\%$, $PV = 1,000$, $FV = -200$

$$\text{PMT} = 194.78$$

**5. Sinking Fund and Waterfall Structures**

- **Sinking Fund:** Issuer repays fixed portions of principal over time.

  Example: $300 m issue, redeem $20m annually from Year 6–20.

  - **Advantage:** Lower credit risk (gradual repayment).
  - **Disadvantage:** Reinvestment risk if rates fall (cash returned early).

- **Waterfall (ABS/MBS):** Cash flows distributed hierarchically by tranche.
  - Senior tranches repaid first.
  - Junior tranches absorb default losses.

**6. Coupon Structures Overview**

**7. Inflation-Linked Bonds**

- **Interest-Indexed:** Coupon adjusts for inflation; principal fixed.
- **Capital-Indexed (TIPS):** Coupon fixed; principal adjusts by inflation rate.
- Example: 3% coupon, semiannual, inflation +1%:

$$\text{New Principal} = 1{,}000(1.01) = 1{,}010, \quad \text{Semiannual Coupon} = 1.5\% \times 1{,}010 = 15.15$$

- Protects real purchasing power: real rate = 3%.

Exhibit 3: Coupon Structures

| Type | Definition | Example / Note |
|---|---|---|
| **Fixed-Rate** | Coupon fixed at issue date. | Most common. Predictable cash flows. |
| **Floating-Rate (FRN)** | Coupon = MRR + Margin (spread). | Quarterly coupon: (2.3% + 0.75%)/4 = 0.7625% of par. |
| **Step-Up** | Coupon increases by schedule. | Protects investors if rates rise. |
| **Credit-Linked** | Coupon adjusts with credit rating. | MRR + 2.5% → MRR + 3% if leverage ¿ 3×. |
| **PIK (Payment-in-Kind)** | Interest paid by issuing more debt instead of cash. | Used by highly leveraged issuers; high yield. |
| **Green Bonds** | Coupon rises if environmental targets not met. | Example: $CO_2$ reduction failure ⇒ coupon +25bps. |
| **Inflation-Linked** | Coupon or principal linked to CPI or similar. | Protects real returns; e.g., U.S. TIPS. |

**8. Deferred and Zero-Coupon Bonds**

- **Zero-Coupon:** Single payment at maturity; trades below par.
- **Deferred Coupon:** No interest initially; coupons begin later.
- Used by issuers expecting delayed revenue (e.g., project financing).

**9. Embedded Contingency Provisions (Embedded Options)**

- **Callable Bonds:** Option to issuer.
- **Putable Bonds:** Option to investor.
- **Convertible Bonds:** Option to investor to convert to shares.
- Bonds without options = **straight bonds**.

**Callable Bonds**

- Issuer may redeem early at preset price (call price).
- **Example:**
  - Callable after 5 years at 102% of par.
  - Price cap in market ⇒ higher yield, lower price.
- **Call Risk:** Reinvestment risk for investors when rates fall.

If yield falls from 6% to 4%, issuer can refinance at lower cost.

**Putable Bonds**

- Bondholder can sell bond back at par before maturity.
- Protects investor when yields rise or credit worsens.
- **Put Option Value** $\Rightarrow$ higher price, lower yield vs. straight bond.

**Convertible Bonds**

- Bondholder may convert bond into equity.
- **Formulas:**

$$\text{Conversion Ratio} = \frac{\text{Par Value}}{\text{Conversion Price}}$$

$$\text{Conversion Value} = (\text{Share Price}) \times (\text{Conversion Ratio})$$

- **Example:** \$1,000 bond, conversion price = \$40:

$$\text{Ratio} = 25 \text{ shares}, \quad \text{Share Price} = 50 \Rightarrow V_c = 25 \times 50 = 1{,}250$$

**Warrants**

- Detachable right to buy stock at a fixed price.
- Example: Warrant strike \$40 $\rightarrow$ value if share price $> 40$.

**Contingent Convertible Bonds (CoCos)**

- Convert to equity automatically if trigger (e.g., bank's capital ratio) is breached.
- Used by banks to meet regulatory capital requirements.
- Benefit issuer: automatic recapitalization without new issue.

## LOS 50.b: Legal, Regulatory, and Tax Considerations

### 1. Bond Market Classifications

### 2. Eurobond Features

- Typically less regulated; may be bearer or registered form.
- **Naming convention:** by currency (Eurodollar, Euroyen, etc.).
- Currency denomination primarily determines yield differences due to domestic interest rates.

Exhibit 4: Bond Issuance Categories

| Type | Definition | Example |
|---|---|---|
| **Domestic Bond** | Issued and traded in issuer's home market. | Deutsche Bank issuing euro bonds in Germany. |
| **Foreign Bond** | Issued by foreign entity in domestic market. | Toyota issues USD bonds in the U.S. (Yankee bond). |
| **Eurobond** | Issued outside jurisdiction of any single country, any currency. | Chinese firm issues yen-denominated Euroyen bonds in London. |
| **Global Bond** | Trades in both domestic and Eurobond markets. | Apple global USD bond traded in U.S. and London. |

**3. Islamic (Sukuk) Bonds**

- Must comply with Sharia (no interest).
- Investors earn **rent income** from underlying tangible assets.
- Funds must be used for permissible (halal) purposes.

**4. Taxation Considerations**

- **Interest Income:** Taxed as ordinary income.
- **Municipal Bonds (U.S.):** Usually exempt from federal and in-state taxes.
- **Capital Gains:** Taxed upon sale; lower rate for long-term holdings.
- **Original Issue Discount (OID):**
  - Zero-coupon or deep-discount bonds.
  - Accreted interest taxed annually even with no cash payment.
- **Premium Bonds:** Amortized premium can reduce taxable coupon income.

## Key Takeaways Summary

**Conceptual Summary:**

- **Structure defines timing of cash flows.**
- **Coupon design** affects sensitivity to rates, inflation, and credit events.
- **Embedded options** shift risk–reward between issuer and holder.
- **Legal jurisdiction** impacts regulation, taxation, and yield levels.

Exhibit 5: Module 50.1 Summary Overview

| LOS | Core Insights |
|---|---|
| 50.a | Bonds differ by cash flow structure (bullet, amortizing, sinking fund) and coupon design (fixed, floating, step-up, PIK, inflation-linked). Embedded options (call, put, convertibility) redistribute value between issuer and investor. |
| 50.b | Legal/regulatory frameworks define issuance categories (domestic, foreign, Eurobond, global). Tax regimes affect after-tax returns. Sukuk and OID bonds have specialized treatment. |

# Module 51.1: Fixed-Income Issuance and Trading

## LOS 51.a: Fixed-Income Market Segments and Participants

**1. Global Bond Market Segmentation** Bond markets can be segmented by several criteria:

- **Type of Issuer:** Governments, Corporates, and Special Purpose Entities (SPEs) issuing Asset-Backed Securities (ABSs).
- **Credit Quality:** Investment-grade vs. High-yield (speculative/junk).
- **Maturity:** Short-term ( 1 year), Intermediate (1–10 years), Long-term (¿ 10 years).
- **Other Classifications:** Currency, Geography, and ESG features.

Exhibit 1: Issuer Types and Features

| Issuer Type | Description | Example / Notes |
|---|---|---|
| **Sovereign Governments** | National governments issuing Treasuries or sovereign bonds. | U.S. Treasury, German Bunds; lowest credit risk. |
| **Non-Sovereign Governments** | Regional or municipal authorities. | U.S. municipal bonds, German Länder bonds. |
| **Corporates** | Private sector issuers for funding operations, acquisitions, or investment. | Investment-grade: Apple Inc. / High-yield: Ford Motor Co. |
| **Special Purpose Entities (SPEs)** | Issue Asset-Backed or Mortgage-Backed Securities (ABS/MBS). | Cash flows come from underlying financial assets (e.g., auto loans, mortgages). |

**2. Issuer Categories**

Exhibit 2: Credit Rating Tiers

| Category | S&P | Moody's | Comment |
|---|---|---|---|
| Investment Grade | AAA–BBB– | Aaa–Baa3 | Suitable for institutional portfolios. |
| High Yield / Speculative | BB+ and below | Ba1 and below | Also called "junk bonds." |
| Fallen Angels | Downgraded from IG to HY | — | Example: BBB → BB downgrade. |

**3. Credit Quality Classifications**

**4. Maturity Segments**

- **Money Market:** 1 year (e.g., Treasury bills, commercial paper, repos).
- **Intermediate-Term:** 1–10 years (e.g., notes, mid-term corporate debt).
- **Long-Term:** ¿ 10 years (e.g., Treasury bonds, corporate debentures, mortgages).

**5. Example: Corporate Issuance Across Maturities**

- Short-term → **Commercial Paper** for seasonal working capital.
- Medium-term → **Bank Syndicated Loans** for equipment or expansion.
- Long-term → **Bond Issuance** for capital investment projects.

Exhibit 3: Investor Positioning by Risk and Maturity

| Investor Type | Preferred Segment | Rationale / Objective |
|---|---|---|
| **Pension Funds / Insurers** | Long-term, investment grade | Match long-term liabilities (pensions, insurance claims). |
| **Corporations** | Short-term, high credit quality | Manage working capital, excess liquidity (e.g., CP, repos). |
| **Central Banks** | Intermediate-term sovereign bonds | Use for monetary policy and liquidity control. |
| **Bond Funds / ETFs** | Intermediate, investment grade (ex-Treasuries) | Diversified fixed-income portfolios for stable returns. |
| **Asset Managers / Hedge Funds** | Intermediate, high-yield or distressed | Seek higher yield via credit risk. |
| **Banks / Financial Intermediaries** | Treasuries across maturity spectrum | Manage interest rate and liquidity risk. |

### 6. Investor Positioning Along the Credit/Maturity Spectrum

## LOS 51.b: Types of Fixed-Income Indexes

### 1. Key Distinctions from Equity Indexes

1. **More Constituents:** A single issuer can have hundreds of bond issues.
2. **Higher Turnover:** Due to frequent issuance, maturities, and redemptions.
3. **Sector Weights Shift:** Government and corporate issuance patterns affect index weights over time.

Exhibit 4: Major Fixed-Income Index Types

| Index Type | Description | Examples |
|---|---|---|
| **Aggregate Index** | Broad-based; includes multiple sectors, maturities, and currencies. | Bloomberg Barclays Aggregate Index (excludes high-yield/unrated). |
| **Geographic Index** | Bonds from specific countries or regions. | ICE BofA U.S. Corporate Index, Euro Aggregate Index. |
| **Credit Quality Index** | Classified by rating (IG vs. HY). | Bloomberg U.S. High Yield Index. |
| **Sector Index** | Focused on specific issuers (corporate, sovereign, ABS). | JPMorgan Emerging Market Bond Index Plus. |
| **ESG / Sustainable Index** | Exclude certain industries; minimum ESG score. | Bloomberg Barclays MSCI Euro Corporate Sustainable SRI Index. |

### 2. Aggregate and Sector Indexes

### 3. ESG-Integrated Index Example

- Excludes firms in non-sustainable sectors (e.g., alcohol, coal).
- Requires **min credit rating = BBB– (Baa3)** and **min ESG rating = BBB**.

### 4. Index as Benchmark

- Benchmark must align with portfolio exposure:
  - Sector, maturity, currency, and credit quality.
  - Example: Emerging-market fund → JPM EMBI+ benchmark.

## LOS 51.c: Primary vs. Secondary Markets

### 1. Primary Market

- **Definition:** Issuance of new bonds → issuer receives new capital.
- **Two Forms:**
  1. **Public Offering:** Registered with regulator; open to all investors.
  2. **Private Placement:** Sold to select investors (e.g., institutions).

Exhibit 5: Primary Market Issuance Types

| Method | Description | Example / Implication |
|---|---|---|
| **Underwritten Offering** | Investment bank guarantees sale price to issuer. | Used by large investment-grade corporates; lower risk for issuer. |
| **Best-Efforts Offering** | Bank sells bonds on commission basis (no guarantee). | Used for smaller or high-yield issues; higher issuance risk. |
| **Shelf Registration** | Bonds pre-approved by regulator, issued over time. | Flexible funding for frequent issuers. |
| **Auction (Government)** | Bonds sold to bidders in competitive/noncompetitive auctions. | Common for Treasuries and sovereign debt. |

### 2. Issuance Methods

### 3. Example: Debut vs. Repeat Issuer

- **Debut Issue:** Requires marketing roadshows; time-intensive.
- **Repeat Issue:** For frequent issuers with shelf registrations, issuance may complete in hours.

### 4. Secondary Market

- **Definition:** Trading of existing bonds among investors.
- **Structure:** Primarily **dealer (OTC) markets**, not exchanges.
- **Dealer Quote:** Bid (buy) and Ask (sell) prices → spread = dealer's margin.

### 5. Bid–Ask Spread by Liquidity

Exhibit 6: Liquidity and Spreads

| **Bond Type** | **Typical Spread (bps)** | **Liquidity Note** |
|---|---|---|
| On-the-run Sovereigns | $< 1$ | Very liquid; high trading volume. |
| Investment-Grade Corporate | 2–5 | Actively traded, lower risk. |
| High-Yield / Seasoned Bonds | 10–20+ | Less liquid; wider spreads. |
| Distressed Debt | 50–200+ | Illiquid; speculative trading. |

**6. Distressed Debt Market**

- Bonds of issuers near or in bankruptcy.
- Bought by specialized investors (hedge or distress funds).
- Value from potential recovery or restructuring.
- Temporary surge in trading volume when distress emerges.

## Key Takeaways Summary

Exhibit 7: Summary of Module 51.1 LOS Concepts

| **LOS** | **Core Insights** |
|---|---|
| 51.a | Bond markets segmented by issuer, credit quality, and maturity. Issuers: sovereigns, corporates, SPEs. Investors allocate based on desired risk, return, and liability matching. |
| 51.b | Fixed-income indexes differ from equities: more constituents, higher turnover, and sector shifts. Aggregate and ESG indexes cover global, regional, or sector exposures. |
| 51.c | Primary market = issuance (underwritten, best-efforts, shelf, auction). Secondary market = OTC trading with bid–ask spreads by liquidity. Distressed debt involves high-risk, high-return repositioning. |

**Conceptual Summary:**

- **Issuers:** Governments, Corporates, SPEs.
- **Investors:** Position along maturity and risk curve based on objectives.
- **Indexes:** Track market segments; used for benchmarking.
- **Primary Market:** Capital-raising via offerings or auctions.
- **Secondary Market:** Liquidity via dealer networks.
- **Liquidity Indicator:** Bid–ask spread $\uparrow \rightarrow$ liquidity $\downarrow$.

# Module 52.1: Fixed-Income Markets for Corporate Issuers

## LOS 52.a: Short-Term Funding Alternatives

**1. Overview** Corporations and financial institutions use short-term financing to meet **working capital** needs, manage liquidity, or bridge to long-term funding. Funding can be classified as:

- **Loan-based financing (bank credit lines)**
- **Security-based financing (commercial paper, repos, ABCP)**

### A. Short-Term Funding for Nonfinancial Corporations

Exhibit 1: Bank Lines of Credit

| Type | Definition / Features | Example / Notes |
|---|---|---|
| **Uncommitted Line of Credit** | Bank offers short-term funds at MRR + spread but may refuse to lend. Flexible but unreliable. | Suitable for stable firms maintaining deposits with bank. No commitment fees. |
| **Committed Line of Credit** | Bank commits funds for a defined period (e.g., 1 year). Borrower pays a commitment fee ($\approx 50$ bps). | Reliable; subject to renewal risk. Syndicated among banks to spread risk. |
| **Revolving Line of Credit (Revolver)** | Multi-year facility allowing repeated borrowing and repayment. | More reliable; includes restrictive covenants (e.g., leverage ratios). |

**2. External Loan Financing**

**3. Secured Loans and Factoring**

- **Secured Loan:** Borrower pledges collateral (e.g., receivables, inventory, fixed assets).
- **Factoring:** Sale of receivables to a "factor" at a discount $\rightarrow$ immediate cash inflow.

$$\text{Discount} = \text{Face Value} - \text{Proceeds}$$

- Discount reflects customer credit quality + collection costs.

### 4. Security-Based Financing (Commercial Paper)

- **Commercial Paper (CP):** Unsecured, short-term promissory notes issued by highly rated firms.
- **Maturity:** Usually $< 270$ days (often 30–90 days).
- **Cost:** Lower than bank loans (no intermediation margin).
- **Rollover Risk:** Risk that CP cannot be refinanced at maturity.
- **Mitigation:** Backup credit lines (*liquidity enhancement lines*) with banks.

**Example: CP Issuance**

$$\text{Face Value} = \$10{,}000{,}000, \quad \text{Term} = 90 \text{ days}, \quad y = 5\%$$

$$P = 10{,}000{,}000 \times \left(1 - 0.05 \times \frac{90}{360}\right) = 9{,}875{,}000$$

Issuer receives \$9.875m today, repays \$10m at maturity.

### 5. Eurocommercial Paper (ECP)

- Issued in international markets outside home jurisdiction.
- Typically denominated in major currencies (USD, EUR, GBP, JPY).

## B. Short-Term Funding for Financial Institutions

Exhibit 2: Bank Deposit Types

| Deposit Type | Description | Notes |
|---|---|---|
| **Demand Deposits** | Checking accounts; withdrawable anytime; usually non-interest-bearing. | Provide transaction services and liquidity. |
| **Operational Deposits** | Large clients' deposits used for clearing/custody. | Integral to daily operations. |
| **Savings Deposits / CDs** | Pay interest; fixed maturity. | **Nonnegotiable CD:** cannot be sold before maturity. **Negotiable CD:** tradable before maturity. |

### 6. Deposit Funding

**7. Interbank and Central Bank Funding**

- **Interbank Funds:** Loans between banks (1 day–1 year) at interbank MRR (e.g., SOFR, €STR).
- **Central Bank Funds Market:** Banks lend/borrow reserves held at central bank.
- **Discount Window:** Lender of last resort facility; higher rate and regulatory scrutiny.

**8. Asset-Backed Commercial Paper (ABCP)**

1. Financial institution sells loans to a Special Purpose Entity (SPE).
2. SPE issues short-term securities backed by these loans.
3. Investors buy ABCP; SPE repays them with loan cash flows.
4. Sponsor bank provides liquidity line or credit enhancement.

## LOS 52.b: Repurchase Agreements (Repos)

**1. Definition**

- A **repo** = sale of securities with an agreement to repurchase later at higher price.
- Economically = short-term **collateralized loan**.

**2. Key Formulas**

$$\text{Repo Rate} = \frac{P_{\text{repurchase}} - P_{\text{initial}}}{P_{\text{initial}}} \times \frac{360}{\text{Days}}$$

$$\text{Initial Margin} = \frac{\text{Collateral Value}}{\text{Loan Amount}} \times 100\%$$

$$\text{Haircut} = 1 - \frac{1}{\text{Initial Margin}}$$

**Example: Repo Calculation**

- Collateral Value = \$1,000,000
- Initial Margin = 103%
- Repo Rate = 2%, Term = 90 days

$$\text{Purchase Price} = \frac{1{,}000{,}000}{1.03} = 970{,}874$$

$$\text{Repurchase Price} = 970{,}874[1 + 0.02(90/360)] = 975{,}728$$

$$\text{Haircut} = 1 - \frac{970{,}874}{1{,}000{,}000} = 2.91\%$$

**3. Margining**

- If collateral value $\downarrow \rightarrow$ lender requests **variation margin**.
- If collateral $\uparrow \rightarrow$ borrower may request release of excess collateral.

Exhibit 3: Types of Repo Agreements

| Type | Description | Comment |
|---|---|---|
| **Overnight Repo** | Maturity = 1 day. | Used for daily liquidity management. |
| **Term Repo** | Maturity ¿ 1 day (up to 1 year). | Common durations: 1 week, 1 month, 3 months. |
| **General Collateral Repo** | Collateral = any eligible security. | Common for liquidity operations. |
| **Special Repo** | Specific hard-to-borrow security as collateral. | Repo rate may be very low or even negative. |
| **Tri-Party Repo** | Third-party custodian manages collateral. | Reduces settlement margining risk. |
| **Bilateral Repo** | Two-party direct agreement. | Greater counterparty risk. |

**4. Repo Types**

**5. Repo Applications**

- **Borrowers:** Finance trading positions.
- **Lenders:** Earn repo rate on excess liquidity.
- **Central Banks:** Monetary policy tool.
- **Short Sellers:** Borrow securities for short sale via reverse repo.

**6. Repo Rate Drivers**

- **Higher** when: term longer, collateral quality lower, undercollateralized.
- **Lower** when: collateral in high demand, overcollateralized, short term.

**7. Risks**

- Default risk (counterparty fails to repurchase).
- Collateral risk (decline in collateral value).

- Margining risk (incorrect/late margin).
- Legal risk (unenforceable contract).
- Netting/settlement risk (cross-contract payment failures).

## LOS 52.c: Long-Term Funding—Investment Grade vs. High Yield

### 1. General Observations

- **Normal Yield Curve:** Longer maturity → higher yield.
- Spread between short vs. long maturity is **wider for high-yield** issuers.

Exhibit 4: Investment Grade vs. High Yield Debt Characteristics

| Feature | Investment Grade | High Yield |
|---|---|---|
| **Default Risk** | Low; focus on downgrade risk. | High; focus on default and recovery rate. |
| **Credit Spread Component** | Small portion of total yield. | Dominant component of total yield. |
| **Covenants** | Limited, mainly restricting liens and sale/leaseback. | Extensive; include leverage and dividend restrictions. |
| **Collateral Requirement** | Typically unsecured. | Often secured or asset-backed. |
| **Maturity** | Broad range, often ¿10 years. | Shorter, typically 10 years. |
| **Standardization** | Highly standardized, benchmark issues. | Customized; less standardized, higher legal complexity. |
| **Flexibility** | Easy to refinance when rates fall. | Limited; higher refinancing cost. |
| **Optionality** | Few embedded options. | Often callable or prepayable. |
| **Return Profile** | Bond-like, predictable. | Equity-like; higher volatility. |

### 2. Comparison Table

### 3. Example: Yield Composition

$$\text{Yield}_{IG} = \text{Risk-free} + \text{Spread}_{IG} = 3\% + 1\% = 4\%$$

$$\text{Yield}_{HY} = \text{Risk-free} + \text{Spread}_{HY} = 3\% + 5\% = 8\%$$

### 4. Callable High-Yield Bonds

- Allow issuer to redeem early when credit improves.
- Benefit to issuer: refinancing flexibility.
- Cost to investor: reinvestment and call risk → higher required yield.

**5. Summary of Funding Strategy**

- **Investment Grade:**
  - Long-term maturities.
  - Broad investor base.
  - Benchmark curve issuance.
- **High Yield:**
  - Shorter-term maturities.
  - Heavier covenant and collateral structures.
  - Issuance driven by market timing and refinancing needs.

## Key Takeaways Summary

Exhibit 5: Module 52.1 Summary Overview

| LOS | Core Insights |
|---|---|
| 52.a | Nonfinancial firms: credit lines, secured loans, factoring, commercial paper. Financial institutions: deposits, CDs, interbank funds, central bank borrowing, and ABCP. |
| 52.b | Repos are collateralized loans; repo rate determined by collateral quality, term, and market liquidity. Tri-party repos mitigate margining and settlement risk. |
| 52.c | Investment-grade debt = longer maturities, fewer covenants, lower spreads. High-yield debt = shorter maturities, higher spreads, more covenants, often callable. |

**Conceptual Summary:**

- **Short-term corporate funding** = flexibility vs. reliability trade-off.
- **Repos** provide efficient, low-cost, collateralized funding.
- **Credit quality** drives structure, maturity, and investor protections in long-term debt markets.

# Module 53.1: Fixed-Income Markets for Government Issuers

## LOS 53.a: Funding Choices by Sovereign, Nonsovereign, Quasi-Government, and Supranational Entities

### 1. Overview of Sovereign Debt

- **Definition:** Bonds issued by national governments to finance expenditures on public goods, infrastructure, and fiscal deficits.
- **Backing:** Full faith and credit of the government, supported by **tax-raising power**.
- **Issuer Profile:** Largest and typically highest credit quality issuer within domestic markets.

### 2. Accounting and Economic Perspective

- Government financial reports use **cash-based accounting** rather than accruals.
- Analysts should view governments through an **"economic balance sheet"**:
  - **Implied Assets:** Future tax revenues.
  - **Implied Liabilities:** Future social expenditures and obligations.

Exhibit 1: Comparison of Developed and Emerging Market Sovereign Issuers

| Aspect | Developed Market Sovereign | Emerging Market Sovereign |
|---|---|---|
| Economic Base | Diversified, stable, transparent fiscal policy. | Concentrated, volatile, often commodity-dependent. |
| Currency | Reserve currency (e.g., USD, EUR, RMB). | Domestic or foreign reserve currency. |
| Debt Type | Domestic debt dominates. | Mix of domestic and external debt. |
| Investor Base | Predominantly domestic. | Often foreign due to limited domestic capacity. |
| Fiscal Stability | Predictable, credible. | Sensitive to commodity cycles and exchange rates. |
| Credit Rating | High (AAA to A). | Variable; may face downgrade or restructuring risk. |

### 3. Developed vs. Emerging Market Sovereigns

**4. Types of Sovereign Debt**

- **Domestic Debt:** Issued in local currency to domestic investors.
  - May face *liquidity* and *convertibility* constraints.
- **External Debt:** Issued to foreign investors, denominated in domestic or foreign (reserve) currency.
  - Investors face **indirect currency risk** — repayment depends on the issuer's ability to generate foreign currency.

**5. Fiscal Policy and Debt Issuance**

- Government issues debt to finance fiscal deficits.
- Fiscal expansion ($\uparrow$ spending or $\downarrow$ taxes) $\rightarrow$ higher debt issuance.
- Debt management policy targets stable maturity distribution between short- and long-term debt.

**6. Ricardian Equivalence (Theoretical Reference)**

- Suggests taxpayers are indifferent between:

  Higher taxes now vs. issuing debt now (taxes later).

- Holds only if:
  - Rational expectations and no capital market frictions.
  - Taxpayers can perfectly smooth consumption and intergenerational transfer.
- In practice, these assumptions fail $\rightarrow$ governments must **manage maturity mix**.

**7. Benefits of Debt Across Maturities**

- **Short-term debt:** Safe, liquid, alternative to bank deposits. But entails **rollover risk**.
- **Long-term debt:** Provides benchmark yields, supports monetary policy, and hedges interest rate risk.

**8. Use of Government Bonds in Financial System**

- Used as **collateral** in repo markets.
- Define **risk-free benchmark yield curve**.
- **Central banks** use sovereign bonds for open-market operations.

### B. Nonsovereign and Quasi-Government Issuers

#### 1. Nonsovereign Governments

- Include states, provinces, municipalities, and local authorities.
- Issue debt to fund public works and infrastructure.

Exhibit 2: Types of Nonsovereign Bonds

| Type | Definition | Repayment Source |
|---|---|---|
| **General Obligation (GO) Bonds** | Backed by full taxing power of the issuer. | Local/regional tax revenues. |
| **Revenue Bonds** | Finances specific projects (e.g., toll roads, hospitals). | Fees or user charges from funded project. |

#### 2. Quasi-Government (Agency) Bonds

- Issued by **government-sponsored entities (GSEs)** to fulfill specific mandates:
  - Examples: Ginnie Mae (U.S.), KfW (Germany), Japan Finance Corp.
- Often enjoy **implicit or explicit sovereign guarantee**.
- **Yield** typically close to sovereign benchmark.

#### 3. Supranational Agencies

- Formed by multiple governments to promote economic development.
- **Examples:** World Bank (IBRD), IMF, Asian Development Bank, European Investment Bank.
- **Credit Quality:** Extremely high (AA–AAA).
- **Purpose:** Infrastructure, poverty reduction, financial stability.

## LOS 53.b: Issuance and Trading of Government vs. Corporate Debt

#### 1. Issuance Method: Auctions vs. Underwriting

- **Sovereign Debt:** Issued via **public auctions**.
- **Corporate Debt:** Issued via **underwritten offerings** or **private placements**.

**2. Government Bond Auctions**

- **Bid Types:**
  - **Competitive Bid:** Investor specifies price/yield.
  - **Noncompetitive Bid:** Investor accepts auction yield and is guaranteed allocation.
- **Allocation Process:**
  1. Noncompetitive bids filled first at auction yield.
  2. Competitive bids ranked from highest price (lowest yield) downward.
  3. Allocation continues until issue amount met.

Exhibit 3: Auction Mechanisms

| Auction Type | Mechanism | Implication |
|---|---|---|
| **Single-Price (Uniform)** | All successful bidders pay the same price — corresponding to the **cut-off yield**. | Reduces yield volatility; encourages broader participation. |
| **Multiple-Price (Discriminatory)** | Each bidder pays the price they actually bid. | Yields closer together; encourages large, informed bids. |

**3. Auction Pricing Types**

**4. Primary Dealers**

- Licensed institutions required to:
  - Submit competitive bids in auctions.
  - Act as counterparties for central bank open-market operations.
  - Distribute government bonds to the broader market.

**5. Secondary Market Trading**

- **Market Type:** OTC quote-driven dealer markets.
- **On-the-Run Bonds:** Most recent issue of a given maturity; most liquid and used as **benchmark yields**.
- **Off-the-Run Bonds:** Older issues; less liquid, trade at higher yields.

**6. Investor Types and Objectives**

- **Economic Investors:** Seek yield and total return (e.g., mutual funds, pension funds).
- **Non-Economic Investors:**
  - Central banks (for monetary policy operations).
  - Foreign governments (reserve holdings).
  - Financial institutions (regulatory liquidity requirements).

**7. Yield Effects**

- Non-economic demand (central banks, regulators) pushes yields below equilibrium.
- Government debt thus serves as **low-risk benchmark** for corporate credit spreads.

## Key Formula and Example: Cut-Off Yield Determination

Cut-off Yield = Yield of the lowest-price (highest-yield) successful bid.

**Example:** A Treasury auction issues $100 million of 10-year bonds.

Exhibit 4: Sample Auction Bids

| Bidder | Price (% of Par) | Bid Amount ($m) |
|---|---|---|
| A | 101.00 | 40 |
| B | 100.75 | 30 |
| C | 100.50 | 50 |
| D | 100.25 | 40 |

- Allocation proceeds from highest price down.
- Total $100m issued → allocations to A ($40m) + B ($30m) + C ($30m).
- **Cut-off yield** corresponds to price = 100.50.
- Under a **single-price auction**, all pay 100.50.
- Under a **multiple-price auction**, each pays their bid price.

Exhibit 5: Summary of Government Debt Characteristics

| Issuer Type | Key Features |
|---|---|
| **Sovereign Government** | Backed by taxation power; issues across maturity spectrum; benchmark yield curve; issued via public auctions. |
| **Nonsovereign Government** | States, provinces, municipalities; issue general obligation or revenue bonds; rely on local taxes or project fees. |
| **Agency / Quasi-Government** | Created to serve specific policy goals (e.g., housing, infrastructure); often sovereign-backed. |
| **Supranational Agency** | Multinational entities (World Bank, IMF); high credit quality; foster economic development. |

## Key Summary Table

### Conceptual Takeaways

- Sovereign and supranational issuers anchor global fixed-income markets.
- Governments issue across maturities to balance cost and rollover risk.
- Auctions ensure transparency and wide investor participation.
- On-the-run bonds define the **risk-free yield curve** for valuation.
- Non-economic investors contribute to lower sovereign yields relative to corporate bonds.

# Module 54.1: Fixed-Income Bond Valuation — Prices and Yields

## LOS 54.a: Calculating a Bond's Price Given a Yield-to-Maturity (YTM)

### 1. Bond Pricing Fundamentals

- **Definition:** The price of a bond equals the present value (PV) of its future cash flows, discounted at the bond's yield to maturity (YTM).
- **YTM:** The internal rate of return (IRR) assuming:
  - The bond is held to maturity.
  - All coupons are paid in full and reinvested at the same YTM.

**2. Formula: Annual Coupon Bond**

$$P = \sum_{t=1}^{N} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^N}$$

where: $P$ = price of bond (per \$100 par)
$C$ = annual coupon payment ($C$ = coupon rate $\times F$)
$F$ = face value (par)
$r$ = yield per period (YTM)
$N$ = number of periods

Exhibit 1: 5-Year, 10% Annual Coupon, \$100 Par Bond

| YTM | Price (PV) | Comment |
|---|---|---|
| 10% | 100.00 | Par bond (coupon = yield). |
| 8% | 107.99 | Premium bond (coupon ¿ yield). |
| 12% | 92.79 | Discount bond (coupon ¡ yield). |

**Example 1: Annual-Pay Bond (Par, Premium, Discount) Interpretation:**

- ↓ YTM ↑ Bond Price (Inverse relationship).
- Price increase for 2% ↓ yield (from 10% to 8%) = +7.99.
- Price decrease for 2% ↑ yield (from 10% to 12%) = 7.21.
- **Asymmetry shows convexity.**

**3. Semiannual Coupon Bond Pricing**

$$P = \sum_{t=1}^{2N} \frac{C/2}{(1+r/2)^t} + \frac{F}{(1+r/2)^{2N}}$$

**Example:** 5-year, 10% coupon, semiannual payments, YTM = 8%.

$$N = 10; \quad PMT = 5; \quad FV = 100; \quad I/Y = 4\%.$$

$$PV = -108.11.$$

**Result:** Price = 108.11 → Premium bond.

**4. Calculating YTM Given Price**

$$N = 10; \quad PMT = 5; \quad FV = 100; \quad PV = -105.$$

$$\text{Solve: } I/Y = 4.37\% \quad \Rightarrow \text{YTM} = 4.37\% \times 2 = 8.74\%.$$

**Interpretation:** If price ¿ par → YTM ¡ coupon rate.

**5. Accrued Interest, Flat Price, and Full Price Concepts:**

- **Accrued Interest (AI)** = interest earned between coupon payments.

$$\text{AI} = \text{Coupon Payment} \times \frac{\text{Days Since Last Coupon}}{\text{Days in Coupon Period}}$$

- **Flat (Clean) Price:** Quoted price excluding accrued interest.
- **Full (Dirty) Price:** Invoice price including accrued interest.

$$\text{Full Price} = \text{Flat Price} + \text{Accrued Interest}$$

**6. Day-Count Conventions**

- **Actual/Actual:** Use actual number of days.
- **30/360:** Assume 30 days per month, 360 days per year.

**Example 2: Accrued Interest Calculation Bond:** 4% annual coupon, coupon date May 15, settlement Aug 10.

$$\text{Coupon} = 4\% \times 100 = \$4.$$

**30/360 method:** Days = 15 + 30 + 30 + 10 = 85 days

$$AI = 4 \times \frac{85}{360} = 0.944.$$

**Actual/Actual method:** Days = 16 + 30 + 31 + 10 = 87 days

$$AI = 4 \times \frac{87}{365} = 0.954.$$

Exhibit 2: Semiannual 5% Bond, YTM = 4%, Settlement Aug 21

| **Step** | **Calculation** |
|---|---|
| 1. PV on last coupon date | $N = 4$; $PMT = 2.5$; $FV = 100$; $I/Y = 2\% \Rightarrow PV = 101.904$. |
| 2. Days since last coupon | 67 days; total coupon period = 183 days. |
| 3. Full price | $101.904 \times (1.02)^{67/183} = 102.645$. |
| 4. Accrued interest | $2.5 \times \frac{67}{183} = 0.915$. |
| 5. Flat price | $102.645 - 0.915 = 101.73$. |

**Example 3: Full vs. Flat Price**

**Interpretation:** Flat price ¡ value at last coupon date due to partial period discounting.

## LOS 54.b: Relationships Among Price, Coupon, Maturity, and Yield-to-Maturity

### 1. Fundamental Relationships

- Price and YTM are **inversely related**.
- Lower coupon rate $\rightarrow$ **higher price sensitivity**.
- Longer maturity $\rightarrow$ **higher duration and price sensitivity**.
- Price-Yield relationship is **convex**.

### 2. Convexity Illustration

- For equal yield changes:
$$|\Delta P_{\text{down}}| > |\Delta P_{\text{up}}|$$
- Example: 2% $\downarrow$ yield increases price more than 2% $\uparrow$ yield decreases it.

### 3. Pull-to-Par (Constant-Yield Price Trajectory)

- Regardless of coupon rate or YTM, bond price $\rightarrow$ par as maturity approaches.
- This convergence is called the **pull to par**.

Exhibit 3: Bond Price vs. Yield Relationship

| **Scenario** | **Outcome** |
|---|---|
| Coupon = YTM | Price = Par ($P = 100$). |
| Coupon ¿ YTM | Price ¿ Par (Premium Bond). |
| Coupon ¡ YTM | Price ¡ Par (Discount Bond). |

### 4. Graphical Summary

- **X-axis:** Yield to Maturity.
- **Y-axis:** Bond Price.
- The curve slopes downward and is convex.

$$\text{Convexity: } \frac{d^2P}{dr^2} > 0$$

### 5. Example: Pull-to-Par Illustration 3-year 6% semiannual bond:

**Interpretation:** Prices converge to par even though yields differ.

Exhibit 4: Convergence Toward Par Over Time

| Time to Maturity | YTM = 3% | YTM = 6% | YTM = 12% |
|---|---|---|---|
| 3 years | 107.7 | 100.0 | 85.8 |
| 2 years | 105.1 | 100.0 | 90.6 |
| 1 year | 102.4 | 100.0 | 95.3 |
| At maturity | 100.0 | 100.0 | 100.0 |

## LOS 54.c: Matrix Pricing

### 1. Concept

- Used when bonds are **illiquid or not traded**.
- Estimates YTM or price using **comparable bonds** (same credit quality and maturity).
- Involves **linear interpolation** between yields of similar traded issues.

### 2. Step-by-Step Process

1. Identify traded bonds with same rating and similar maturity.
2. Average yields if multiple bonds exist per maturity.
3. Interpolate YTM for the desired maturity.
4. Use interpolated YTM to calculate bond price.

Exhibit 5: Interpolation for A+ 3-Year Bond

| Traded Bonds | YTM |
|---|---|
| 2-year A+ bond | 4.3% |
| 5-year A+ bonds (avg) | 5.2% |

**Example 1: Pricing an Illiquid Bond**

$$\text{Interpolated 3-year YTM} = 4.3 + (5.2 - 4.3) \times \frac{3-2}{5-2} = 4.6\%.$$

$$N = 3;\ PMT = 4;\ FV = 100;\ I/Y = 4.6 \Rightarrow PV = 98.354.$$

**Estimated Value:** \$98.35 per \$100 par.

**3. Example 2: Spread-Based Matrix Pricing for New Issues Given:**

$$\text{4-year Treasury YTM} = 1.48\%$$
$$\text{6-year Treasury YTM} = 2.15\%$$
$$\text{5-year A-rated corporate YTM} = 2.64\%$$

**Interpolated 5-year Treasury YTM:**

$$1.48 + (2.15 - 1.48) \times \frac{5-4}{6-4} = 1.815\%.$$

**Spread:**

$$2.64 - 1.815 = 0.825\%.$$

**Required YTM for 6-year A-rated:**

$$2.15 + 0.825 = 2.975\%.$$

**4. Practical Uses**

- Valuing infrequently traded corporate or municipal bonds.
- Estimating new issue pricing spreads.
- Calibrating bond curves for valuation models.

## Key Takeaways Summary

Exhibit 6: LOS Summary Table

| LOS | Core Insights |
|---|---|
| 54.a | Price = PV of all future CFs discounted at YTM. Flat price excludes accrued interest; full price includes it. Accrued interest = coupon × fraction of period elapsed. |
| 54.b | Price–yield relationship is inverse and convex. Lower coupon and longer maturity → higher sensitivity. All bond prices converge to par as maturity nears (pull to par). |
| 54.c | Matrix pricing estimates yield or price for illiquid issues by interpolating between yields of traded bonds with similar rating and maturity. |

**Conceptual Summary:**

- Bonds = discounted CF models.
- Prices rise when yields fall, but asymmetrically (convexity).
- Quoted price (flat) + accrued interest = full (dirty) price.
- Matrix pricing bridges illiquid and benchmark bond valuations.

# Module 55.1: Yield and Yield Spread Measures for Fixed-Rate Bonds

## LOS 55.a: Calculate Annual Yield on a Bond for Varying Compounding Periods

**1. Definition: Yield to Maturity (YTM)**

- The **YTM** is the discount rate that equates the present value of a bond's cash flows to its current market price:

$$P = \sum_{t=1}^{N} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^N}$$

- It is the internal rate of return (IRR) assuming:
  - The bond is held to maturity.
  - All coupon payments are made and reinvested at the same YTM.

**2. Example: Annual vs. Semiannual Yield** **Given:** 5-year, 7% annual-pay bond, price = 102.078.

$$N = 5,\ PMT = 7,\ FV = 100,\ PV = -102.078 \Rightarrow I/Y = 6.5\%.$$

**Interpretation:** Bond trades at a premium (price ¿ par, YTM ¡ coupon).

**Semiannual version:**

$$\text{YTM (semiannual)} = 3.253\% \times 2 = 6.506\%.$$

**3. Effective Annual Yield (EAY)**

$$\boxed{\text{EAY} = \left(1 + \frac{\text{YTM}}{n}\right)^n - 1}$$

where $n$ = number of coupon periods per year (periodicity).

Exhibit 1: Effective Annual Yield Comparison

| Periodicity (n) | Quoted YTM | Effective Annual Yield (EAY) |
|---|---|---|
| 1 (Annual) | 10.00% | 10.00% |
| 2 (Semiannual) | 10.00% | 10.25% |
| 4 (Quarterly) | 10.00% | 10.38% |

**4. Adjusting Yields for Periodicity** **Example:** Atlas Corp. bond quoted at 4% (semiannual).

$$\text{Semiannual rate} = 2\% \text{ per 6 months.}$$

**Equivalent Annual (EAY):**

$$(1.02)^2 - 1 = 4.04\%.$$

**Equivalent Quarterly Quoted Yield:**

$$(1.02)^{1/2} - 1 = 0.995\% \Rightarrow 4 \times 0.995 = 3.98\%.$$

**Conclusion:**

- Annual-pay bond at 4.04% EAY.
- Semiannual-pay bond at 4.00% nominal.
- Quarterly-pay bond at 3.98% nominal.

**5. Street vs. True Yield**

- **Street convention:** Uses stated coupon dates.
- **True yield:** Adjusts for actual payment dates (weekends/holidays). $\Rightarrow$ True yield is slightly lower.

**6. Current Yield (CY)**

$$\boxed{\text{Current Yield} = \frac{\text{Annual Coupon Payment}}{\text{Flat Price}}}$$

**Example:** 20-year, 6% semiannual bond, price = 802.07.

$$CY = \frac{60}{802.07} = 7.48\%.$$

**7. Simple Yield**

$$\boxed{\text{Simple Yield} = \frac{\text{Annual Coupon} \pm \text{Amortized Discount/Premium}}{\text{Price}}}$$

**Example:** 3-year, 8% semiannual bond, price = 90.165.

$$\text{Discount} = 100 - 90.165 = 9.835; \quad \text{Amortization per year} = \frac{9.835}{3} = 3.278.$$

$$\text{Simple Yield} = \frac{8 + 3.278}{90.165} = 12.47\%.$$

### 8. Yield to Call (YTC) and Yield to Worst (YTW)

- **YTC:** Yield assuming bond is called at a specific date and price.
- **YTW:** The lowest of YTM and all YTC values.

**Example:** 6% bond, price = 102, callable:

- At 102 in 3 years.
- At 101 in 4 years.

$$\text{YTM} = 5.54\%, \quad \text{YTC}_1 = 5.59\%, \quad \text{YTC}_2 = 5.56\%.$$

**YTW** = 5.54% (lowest value).

### 9. Option-Adjusted Yield (OAY)

$$\text{Callable bond value} = \text{Straight bond value} - \text{Call option value}.$$

$$\Rightarrow \text{Option-adjusted price} = \text{Callable bond price} + \text{Option value}.$$

$$\text{Option-adjusted yield (OAY)} < \text{Callable bond YTM}.$$

**Interpretation:** OAY "removes" option effects — yield as if option-free.

## LOS 55.b: Compare, Calculate, and Interpret Yield and Yield Spread Measures

### 1. Definition: Yield Spread

$$\boxed{\text{Yield Spread} = \text{Bond Yield} - \text{Benchmark Yield}}$$

- Measured in **basis points (bps)**.
- Reflects compensation for credit, liquidity, and optionality risk.

### 2. G-Spread (Government Spread)

$$\text{G-Spread} = \text{YTM}_{\text{bond}} - \text{YTM}_{\text{govt (same maturity)}}.$$

**Example:**

- Bond YTM = 6.82%.
- Interpolated Treasury yield (3-year) = 4.33%.

$$G\text{-spread} = 6.82 - 4.33 = 2.49\% = 249 \text{ bps}.$$

### 3. I-Spread (Interpolated Swap Spread)

$$\text{I-Spread} = \text{YTM}_{\text{bond}} - \text{Swap rate}_{\text{same tenor}}.$$

**Interpretation:** Excess yield over interbank reference rate (e.g., EURIBOR or SOFR).

### 4. Interpreting Yield Spreads

- **Macro factors:** Affect benchmark yield (risk-free rate, inflation).
- **Micro factors:** Affect credit/liquidity → spread widens.

Exhibit 2: Interpreting Spread Movements

| **Observation** | **Interpretation** |
|---|---|
| Bond yield ↑, spread unchanged | Benchmark yield ↑ → macro factors. |
| Bond yield ↑, spread ↑ | Bond-specific risk ↑ → micro factors. |
| Bond yield ↓, spread ↓ | Improved credit/liquidity. |

### 5. Z-Spread (Zero-Volatility Spread)

$$\boxed{\text{Z-spread} = \text{Constant spread added to benchmark spot curve to match bond price.}}$$

$$P = \sum_{t=1}^{N} \frac{CF_t}{(1 + s_t + ZS)^t}$$

where $s_t$ = spot rate at time $t$.

**Example:**

- 3-year, 9% coupon, price = 89.464.
- Treasury spot rates = 4%, 8.167%, 12.377%.

$$\text{G-spread} = 13.5\% - 12.0\% = 1.5\%.$$

Find $Z$-spread by trial-and-error such that PV = 89.464.

### 6. OAS (Option-Adjusted Spread)

$$\boxed{\text{OAS} = \text{Z-spread} - \text{Option Value (bps)}}$$

- For **callable bonds:** OAS < Z-spread.
- For **putable bonds:** OAS > Z-spread.

**Example:**

$$\text{Z-spread} = 180 \text{ bps}, \ \text{Option value} = 60 \text{ bps}. \Rightarrow \text{OAS} = 120 \text{ bps}.$$

**Interpretation:**

- Z-spread = total yield premium (credit + liquidity + optionality).
- OAS = yield premium excluding optionality.

Exhibit 3: Types of Yield Spreads

| **Type** | **Benchmark** | **Interpretation** |
|---|---|---|
| G-spread | Government yield | Simple yield difference vs. Treasury. |
| I-spread | Swap rate | Spread vs. interbank curve. |
| Z-spread | Spot yield curve | Spread constant over maturities. |
| OAS | Spot yield curve (option-free) | Spread net of embedded option. |

## Key Relationships and Formulas

**Effective Yield Conversions:**

$$\text{EAY} = (1 + \frac{r_{nom}}{n})^n - 1$$

**Current Yield:**

$$CY = \frac{\text{Annual Coupon}}{\text{Price}}$$

**Simple Yield:**

$$SY = \frac{\text{Annual Coupon} \pm \frac{(100-P)}{T}}{P}$$

**Yield to Worst:**

$$YTW = \min(\text{YTM}, \text{YTC}_1, \text{YTC}_2, \ldots)$$

**Z-Spread vs. OAS Relationship:**

$$Z\text{-spread} = OAS + \text{Option Value}$$

## Summary Table

**Key Insights:**

- **Higher compounding frequency => higher effective yield.**
- **Callable bonds:** YTW = minimum of YTM and YTCs.
- **Z-spread vs. OAS:** OAS = Z-spread minus option value.
- **Macro vs. micro factors:** Benchmark vs. spread changes.

Exhibit 4: LOS 55 Summary

| LOS | Summary |
|---|---|
| 55.a | YTM depends on compounding frequency. Effective annual yield rises with periodicity. Street yields assume nominal dates; true yields adjust for holidays. Current yield = coupon / price. Simple yield includes amortized discount/premium. Yield to worst = lowest YTM or YTC. Option-adjusted yield removes embedded option effect. |
| 55.b | Yield spreads measure risk premium vs. benchmark (G, I, Z, or OAS). Z-spread adjusts for the term structure of spot rates. OAS removes option impact, isolating credit + liquidity effects. G-spread and I-spread use YTMs; Z-spread and OAS use spot curves. |

# Module 56.1: Yield and Yield Spread Measures for Floating-Rate Instruments

## LOS 56.a: Calculate and Interpret Yield Spread Measures for Floating-Rate Notes (FRNs)

### 1. Overview of Floating-Rate Notes (FRNs)

- **Definition:** A bond whose coupon payments adjust periodically according to a variable market reference rate (MRR) plus a fixed margin.
- **General Formula:**

$$\text{Coupon Rate}_t = \text{MRR}_t + \text{Quoted Margin (QM)}$$

- **Coupon Reset:** Based on current market reference rate at the beginning of each coupon period; interest is paid **in arrears**.

Exhibit 1: Floating-Rate Note Margin Definitions

| Term | Definition / Interpretation |
|---|---|
| **Market Reference Rate (MRR)** | Short-term market rate (e.g., LIBOR, EURIBOR, SOFR, etc.). |
| **Quoted Margin (QM)** | Fixed spread over MRR specified at issuance. |
| **Discount Margin (DM)** | Required margin over MRR that equates market price to par (reflects current credit risk). |
| **Required Margin** | Same as discount margin — the yield premium investors currently demand. |

### 2. Key Terminology

Exhibit 2: FRN Valuation Logic

| Condition | Relationship Between QM and DM | Price Behavior |
|---|---|---|
| At issuance | QM = DM | FRN trades at par. |
| Credit quality worsens | QM ¡ DM | FRN trades at a discount. |
| Credit quality improves | QM ¿ DM | FRN trades at a premium. |

### 3. Par, Premium, and Discount Pricing Relationships

### 4. Analogy to Fixed-Rate Bonds

- Fixed-rate bond analogy:

$$\text{Coupon vs. YTM} \Longleftrightarrow \text{Quoted Margin vs. Discount Margin}$$

- If yield demanded > coupon → bond trades below par. Similarly, if DM > QM → FRN trades below par.

### 5. Simplified Valuation on Reset Date

$$P = \sum_{t=1}^{N} \frac{CF_t}{(1 + \text{MRR} + \text{DM})^t}$$

where $CF_t = (\text{MRR} + \text{QM}) \times$ Face Value.

**Example 1: FRN Valuation Given:**

- Par value = \$100,000.
- Semiannual coupon = MRR + 120 bps.
- MRR = 3.0% (annualized), DM = 1.5% (annualized).
- Remaining maturity = 5 years → 10 semiannual periods.

**Step 1: Compute coupon per period**

$$\text{Coupon rate} = (3.0 + 1.2)\% = 4.2\% \Rightarrow \text{semiannual coupon} = 2.1\%.$$

**Step 2: Compute discount rate per period**

$$\text{Discount rate} = (3.0 + 1.5)\%/2 = 2.25\%.$$

**Step 3: Calculate PV**

$$N = 10;\ PMT = 2.1;\ FV = 100;\ I/Y = 2.25; \Rightarrow PV = 98.67.$$

**Result:** FRN Value = \$98,670 → trades below par since DM ¿ QM.

**6. Interpretation Summary**

- **If DM ¿ QM:** Investors demand higher yield → price < 100 (discount).
- **If DM ¡ QM:** Investors accept lower yield → price > 100 (premium).
- **If DM = QM:** Price = 100 (at par on reset dates).

## LOS 56.b: Calculate and Interpret Yield Measures for Money Market Instruments

**1. Overview**

- **Money Market Instruments:** Short-term debt (maturity  1 year).
- Examples: Treasury bills, commercial paper, CDs, repos.
- Yields vary by:
  - Quotation basis (add-on vs. discount).
  - Day-count convention (360-day vs. 365-day year).

**2. Add-On Yield (AOY) — Interest on Principal**

$$\boxed{\text{Quoted Add-On Yield} = \text{HPY} \times \frac{\text{Year Days}}{\text{Days to Maturity}}}$$

where:

$$\text{HPY} = \frac{\text{Interest Earned}}{\text{Price Paid}}.$$

**Example:** 100-day CD, AOY = 1.5% (365-day year)

$$\text{HPY} = 1.5\% \times \frac{100}{365} = 0.41\%.$$

$$\text{Future Value} = 1000(1.0041) = 1004.10.$$

**3. Discount Yield (DY) — Discount from Face Value**

$$\boxed{\text{Quoted Discount Yield} = \text{Discount} \times \frac{360}{\text{Days to Maturity}}}$$

$$\text{Discount} = 1 - \frac{P}{F}.$$

**Example:** 180-day T-bill quoted at 2.2% on a 360-day basis.

$$\text{Discount} = 0.022 \times \frac{180}{360} = 0.011.$$

$$\text{Price} = 1000(1 - 0.011) = 989.$$

$$\text{HPY} = \frac{1000 - 989}{989} = 1.11\%.$$

**4. Converting Between Yield Conventions**

- Required to compare securities quoted on different bases.

Exhibit 3: Money Market Yield Conversions

| **Conversion** | **Formula** |
|---|---|
| Discount yield → Price | $P = F\left(1 - \frac{d \times t}{360}\right)$ |
| Price → Add-on yield | $r = \frac{F-P}{P} \times \frac{365}{t}$ |
| Add-on yield (360) → Bond Equivalent Yield (BEY) | $r_{BEY} = r_{360} \times \frac{365}{360}$ |

**5. Example: Comparing Different Money Market Quotes** **Case 1: 90-day T-bill** Discount = 1.2%, 360-day basis.

$$P = 1000(1 - 0.012 \times 90/360) = 997.$$

$$\text{Add-on yield (BEY)} = \frac{3}{997} \times \frac{365}{90} = 1.22\%.$$

**Case 2: 120-day CD** Add-on yield = 1.4% (365-day).

$$\text{Interest} = 0.014 \times \frac{120}{365} = 0.004603.$$

$$\text{Maturity value} = 1,000,000(1.004603) = 1,004,603.$$

**Case 3: 100-day deposit** Add-on yield = 1.5% (360-day).

$$\text{Bond equivalent yield (BEY)} = 1.5 \times \frac{365}{360} = 1.5208\%.$$

$$\text{HPY} = 1.5\% \times \frac{100}{360} = 0.4167\%.$$

$$\text{Effective annual yield} = (1.004167)^{3.65} - 1 = 1.5294\%.$$

$$\text{Semiannual bond yield} = 2 \times \left((1.015294)^{1/2} - 1\right) = 1.5236\%.$$

**6. Bond Equivalent Yield (BEY)**

- **Definition:** Add-on yield annualized using 365 days — allows direct comparison with bond yields.
- **Formula:**

$$\boxed{\text{BEY} = \frac{\text{HPY} \times 365}{\text{Days to Maturity}}}$$

**7. Interpretation of Yield Basis**

Exhibit 4: Quotation Conventions in Money Markets

| **Instrument** | **Quotation Basis** | **Year Convention** |
|---|---|---|
| Treasury bills, commercial paper | Discount yield | 360-day |
| Bank CDs, repos, interbank deposits | Add-on yield | 365-day |
| Bond equivalent yields (BEY) | Add-on yield | 365-day |

Exhibit 5: Summary of Core Yield Formulas

| **Concept** | **Formula** |
|---|---|
| Floating-Rate Coupon | $\text{Coupon} = (\text{MRR} + \text{QM}) \times F$ |
| FRN Discount Rate | $r = \text{MRR} + \text{DM}$ |
| Add-On Yield | $r_{AOY} = \frac{F-P}{P} \times \frac{\text{Year Days}}{\text{Days to Maturity}}$ |
| Discount Yield | $r_{DY} = \frac{F-P}{F} \times \frac{360}{\text{Days to Maturity}}$ |
| Bond Equivalent Yield (BEY) | $r_{BEY} = \text{HPY} \times \frac{365}{\text{Days to Maturity}}$ |
| FRN Price Approximation | $P = \sum_{t=1}^{N} \frac{CF_t}{(1+\text{MRR}+\text{DM})^t}$ |

Exhibit 6: Conceptual Summary

| **LOS** | **Core Insights** |
|---|---|
| 56.a | FRNs pay MRR + quoted margin (QM). Required margin (DM) reflects investor credit demands. FRN trades at par if QM = DM, at discount if QM ¡ DM, and at premium if QM ¿ DM. Value estimated using MRR + DM as discount rate. |
| 56.b | Money market yields differ by quotation (add-on vs. discount) and year convention (360 or 365). Bond equivalent yield (BEY) = add-on yield using 365-day basis. Yield conversions allow cross-comparison of securities. |

## Key Formulas Summary

## Summary Table: LOS 56.a–56.b

**Key Takeaways:**

- FRNs reset coupons periodically, making price less volatile than fixed-rate bonds.
- The relationship QM vs. DM drives FRN price deviations from par.
- Money market yields require standardization (BEY) for comparison across instruments.
- Discount instruments quote yields below face; add-on instruments quote above price paid.

# Module 57.1: The Term Structure of Interest Rates — Spot, Par, and Forward Curves

## LOS 57.a: Define Spot Rates and the Spot Curve; Calculate the Price of a Bond Using Spot Rates

### 1. Definition of Spot Rates

- A **spot rate** ($S_t$) is the market discount rate for a single future payment at time $t$.
- Also called **zero-coupon rate** or **zero rate**.
- Represents yield on a zero-coupon bond maturing at $t$.

### 2. Bond Valuation Using Spot Rates

$$\boxed{P_0 = \sum_{t=1}^{N} \frac{CF_t}{(1+S_t)^t}}$$

where $CF_t$ = coupon or principal at time $t$, discounted by the corresponding spot rate $S_t$.

### 3. Example: Bond Valuation Using Spot Rates Given:

$$S_1 = 3\%, \quad S_2 = 4\%, \quad S_3 = 5\%, \quad \text{Coupon rate} = 5\%, \quad FV = 100$$

$$P = \frac{5}{1.03} + \frac{5}{(1.04)^2} + \frac{105}{(1.05)^3} = 100.18$$

**Interpretation:**

- Bond value slightly > par → YTM slightly < coupon (4.93%).
- Price derived from spot rates = **no-arbitrage price**.

### 4. Conceptual View

- Each cash flow discounted at its own appropriate zero-coupon yield.
- YTM is a weighted average of spot rates.
- Spot curve: graphical plot of spot rates ($y$-axis) vs. maturity ($x$-axis).

Exhibit 1: Spot Rates vs. Bond Pricing

| **Scenario** | **Relative Spot/YTM Levels** | **Resulting Bond Price** |
|---|---|---|
| Spot rates increase with maturity (normal) | YTM slightly below long-term spot | Price $\approx$ par |
| Flat spot curve | All spot rates equal to YTM | Price = par |
| Inverted spot curve | YTM higher than early spot rates | Price $<$ par |

## LOS 57.b: Define Par and Forward Rates; Compute Par Rates, Forward Rates, and Spot Rates

### 1. Par Yield (or Par Rate)

- The **par yield** is the coupon rate that causes a bond's price to equal its par value.
- For an $N$-year bond:

$$100 = \sum_{t=1}^{N} \frac{PMT}{(1+S_t)^t} + \frac{100}{(1+S_N)^N}$$

  Solve for $PMT \rightarrow$ the **par yield**.

**Example:** Given spot rates $S_1 = 1\%, S_2 = 2\%, S_3 = 3\%$:

$$\frac{PMT}{1.01} + \frac{PMT}{(1.02)^2} + \frac{100 + PMT}{(1.03)^3} = 100$$

$$\Rightarrow PMT = 2.96\%$$

**Interpretation:** 3-year par bond has coupon = 2.96% to trade at par.

### 2. Forward Rates (Notation and Concept)

- Forward rate *ayby*: rate for a $b$-year loan beginning $a$ years from now.

  Example: $2y1y$ = 1-year rate beginning 2 years from today.

- Forward rates are implied future rates ensuring no-arbitrage between investing now vs. investing later.

### 3. Relationship Between Spot and Forward Rates

$$\boxed{(1+S_N)^N = \prod_{t=1}^{N}(1+f_{t-1,t})}$$

where $f_{t-1,t}$ = forward rate for period $t$.

**Example 1: Compute Spot from Forward Rates**

$$S_1 = 2\%, \quad 1y1y = 3\%, \quad 2y1y = 4\%$$

$$(1+S_3)^3 = (1.02)(1.03)(1.04) \Rightarrow S_3 = (1.02 \times 1.03 \times 1.04)^{1/3} - 1 = 2.997\%$$

**Interpretation:** The 3-year spot rate is the geometric mean of one 1-year spot and two forward rates.

**4. Computing Forward Rate from Spot Rates**

$$\boxed{(1+S_2)^2 = (1+S_1)(1+1y1y)} \Rightarrow 1y1y = \frac{(1+S_2)^2}{(1+S_1)} - 1$$

**Example 2:**

$$S_1 = 4\%, \quad S_2 = 8\% \Rightarrow 1y1y = \frac{1.08^2}{1.04} - 1 = 12.154\%.$$

**Meaning:** Investors earn 4% first year, then 12.15% next year, equivalent to 8% annualized for two years.

**5. 3-Year Extension**

$$(1+S_3)^3 = (1+S_2)^2(1+2y1y) \Rightarrow 2y1y = \frac{(1+S_3)^3}{(1+S_2)^2} - 1$$

**Example 3:**

$$S_1 = 4\%, \quad S_2 = 8\%, \quad S_3 = 12\% \Rightarrow 2y1y = \frac{1.12^3}{1.08^2} - 1 = 20.45\%.$$

**Quick Approximation:**

$$2y1y \approx (3 \times 12) - (2 \times 8) = 20\%.$$

**6. Multi-Period Forward Rates**

$$(1+S_4)^4 = (1+S_2)^2(1+2y2y)^2 \Rightarrow 2y2y = \sqrt{\frac{(1+S_4)^4}{(1+S_2)^2}} - 1$$

**Example 4:**

$$S_2 = 6\%, \quad S_4 = 8\% \Rightarrow 2y2y = 10.04\%.$$

**Approximation:** $(4 \times 8 - 2 \times 6)/2 = 10\%$.

**7. Valuing a Bond Using Forward Rates Given:**

$$S_1 = 4\%,\ 1y1y = 5\%,\ 2y1y = 6\%; \quad \text{3-year, 5\% coupon, } FV = 1000.$$

**Discount Factors:**

$$DF_1 = \frac{1}{1.04}, \quad DF_2 = \frac{1}{(1.04)(1.05)}, \quad DF_3 = \frac{1}{(1.04)(1.05)(1.06)}.$$

$$P = 50(DF_1 + DF_2 + DF_3) + 1000(DF_3) = 1000.7.$$

**Interpretation:** Forward-based valuation = spot-based valuation $\rightarrow$ both yield **no-arbitrage price**.

## LOS 57.c: Compare the Spot Curve, Par Curve, and Forward Curve

### 1. Spot Curve (Zero Curve)

- Plots spot rates ($S_t$) vs. maturities.
- Derived from zero-coupon or stripped Treasury bonds.
- Represents pure time-value yields.

### 2. Par Curve

- Plots **par yields** — coupon rates for bonds trading exactly at par.
- Constructed using spot rates.
- Less affected by illiquidity and taxation distortions than direct yield curves.

### 3. Forward Curve

- Plots forward rates ($f_t$) for successive periods.
- Represents expected future short-term rates.

Exhibit 2: Curve Relationship Summary

| **Curve Type** | **Description** |
|---|---|
| Spot Curve | Set of yields on zero-coupon bonds (spot rates). |
| Par Curve | Hypothetical yields of par bonds for each maturity. |
| Forward Curve | Implied future short-term rates consistent with spot curve. |

### 4. Relationships Among Curves

### 5. Comparative Dynamics

- **Upward-sloping (normal) yield curve:**

$$\text{Forward rates} > \text{Spot rates} > \text{Par yields}.$$

- **Downward-sloping (inverted) yield curve:**

$$\text{Forward rates} < \text{Spot rates} < \text{Par yields}.$$

- **Flat curve:**

$$\text{Forward rates} = \text{Spot rates} = \text{Par yields}.$$

**6. Example: Normal Yield Curve Intuition**

$$S_1 = 1\%,\ 1y1y = 3\%. \Rightarrow S_2 \approx (1 + 0.01)(1 + 0.03)^{1/2} - 1 \approx 2\%.$$

**Interpretation:**

- Forward curve steeper than spot curve.
- Par yield curve smoother (weighted average of spots).

Exhibit 3: Comparative Behavior of Yield Curves

| Yield Curve Type | Forward Curve vs. Spot Curve | Par Curve vs. Spot Curve |
|---|---|---|
| Upward-sloping | Above | Slightly below |
| Flat | Equal | Equal |
| Downward-sloping | Below | Slightly above |

## Summary Table: LOS 57.a–57.c

Exhibit 4: Summary of Core Concepts

| LOS | Summary |
|---|---|
| 57.a | Spot rates are zero-coupon yields for specific maturities. Bond prices are found by discounting each cash flow at its respective spot rate (no-arbitrage price). |
| 57.b | Par yields are coupon rates that make bonds price at par. Forward rates are implied future short-term rates consistent with the spot curve. Relations: $(1+S_2)^2 = (1+S_1)(1+1y1y)$ and $(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$. |
| 57.c | Spot, par, and forward curves are interrelated: forward rates drive spot rates; spot rates drive par yields. In a normal curve, forward $>$ spot $>$ par. In an inverted curve, forward $<$ spot $<$ par. |

**Key Takeaways:**

- Spot rates = pure time-value yields from zero-coupon bonds.
- Forward rates = implied future short-term borrowing/lending rates.
- Par yields = hypothetical coupon rates that price bonds at par.
- Forward $\rightarrow$ Spot $\rightarrow$ Par sequence defines yield curve dynamics.

# Module 58.1: Interest Rate Risk and Return

## LOS 58.a: Sources of Return from Fixed-Rate Bonds

**1. Overview** A fixed-rate bond provides three primary sources of return:

1. **Coupon and Principal Payments:** Fixed periodic cash inflows promised by the issuer.
2. **Reinvestment Income:** Interest earned on reinvested coupon payments.
3. **Capital Gain or Loss:** Difference between sale price and purchase price if sold before maturity.

**2. Key Assumptions**

- The bond makes all payments as promised (no default).
- Coupons are reinvested at the bond's original YTM.
- YTM remains constant unless otherwise stated.

**3. Five Key Results**

(1) Holding to maturity with constant YTM $\rightarrow$ realized return = YTM.

(2) Selling before maturity with unchanged YTM $\rightarrow$ realized return = YTM.

(3) YTM rises $\rightarrow$ realized return $>$ original YTM (if held to maturity).

(4) YTM rises $\rightarrow$ realized return $<$ original YTM (if short horizon).

(5) YTM falls $\rightarrow$ realized return $<$ original YTM (if long horizon).

## Unchanged YTM — Bond Held to Maturity

**Example 1:**

$$6\% \text{ annual-pay, 3-year bond, } YTM = 7\%$$

$$N = 3; I/Y = 7; PMT = 6; FV = 100; \Rightarrow PV = -97.376$$

**Coupon and Reinvestment Income:**

$$FV_{\text{coupons}} = 6(1.07)^2 + 6(1.07) + 6 = 19.289$$

Reinvestment income $= 19.289 - 18 = 1.289$

**Total Future Value:**

$$119.289 = 97.376(1.07)^3$$

**Conclusion:** Realized annual return = YTM = 7%.

## Unchanged YTM — Bond Sold Before Maturity

**Concept:** If YTM remains constant, bond value converges to par as maturity approaches. Intermediate values = **carrying values** (on the constant-yield trajectory).

### Example 2: Capital Gain/Loss

$$\text{20-year bond, 5\% semiannual coupon, } YTM = 6\%.$$

Sold after 5 years (15 years remaining).

**Carrying value:**

$$N = 30; I/Y = 3; PMT = 2.5; FV = 100; \Rightarrow PV = -90.20$$

Sold at 91.40 $\rightarrow$ Capital gain = 91.40 - 90.20 = 1.20.

### Conceptual Note:

- Bonds held or sold at constant YTM have no true capital gain/loss.
- Price converges toward par through amortization of premium/discount.

## Changed YTM — Bond Held to Maturity

### Example 3: YTM Change After Purchase

$$\text{3-year 6\% bond, } YTM = 7\%, \text{Price} = 97.376.$$

**Case 1: YTM rises to 8%**

$$FV_{\text{coupons}} = 6(1.08)^2 + 6(1.08) + 6 = 19.478 \Rightarrow \text{Realized return } > 7\%.$$

**Case 2: YTM falls to 6%**

$$FV_{\text{coupons}} = 6(1.06)^2 + 6(1.06) + 6 = 19.102 \Rightarrow \text{Realized return } < 7\%.$$

### Interpretation:

- Reinvestment income dominates for long horizons.
- Higher YTM $\Rightarrow$ more reinvestment income $\Rightarrow$ higher return.

## Changed YTM — Bond Sold Before Maturity

**Example 4: One-Year Horizon**

$$\text{3-year 6\% bond, purchased at 97.376, horizon} = 1 \text{ year.}$$

**Case 1: YTM increases to 8%**

$$N = 2; I/Y = 8; PMT = 6; FV = 100; PV = -96.433.$$

Return $< 7\%$ (price loss dominates).

**Case 2: YTM decreases to 6%**

$$N = 2; I/Y = 6; PMT = 6; FV = 100; PV = -100.00.$$

Return $> 7\%$ (price gain dominates).

Exhibit 1: Price vs. Reinvestment Risk Tradeoff

| Investment Horizon | Dominant Risk | Effect of YTM Change |
|---|---|---|
| Short-term | Price risk | YTM↑ → Return↓ (price loss) |
| Long-term | Reinvestment risk | YTM↑ → Return↑ (more coupon reinvestment income) |
| At balance (duration) | Equal | Return unaffected by small YTM shifts |

**Key Idea: Price Risk vs. Reinvestment Risk**

## LOS 58.b: Relationship Among Holding Period Return, Macaulay Duration, and Investment Horizon

**Conceptual Link:**

- **Holding Period Return (HPR):** Realized yield over investor's horizon.
- **Macaulay Duration:** Time-weighted average of cash flows; balance point of price vs. reinvestment risk.
- **Investment Horizon:** Time investor plans to hold bond.

**Relationship Summary:**

$$\boxed{\text{Duration Gap} = D_{Mac} - \text{Investment Horizon}}$$

- Positive duration gap → price risk dominates (YTM↑ → loss).
- Negative duration gap → reinvestment risk dominates (YTM↓ → loss).
- Zero duration gap → risks offset → horizon yield = original YTM.

## LOS 58.c: Macaulay Duration — Definition, Calculation, and Interpretation

**Definition:**

$$\boxed{D_{Mac} = \sum_{t=1}^{N} \left( \frac{t \times PV(CF_t)}{P_0} \right)}$$

where $PV(CF_t) = \dfrac{CF_t}{(1+y)^t}$, and $P_0 = \sum PV(CF_t)$.

**Interpretation:** Average time (in years) it takes to receive bond's cash flows (weighted by PVs).

**Example 5: Macaulay Duration Calculation Bond:** 5-year, 11% annual coupon, Price = 86.59, YTM = 15%.

Exhibit 2: Present Values and Weights for Macaulay Duration

| Year | Cash Flow ($) | PV(CF) | Weight | t × Weight |
|---|---|---|---|---|
| 1 | 11 | 9.57 | 0.1105 | 0.1105 |
| 2 | 11 | 8.32 | 0.0961 | 0.1922 |
| 3 | 11 | 7.23 | 0.0835 | 0.2505 |
| 4 | 11 | 6.29 | 0.0726 | 0.2904 |
| 5 | 111 | 55.18 | 0.6373 | 3.1865 |
| | | | **Macaulay Duration:** | **4.03 years** |

**Interpretation:**

- Investment horizon = 4 years → horizon yield = original YTM (15%) regardless of rate change.
- Confirms duration as point of risk neutrality.

## Example: Horizon Yield Stability (Duration Balance)

**Bond:** 5-year, 11% coupon, Price = 86.59, YTM = 15%, Horizon = 4 years.

Exhibit 3: Horizon Yield Outcomes

| YTM Change | Sale Price (PV) | Horizon Yield |
|---|---|---|
| Falls to 14% | 97.368 | 15.0% |
| Rises to 16% | 95.690 | 15.0% |

**Conclusion:**

$$\boxed{\text{When Horizon} = D_{Mac},\ \text{Price Risk} = \text{Reinvestment Risk.}}$$

## Summary Tables

Exhibit 4: LOS Summary

| LOS | Summary |
|---|---|
| 58.a | Three return components: coupon/principal, reinvested coupons, capital gain/loss. Holding to maturity at constant YTM → realized return = YTM. |
| 58.b | Price risk dominates short horizons; reinvestment risk dominates long horizons. Horizon yield equals YTM when investment horizon = Macaulay duration. |
| 58.c | Macaulay duration = weighted average time to receipt of CFs. Duration gap = $D_{Mac}-$ investment horizon. Positive gap → price risk; negative gap → reinvestment risk. |

Exhibit 5: Risk and Horizon Relationship

| Case | Dominant Risk | Effect on Realized Return |
|---|---|---|
| Short Horizon | Price risk | YTM↑ → Return↓, YTM↓ → Return↑ |
| Long Horizon | Reinvestment risk | YTM↑ → Return↑, YTM↓ → Return↓ |
| Horizon = Duration | Balanced | Return = Original YTM (risk offset) |

## Key Takeaways

- **Sources of Return:** Coupon, reinvestment, capital gain/loss.
- **Risks:**
  - Price risk dominates short horizons.
  - Reinvestment risk dominates long horizons.
- **Duration Neutral Point:** When horizon = Macaulay duration, realized return = YTM regardless of rate change.
- **Duration Gap:**

$$D_{Mac} - \text{Investment Horizon} = 0 \Rightarrow \text{Perfect Immunization.}$$

# Module 59.1: Yield-Based Bond Duration Measures and Properties

## LOS 59.a: Define, Calculate, and Interpret Modified Duration, Money Duration, and PVBP

### 1. Concept Overview

- **Macaulay Duration (MacDur):** Weighted average time to receive the bond's cash flows.
- **Modified Duration (ModDur):** Sensitivity measure of the bond's price to a change in YTM.
- **Money Duration (Dollar Duration):** Currency change in bond value per 1% change in YTM.
- **Price Value of a Basis Point (PVBP):** Currency change in bond value per 1 basis point (0.01%) change in YTM.

**2. Modified Duration (ModDur)**

$$\boxed{\text{ModDur} = \frac{\text{MacDur}}{1 + \dfrac{\text{YTM}}{m}}}$$

where $m$ = number of coupon payments per year.

- For annual-pay bonds: $\text{ModDur} = \dfrac{\text{MacDur}}{1 + \text{YTM}}$
- For semiannual-pay bonds: $\text{ModDur} = \dfrac{\text{MacDur}}{1 + \text{YTM}/2}$

**3. Price–Yield Approximation Using ModDur**

$$\boxed{\frac{\Delta P}{P} \approx -\text{ModDur} \times \Delta\text{YTM}}$$

- Estimates percentage price change for a 1% change in YTM.
- Negative sign reflects inverse relationship between price and yield.

**Example 1: Modified Duration Calculation** **Given:** 5-year, 11% annual coupon bond, Price = 86.59, YTM = 15%, MacDur = 4.03.

$$\text{ModDur} = \frac{4.03}{1.15} = 3.50$$

**Interpretation:**

$$\text{Approx.\% price change for 0.5\% yield increase} = -3.50 \times 0.005 = -1.75\%.$$

$$\text{New Price} = 86.59 \times (1 - 0.0175) = 85.09 \text{ (approx.)}$$

**Check:**

$$\text{Exact Price at 15.5\% YTM} = 85.092 \Rightarrow \text{Estimation accuracy confirmed.}$$

### 4. Approximate Modified Duration (Finite Difference Method)

$$\boxed{\text{Approx. ModDur} = \frac{V_- - V_+}{2 \times V_0 \times \Delta\text{YTM}}}$$

where:

- $V_-$: Price if YTM decreases by $\Delta$YTM.
- $V_+$: Price if YTM increases by $\Delta$YTM.
- $V_0$: Current bond price.
- $\Delta$YTM is entered as a decimal (e.g., 0.005 for 50 bps).

**Example 2: Approximate ModDur** **Given:** 5-year, 11% bond, Price = 86.59, $\Delta$YTM = 0.005.

$$V_+ = 85.092, \quad V_- = 88.127.$$

$$\text{Approx. ModDur} = \frac{88.127 - 85.092}{2 \times 86.59 \times 0.005} = 3.505.$$

**Result:** Approximation very close to theoretical ModDur = 3.50.

### 5. Interpretation of ModDur

- ModDur measures **first-order price sensitivity**.
- Linear approximation of convex (curved) price–yield relationship.
- Accurate for small yield changes ( 100 bps).
- Underestimates price increase and overestimates price decrease for larger yield moves due to convexity.

**Figure Concept: Price–Yield Curve**
Curved relationship → convex shape.
ModDur is tangent slope — linear approximation around current YTM.

### 6. Money Duration (Dollar Duration)

$$\boxed{\text{Money Duration} = \text{ModDur} \times \text{Full Price of Bond Position}}$$

- Expressed in currency terms.
- Represents dollar change for a 1% change in YTM.

**Example 3: Money Duration** **Given:** ModDur = 7.42, Full Price = 101.32, Par = \$2,000,000.

$$\text{Money Duration} = 7.42 \times 101.32\% \times 2,000,000 = 15,035,888.$$

**Per \$100 of Par:**

$$7.42 \times 101.32 = 751.79.$$

**Impact of 25 bp Yield Increase:**

$$15,035,888 \times 0.0025 = 37,589.72 \Rightarrow \text{Loss of } \$37{,}589.72.$$

## 7. Price Value of a Basis Point (PVBP)

$$\boxed{\text{PVBP} = \frac{V_- - V_+}{2}}$$

- Change in bond's full price for a 1 bp (0.01%) change in YTM.
- Expressed in currency units per \$100 par.

**Example 4: PVBP Calculation** **Given:** 20-year, 6% annual bond, Price = 101.39, Par = \$1,000,000.

$$YTM = 5.88\%, \quad V_+ = 101.273, \quad V_- = 101.507.$$

$$\text{PVBP per \$100 par} = \frac{101.507 - 101.273}{2} = 0.117.$$

$$\text{PVBP total} = 0.117 \times \$1,000,000 \times 0.01 = \$1,170.$$

**Alternative:**

$$\text{PVBP} = \text{Money Duration} \times 0.0001.$$

Exhibit 1: Summary of Duration and PVBP Relationships

| **Measure** | **Formula / Interpretation** |
|---|---|
| Macaulay Duration | Weighted average time to receive CFs |
| Modified Duration | $\frac{\text{MacDur}}{1 + \text{YTM}/m}$ — % price change per 1% $\Delta$YTM |
| Money Duration | ModDur $\times$ Full Price — \$ change per 1% $\Delta$YTM |
| PVBP | $\frac{V_- - V_+}{2}$ — \$ change per 1 bp $\Delta$YTM |
| Relationship | PVBP = Money Duration $\times$ 0.0001 |

## LOS 59.b: Factors Affecting Interest Rate Risk

### 1. Maturity Effect

- Longer maturity ⇒ greater interest rate risk (usually).
- Distant CFs more sensitive to discount rate changes.
- Exception: for deep-discount bonds, duration increases then decreases with maturity.

### 2. Coupon Rate Effect

- Higher coupon ⇒ lower duration ⇒ less interest rate risk.
- More CFs received sooner, reducing sensitivity to yield changes.
- Zero-coupon bond has highest duration for given maturity.

### 3. Yield Level Effect

- Higher YTM ⇒ lower duration ⇒ lower price sensitivity.
- Due to flatter (less steep) slope of price–yield curve at high yields.

### 4. Coupon Reset (for FRNs)

- Duration ≈ time to next reset date.
- Frequent resets ⇒ low price sensitivity.

### 5. Time Passage Effect (Rolling Down the Curve)

- Between coupon payments, duration decreases smoothly as time passes.
- After coupon payment, duration jumps upward slightly as time to next payment resets.

Exhibit 2: Factors Influencing Duration and Interest Rate Sensitivity

| **Factor** | **Effect on Duration** | **Interpretation** |
|---|---|---|
| Maturity ↑ | Duration ↑ | Long-term bonds = higher price sensitivity. |
| Coupon Rate ↑ | Duration ↓ | High-coupon bonds = lower interest rate risk. |
| YTM ↑ | Duration ↓ | Higher yields = flatter curve, less sensitivity. |
| FRN Reset Frequency ↑ | Duration ↓ | More frequent resets = lower risk. |
| Time (between coupons) | Duration ↓ | Duration declines steadily until coupon date, then resets. |

## Key Takeaways Summary

- **Modified Duration:** Measures % change in price for 1% $\Delta$YTM.
- **Approx. ModDur:** Computed using $V_-$ and $V_+$ around current yield.
- **Money Duration:** Currency change for 1% $\Delta$YTM = ModDur $\times$ Price.
- **PVBP:** Currency change for 1 bp = Money Duration $\times$ 0.0001.
- **Interest Rate Risk Drivers:**
  - Longer maturity $\rightarrow$ higher risk.
  - Lower coupon $\rightarrow$ higher risk.
  - Lower YTM $\rightarrow$ higher risk.
  - FRNs and high-coupon bonds $\rightarrow$ lower risk.

Exhibit 3: Formula Summary

| **Concept** | **Formula** |
|---|---|
| Modified Duration | $\text{ModDur} = \dfrac{\text{MacDur}}{1 + \text{YTM}/m}$ |
| Approx. ModDur | $\dfrac{V_- - V_+}{2V_0\Delta\text{YTM}}$ |
| Money Duration | ModDur $\times$ Full Price |
| PVBP | $\dfrac{V_- - V_+}{2}$ or Money Duration $\times$ 0.0001 |
| Price Change Estimate | $\dfrac{\Delta P}{P} \approx -\text{ModDur} \times \Delta\text{YTM}$ |
| Interest Rate Sensitivity | $\uparrow$ Maturity, $\downarrow$ Coupon, $\downarrow$ YTM $\rightarrow$ $\uparrow$ Duration |

# Module 60.1: Yield-Based Bond Convexity and Portfolio Properties

## LOS 60.a: Calculate and Interpret Convexity and Describe the Convexity Adjustment

### 1. Concept Overview

- Duration gives a **linear** (first-order) estimate of price sensitivity to yield changes.
- The actual price–yield curve is **convex**; thus, duration alone underestimates price increases and overestimates price decreases.

- **Convexity** measures the curvature (second-order sensitivity) of the price–yield relationship.
- Adding convexity to duration improves price-change estimates, especially for larger $\Delta$YTM.

**2. Formula: Convexity of a Single Cash Flow**

$$\boxed{\text{Convexity of CF}_t = \frac{t(t+1)}{(1+r)^{t+2}}}$$

where $r$ is the periodic yield (YTM/$m$).

**Bond Convexity (Weighted Average):**

$$\boxed{\text{Convexity} = \frac{1}{P_0}\sum_{t=1}^{N}\frac{CF_t \times t(t+1)}{(1+r)^{t+2}}}$$

- $CF_t$: Cash flow at period $t$.
- $P_0$: Full price (present value) of the bond.
- For semiannual bonds, divide by $m^2$ (typically $m = 2$) to annualize.

**3. Example 1: Convexity by Cash-Flow Weighting Bond:** 5-year, 11% annual coupon, price = 86.59, YTM = 15%, MacDur = 4.03.

Exhibit 1: Convexity of Each Cash Flow

| Year | Cash Flow ($) | Weight | CF Convexity Term | Weighted Contribution |
|---|---|---|---|---|
| 1 | 11 | 0.1105 | $(1 \times 2)/1.15^2 = 1.512$ | 0.1669 |
| 2 | 11 | 0.0961 | $(2 \times 3)/1.15^4 = 4.537$ | 0.4359 |
| 3 | 11 | 0.0835 | $(3 \times 4)/1.15^6 = 9.074$ | 0.7583 |
| 4 | 11 | 0.0726 | $(4 \times 5)/1.15^8 = 15.123$ | 1.0979 |
| 5 | 111 | 0.6373 | $(5 \times 6)/1.15^{10} = 22.684$ | 14.4560 |
| | | | **Bond Convexity:** | **16.915** |

$$\boxed{\text{Convexity} = 16.915}$$

**4. Approximate Convexity (Finite-Difference Formula)**

$$\boxed{\text{Approx. Convexity} = \frac{V_- + V_+ - 2V_0}{V_0(\Delta YTM)^2}}$$

where

- $V_-$ = bond price if yield decreases by $\Delta$YTM,
- $V_+$ = bond price if yield increases by $\Delta$YTM,
- $V_0$ = current bond price,
- $\Delta$YTM in decimal form (e.g., 0.005 for 50 bps).

**Example 2: Approximate Convexity**

$$V_0 = 86.59138,\ V_+ = 85.09217,\ V_- = 88.12721,\ \Delta YTM = 0.005$$

$$\text{Approx. Convexity} = \frac{88.12721 + 85.09217 - 2(86.59138)}{86.59138(0.005)^2} = 16.92$$

**Interpretation:** Approximation matches true convexity  16.9.

Exhibit 2: Determinants of Convexity

| **Characteristic** | **Effect on Convexity** |
|---|---|
| Longer maturity | Increases convexity |
| Lower coupon rate | Increases convexity |
| Lower YTM | Increases convexity |
| Greater CF dispersion | Higher convexity (for equal duration) |

**5. Characteristics Affecting Convexity**

## LOS 60.b: Estimate Price Change Using Duration and Convexity

**1. Combined Duration–Convexity Approximation**

$$\boxed{\frac{\Delta P}{P} \approx -(\text{ModDur})\Delta y + \tfrac{1}{2}(\text{Convexity})(\Delta y)^2}$$

where $\Delta y$ = change in yield (decimal).

**Example 3: Price Estimate with Duration + Convexity** **Given:** 5-year 11% bond, $P_0 = 86.59138$, ModDur = 3.50, Convexity = 16.9, $\Delta y = -0.005$.

$$\text{Duration Effect} = -3.50(-0.005) = +0.0175 = 1.75\%$$
$$\text{Convexity Effect} = 0.5(16.9)(-0.005)^2 = +0.000211 = 0.0211\%$$
$$\text{Total \% Change} = 1.75\% + 0.0211\% = 1.7711\%$$
$$\text{Estimated New Price} = 86.59138(1.017711) = \mathbf{88.125}$$

**Interpretation:** Convexity adds a small positive adjustment improving accuracy of duration-only estimates.

**2. Money Duration and Money Convexity**

$$\boxed{\begin{array}{c}\text{Money Duration} = \text{ModDur} \times \text{Full Price (Position)} \\ \text{Money Convexity} = \text{Convexity} \times \text{Full Price (Position)}\end{array}}$$

**Example 4: Money Measures for a $10 million Position**

$$P_0 = 0.8659138 \times 10,000,000 = 8,659,138$$

$$\text{MoneyDur} = 3.50 \times 8,659,138 = 30,306,983$$

$$\text{MoneyConv} = 16.9 \times 8,659,138 = 146,339,432$$

**Estimate Change for** $\Delta y = -0.005$:

$$\text{Duration Effect} = -(30,306,983)(-0.005) = +151,534.9$$

$$\text{Convexity Effect} = 0.5(146,339,432)(-0.005)^2 = +1,829.3$$

$$\text{Total Change} = 151,534.9 + 1,829.3 = 153,364.2$$

$$\text{New Value} = 8,659,138 + 153,364.2 = \mathbf{8,812,502}$$

**Consistency Check:** Matches price estimate 88.125 from previous example.

**3. Convexity Adjustment Interpretation**

- The convexity adjustment is always **positive** for a bond with positive convexity.
- **When yields fall:** Duration-only estimate underestimates price increase → convexity correction adds to price.
- **When yields rise:** Duration-only estimate overestimates price drop → convexity correction reduces loss.

**Figure Concept: Duration vs. Actual Price Curve**
– Duration line: tangent, linear.
– True curve: convex upward.
– Convexity term adds curvature, improving both sides of estimate.

## LOS 60.c: Portfolio Duration and Convexity

### 1. Two Approaches

1. **Aggregate Cash-Flow Approach (Theoretical):** Compute a single duration and convexity based on the portfolio's total cash-flow stream.

   Use aggregate CFs to find $P_0$, $D$, and Convexity.

2. **Weighted-Average Approach (Practical):**

$$\boxed{\begin{aligned} D_p &= \sum_{i=1}^{n} w_i D_i \\ C_p &= \sum_{i=1}^{n} w_i C_i \end{aligned}}$$

   where $w_i$ = market-value weight of bond $i$ in portfolio.

### 2. Limitation of Weighted Approach

- Assumes a **parallel shift** in the yield curve — all maturities change by the same $\Delta$YTM.
- In practice, yield-curve changes often involve twists or steepening/flattening.
- For nonparallel shifts, the weighted-average method will misestimate portfolio price change.

Exhibit 3: Portfolio Duration and Convexity Properties

| Approach | Description / Limitation |
|---|---|
| Aggregate CF | Exact but computationally intensive. Requires full cash-flow projection. |
| Weighted Average | Practical; uses individual bond durations/convexities and market-value weights. Assumes parallel yield shifts. |

## Summary Tables and Key Formulas

## Key Takeaways

- **Convexity** captures the curvature of the bond's price–yield relationship and refines duration estimates.
- **Positive convexity** implies that duration-only estimates understate price gains and overstate price losses.

Exhibit 4: Duration–Convexity–Price Relationship

| Measure | Formula / Interpretation |
|---|---|
| Convexity (cash-flow basis) | $\frac{1}{P_0}\sum \frac{CF_t t(t+1)}{(1+r)^{t+2}}$ |
| Approx. Convexity | $\frac{V_- + V_+ - 2V_0}{V_0(\Delta y)^2}$ |
| Price Change (%) | $-\text{ModDur}\Delta y + 0.5(\text{Convexity})(\Delta y)^2$ |
| Money Duration | ModDur × Full Price |
| Money Convexity | Convexity × Full Price |
| Portfolio Duration | $\sum w_i D_i$ (approx., assumes parallel shift) |
| Portfolio Convexity | $\sum w_i C_i$ (approx., assumes parallel shift) |

- **Approx. Price Change:**

$$\frac{\Delta P}{P} \approx -D_{mod}\Delta y + \tfrac{1}{2}C(\Delta y)^2$$

- **Money Measures:**

$$\text{MoneyDur} = D_{mod} \times P_0$$
$$\text{MoneyConv} = C \times P_0$$

- **Portfolio Measures:**

$$D_p = \sum w_i D_i, \quad C_p = \sum w_i C_i$$

  Valid only under parallel yield-curve shifts.

- **Drivers of Convexity:**
  - ↑ Maturity → ↑ Convexity
  - ↓ Coupon → ↑ Convexity
  - ↓ YTM → ↑ Convexity

# Module 61.1: Curve-Based and Empirical Fixed-Income Risk Measures

## LOS 61.a: Why Effective Duration and Effective Convexity Are Appropriate for Bonds with Embedded Options

### 1. Key Concept: Uncertain Cash Flows

- For **option-free bonds**, cash flows and maturities are known ⇒ YTM-based duration is applicable.

- For **bonds with embedded options** (e.g., callable, putable, MBS):
  - Future cash flows and redemption dates are **uncertain**.
  - Exercise of the embedded option changes timing and amount of CFs.
  - These bonds have **no single well-defined YTM**.

**2. Option Equivalence**

- Callable Bond = Straight Bond − Call Option (issuer owns call right).
- Putable Bond = Straight Bond + Put Option (investor owns put right).
- MBS = Straight Bond − Prepayment Option (borrowers can prepay).

**3. Effective Duration (EffDur)**

$$\boxed{\text{EffDur} = \frac{V_- - V_+}{2V_0\Delta\text{Curve}}}$$

where:

- $V_-$ = bond value when benchmark curve decreases by $\Delta$Curve.
- $V_+$ = bond value when benchmark curve increases by $\Delta$Curve.
- $V_0$ = current bond value.
- $\Delta$Curve = change in benchmark yield (decimal, e.g., 0.01 for 1%).

**Interpretation:**

- Measures price sensitivity to **benchmark yield curve shifts**.
- Reflects the option's effect on future cash flows.
- Assumes spreads for credit and liquidity risk remain constant.

**4. Effective Convexity (EffCon)**

$$\boxed{\text{EffCon} = \frac{V_- + V_+ - 2V_0}{V_0(\Delta\text{Curve})^2}}$$

**Interpretation:**

- Captures curvature of price–yield relation for bonds with embedded options.
- Uses changes in **benchmark curve** rather than YTM.

Exhibit 1: Convexity Behavior by Bond Type

| **Bond Type** | **Convexity Type** | **Reason** |
|---|---|---|
| Option-Free | Positive | Price–yield relation always convex. |
| Callable | Can be Negative (at low yields) | Call price caps price increase; issuer likely to call when rates fall. |
| Putable | Always Positive | Put option limits price drop at high yields. |
| MBS | Negative (similar to callable) | Borrowers prepay when rates fall. |

**5. Convexity Profiles by Bond Type**

**6. Price–Yield Relationships**

- Callable bonds flatten upward (negative convexity) when rates fall.
- Putable bonds flatten downward (positive convexity reinforced) when rates rise.

**Figure Concept:** Callable vs. Option-Free vs. Putable Price–Yield Curves
- Callable bond: price capped at call price (negative convexity region).
- Putable bond: price floor near put price (always positive convexity).

**7. Modified vs. Effective Duration (Option-Free Bonds)**

- For option-free bonds, ModDur  EffDur when yield curve is flat.
- In non-flat curves, $\Delta$ in par curve $\Rightarrow$ nonparallel shift in spot curve.
- Hence, ModDur and EffDur differ slightly due to:
  - Nonparallel shifts of government spot rates.
  - Constant credit spread assumption.

## LOS 61.b: Estimate Price Change Using Effective Duration and Convexity

**1. Formula for Price Change (with Respect to Benchmark Curve)**

$$\frac{\Delta P}{P_0} \approx -(\text{EffDur}) \times \Delta\text{Curve} + \frac{1}{2}(\text{EffCon}) \times (\Delta\text{Curve})^2$$

**2. Example 1: Price Change Using Effective Measures** **Given:** EffDur = 10.5, EffCon = 97.3, $\Delta$Curve = $-0.02$ (200 bps).

$$\begin{aligned}\text{Duration Effect} &= -10.5(-0.02) = +0.21 = 21.0\% \\ \text{Convexity Effect} &= 0.5(97.3)(-0.02)^2 = 0.0195 = 1.95\% \\ \text{Total Change} &= 21.0 + 1.95 = \mathbf{22.95}\%\end{aligned}$$

**Interpretation:** Bond price expected to rise 22.95% for a 200 bp curve drop.

Exhibit 2: Effective vs. Modified Duration

| **Aspect** | **Modified Duration** | **Effective Duration** |
|---|---|---|
| Applicable to | Option-free bonds | Bonds with embedded options |
| Yield Input | Bond's YTM | Benchmark yield curve shift ($\Delta$Curve) |
| CF Certainty | Fixed | Uncertain (depends on option exercise) |
| Risk Captured | Total yield change (benchmark + spread) | Benchmark-only change |
| Use of Models | Analytical | Requires bond-pricing model |

**3. Effective vs. Modified Duration Summary**

## LOS 61.c: Key Rate Duration (Partial Duration)

**1. Concept**

- Effective duration assumes **parallel yield curve shifts**.
- Key Rate Duration (KRD) measures price sensitivity to changes in the benchmark yield for a **specific maturity**, with other yields held constant.
- The sum of all key rate durations equals the effective duration.

$$\boxed{D_{\text{Eff}} = \sum_{i=1}^{n} D_{\text{KeyRate},i}}$$

**2. Application: Measuring Shaping Risk**

- Shaping risk = risk from **nonparallel** yield curve shifts (steepening, flattening, twists).
- KRDs allow assessment of sensitivity at individual maturities (e.g., 2y, 5y, 10y, 30y).

**Example 2: Portfolio Key Rate Duration Portfolio:**

- 50% in 5-year zero-coupon bond ($y = 5\%$).
- 50% in 10-year bond ($y = 6\%$).

**Step 1. Compute Modified Durations:**

$$D_5 = \frac{5}{1.05} = 4.762, \quad D_{10} = \frac{10}{1.06} = 9.434$$

**Step 2. Compute Key Rate Durations:**

$$\text{KRD}_5 = 4.762 \times 0.5 = 2.381, \quad \text{KRD}_{10} = 9.434 \times 0.5 = 4.717$$

**Step 3. Apply Yield Changes:**

$$\Delta y_5 = +0.005, \quad \Delta y_{10} = -0.0025$$

$$\text{Impact (5-year)} = -2.381(0.005) = -0.0119 = -1.19\%$$

$$\text{Impact (10-year)} = -4.717(-0.0025) = +0.0118 = +1.18\%$$

$$\text{Total Change} = -1.19\% + 1.18\% = -\mathbf{0.01}\% \text{ (negligible change).}$$

**Interpretation:** Portfolio is well balanced for this nonparallel yield movement.

Exhibit 3: Key Rate Duration Characteristics

| **Feature** | **Explanation** |
|---|---|
| Definition | Sensitivity to yield change at specific maturity |
| Sum Property | $\sum \text{KRD}_i = \text{EffDur}$ |
| Captures | Nonparallel yield curve shifts (shape changes) |
| Uses | Portfolio immunization, yield-curve twist risk control |

## LOS 61.d: Empirical Duration vs. Analytical Duration

### 1. Analytical Duration

- Based on mathematical models and present value relationships.
- Examples: Macaulay, Modified, and Effective Duration.
- Assume:
  - Parallel yield curve shifts.
  - Constant credit spread.

**2. Empirical Duration**

- Derived from **observed historical data**:

$$\text{Empirical Duration} = \frac{\Delta P/P}{\Delta\text{Benchmark Yield}}$$

- Captures the **actual co-movement** of bond prices and benchmark yields.

**3. When Empirical Duration is Preferred**

- When changes in benchmark yields and spreads are **correlated**.
- Especially for **credit-risky bonds** (e.g., corporates).
- In "flight-to-quality" scenarios:
  - Gov't yields ↓ (prices ↑).
  - Corporate spreads ↑ (prices ↓ or stable).
  - Analytical duration overstates sensitivity; empirical duration is smaller.

Exhibit 4: Analytical vs. Empirical Duration

| **Aspect** | **Analytical Duration** | **Empirical Duration** |
|---|---|---|
| Basis | Model / PV relationship | Historical regression data |
| Assumes | Constant spreads, parallel curve shifts | Real-world correlations (spreads + yields) |
| Best for | Government or low-credit-risk bonds | Corporate or spread-sensitive bonds |
| Example Scenario | Flat curve shift | Flight-to-quality (credit spreads widen) |
| Value of Duration | Typically higher | Typically lower |

## Key Takeaways Summary

- **EffDur / EffCon:** Best for option-embedded bonds (uncertain CFs). Measured w.r.t. benchmark yield curve, not YTM.

$$\frac{\Delta P}{P} \approx -(\text{EffDur})\Delta\text{Curve} + 0.5(\text{EffCon})(\Delta\text{Curve})^2$$

- **Callable Bonds:** May exhibit negative convexity at low yields.

- **Putable Bonds:** Always have positive convexity.
- **Key Rate Duration:** Measures partial sensitivity to specific maturities — captures nonparallel shifts.
- **Empirical Duration:** Based on observed data; preferred for corporate bonds when credit spreads move with yields.

Exhibit 5: Summary Formulas

| **Concept** | **Formula** |
|---|---|
| Effective Duration | $\frac{V_- - V_+}{2V_0 \Delta\text{Curve}}$ |
| Effective Convexity | $\frac{V_- + V_+ - 2V_0}{V_0(\Delta\text{Curve})^2}$ |
| Price Change (%) | $-\text{EffDur}(\Delta\text{Curve}) + 0.5(\text{EffCon})(\Delta\text{Curve})^2$ |
| Key Rate Duration (KRD) | Sensitivity to yield at one maturity (others constant) |
| Portfolio Effective Duration | $\sum w_i D_i$ (parallel shift assumption) |
| Empirical Duration | Observed $\frac{\Delta P/P}{\Delta\text{Benchmark Yield}}$ |

# Module 62.1: Credit Risk

## LOS 62.a: Describe Credit Risk and Its Components

### 1. Definition and Nature of Credit Risk

- **Credit risk:** The risk of loss to a fixed-income investor due to a borrower's failure to pay interest or principal (i.e., failure to service its debt).
- A borrower in default fails to meet contractual debt obligations.
- Credit risk can be analyzed from:
  - **Bottom-up (issuer-specific factors).**
  - **Top-down (macroeconomic and systemic factors).**

### 2. Bottom-Up Credit Factors (The 5 Cs of Credit Analysis)

### 3. Top-Down Credit Factors (The 3 Cs)

Exhibit 1: Bottom-Up Credit Analysis – The 5 Cs

| **Factor** | **Explanation** |
|---|---|
| Capacity | Ability to make timely debt payments (e.g., interest coverage, cash flows). |
| Capital | Net worth and other funding sources that reduce reliance on debt. |
| Collateral | Assets pledged as security for debt repayment. |
| Covenants | Legal restrictions protecting lenders (e.g., leverage limits, payout limits). |
| Character | Integrity and reliability of management or borrower. |

Exhibit 2: Top-Down Factors Affecting Creditworthiness

| **Factor** | **Explanation** |
|---|---|
| Conditions | General macroeconomic environment affecting debt servicing ability. |
| Country | Political/legal stability and fiscal institutions of the borrower's country. |
| Currency | FX fluctuations affecting foreign-denominated debt repayment. |

**4. Key Sources of Repayment by Borrower Type**

- **Corporate Debt:**
  - Secured debt → backed by operations + collateral cash flows.
  - Unsecured debt → backed only by operating CFs and assets.
- **Sovereign Debt:**
  - Backed by tax revenues, tariffs, and fees.
  - Secondary sources: privatizations, additional debt issuance.

**5. Liquidity vs. Insolvency**

- **Illiquidity:** Inability to raise cash (temporary shortfall).
- **Insolvency:** Assets < Liabilities (structural deficiency).

**6. Bond Indenture Clauses**

- **Cross-Default:** Default on one issue ⇒ default on all issues.
- **Pari Passu:** All bonds of same type rank equally in default process.
- For secured debt, creditors have claim to pledged assets + general assets.

**7. Measuring Credit Risk**

$$\boxed{\text{Expected Loss (EL)} = \text{Probability of Default (POD)} \times \text{Loss Given Default (LGD)}}$$

- **Probability of Default (POD):** Annualized probability borrower will fail to make payments.

- **Loss Given Default (LGD):** Monetary or percentage loss incurred if default occurs.
- **Recovery Rate:** Fraction of the claim recovered in default.

$$\text{LGD\%} = \text{Expected Exposure} \times (1 - \text{Recovery Rate})$$

**8. Credit Spread as Compensation for Expected Loss**

$$\boxed{\text{Credit Spread} \approx \text{POD} \times \text{LGD\%}}$$

- If actual credit spread > estimated spread → investor overcompensated.
- If actual credit spread < estimated spread → investor undercompensated.

**Example 1: Expected Loss and Credit Spread**

$$\text{POD} = 3\% = 0.03, \quad \text{Recovery Rate} = 75\%$$
$$\text{LGD\%} = 1 - 0.75 = 0.25$$
$$\text{Estimated Spread} = 0.03 \times 0.25 = 0.0075 = 0.75\%$$

**Actual Spread:** $4\% - 2.5\% = 1.5\%$
⇒ Bond fairly overcompensates for credit risk (1.5% ¿ 0.75%).

**9. Relationship Between Credit Quality and Risk Metrics**

- High credit quality ⇒ low POD, strong fundamentals:

  High EBIT margin, high interest coverage, low debt/EBITDA, high CF/net debt.

- LGD depends on:
  - Seniority of issue (secured vs. unsecured).
  - Collateral value.

Exhibit 3: Example Comparison of Credit Risk Characteristics

| **Issuer Type** | **Seniority** | **POD** | **LGD%** |
|---|---|---|---|
| Investment Grade – Unsecured | Senior Unsecured | Low | Moderate |
| High Yield – Secured | Collateralized | Higher POD | Lower LGD (collateral recovery) |

## LOS 62.b: Credit Ratings – Uses and Limitations

**1. Definition and Purpose**

- Ratings reflect forward-looking **creditworthiness** of:
  - Issuer (corporate or sovereign).
  - Specific debt issue.
- Ratings indicate **expected loss** (POD × LGD).

**2. Common Uses of Ratings**

- Compare credit risk across issuers, sectors, maturities.
- Assess **credit migration risk** (downgrade risk).
- Meet **regulatory, statutory, or contractual** investment requirements.

Exhibit 4: Rating Scale Comparison (S&P, Moody's, Fitch)

| **Category** | **S&P/Fitch** | **Moody's** |
|---|---|---|
| Highest Quality | AAA | Aaa |
| High Grade | AA+, AA, AA- | Aa1, Aa2, Aa3 |
| Upper Medium Grade | A+, A, A- | A1, A2, A3 |
| Lower Medium Grade (Investment Grade) | BBB+, BBB, BBB- | Baa1, Baa2, Baa3 |
| Non-Investment Grade (High Yield) | BB+, BB, BB- | Ba1, Ba2, Ba3 |
| Highly Speculative | B+, B, B- | B1, B2, B3 |
| Substantial Risk / Default | CCC–C / D | Caa–C |

**3. Major Credit Rating Agencies and Scales**

- **Investment Grade:** BBB– / Baa3 or higher.
- **High Yield (Junk):** BB+ / Ba1 or lower.

**4. Limitations of Ratings**

1. **Lag Effect:** Ratings adjust slower than market prices/spreads.
2. **Inherent Uncertainty:** Ratings cannot capture all risks (e.g., litigation, disasters, fraud).
3. **Agency Errors:** Historical overrating (e.g., subprime 2008 crisis).
4. **Split Ratings:** Different agencies may assign different ratings.

**Investor Implication:** Rely on independent analysis — not only on ratings.

## LOS 62.c: Factors Influencing Yield Spreads

**1. Definition**

- **Credit spread risk:** The risk that yield spreads widen due to worsening credit conditions.
- Affects both price level and volatility of credit-risky bonds.

**2. Macroeconomic Factors**

- Credit spreads narrow during economic expansions (low POD).
- Credit spreads widen during recessions (high POD).
- **Investment Grade:** Smaller, less volatile spreads.
- **High Yield:** Larger, more volatile spreads (strong cyclical sensitivity).

**Typical Spread Curve Behavior:**

- Expansions → spreads fall and steepen.
- Recessions → spreads rise and flatten (even invert for HY).

**3. Market Factors**

- **Liquidity Risk:** Cost of trading bonds (bid–offer spread width).
- **Broker/Dealer Regulation:** Limits capital available for market making.
- **Funding Stress:** Increases risk aversion, widens spreads.
- **Supply–Demand Imbalances:** Heavy issuance vs. weak demand widens spreads.

**4. Issuer-Specific Factors**

- Issuer profitability, leverage, coverage ratios.
- Worsening financial condition ⇒ spread widens.
- Analysts compare issuer's spread to peer averages.

**5. Example: Decomposing Yield Spread into Credit and Liquidity Components**
**Given:** 10-year bond, 5% coupon, Bid/Ask = 99.5/100.5, Benchmark yield = 3%.

$$\begin{aligned}
\text{Midprice} &= \frac{99.5 + 100.5}{2} = 100 \\
\text{Bond Yield (mid)} &= 5\% \quad \Rightarrow \text{Spread} = 5 - 3 = 2\% \\
\text{Yield}_{bid} &= 5.065\%, \quad \text{Yield}_{ask} = 4.935\% \\
\text{Liquidity Spread} &= 5.065 - 4.935 = 0.13\% \\
\text{Credit Spread} &= 2 - 0.13 = 1.87\%
\end{aligned}$$

$$\boxed{\text{Total Yield Spread} = \text{Credit Spread} + \text{Liquidity Spread}}$$

Exhibit 5: Summary of Spread Determinants

| Factor Type | Effect on Yield Spread |
|---|---|
| Macroeconomic | Widen in recessions, narrow in expansions |
| Market | Widen with low liquidity or funding stress |
| Issuer-Specific | Widen with weak credit metrics |
| Maturity | Longer maturities → higher spreads (greater POD) |
| Credit Quality | Lower quality → wider spreads, higher volatility |

**6. Estimating Price Change from Spread Change**

$$\boxed{\frac{\Delta P}{P} \approx -(\text{Duration}) \times \Delta\text{Spread} + \tfrac{1}{2}(\text{Convexity})(\Delta\text{Spread})^2}$$

*Replace $\Delta YTM$ with $\Delta Spread$ to measure credit spread risk.*

## Key Takeaways Summary

- **Credit Risk =** Probability of Default (POD) × Loss Given Default (LGD).
- **Expected Loss:**
$$\text{EL} = \text{POD} \times (1 - \text{Recovery Rate})$$
- **Credit Spread Compensation:**
$$\text{Credit Spread} \approx \text{POD} \times \text{LGD}\%$$
- **Ratings:** Useful benchmarks but lag market information.
- **Spread Drivers:**
  - Economic cycle (macro factor)
  - Issuer fundamentals (micro factor)
  - Market liquidity and funding (market factor)
- **Decomposition:**
$$\text{Yield Spread} = \text{Credit Spread} + \text{Liquidity Spread}$$

# Module 63.1: Credit Analysis for Government Issuers

## LOS 63.a: Explain Special Considerations in Evaluating Credit of Sovereign and Non-Sovereign Government Debt Issuers

**1. Overview**

Exhibit 6: Formula Summary

| **Concept** | **Formula / Relationship** |
|---|---|
| Expected Loss (EL) | POD × LGD |
| Loss Given Default (%) | Exposure × (1 − Recovery Rate) |
| Credit Spread | POD × LGD% |
| Total Yield Spread | Credit Spread + Liquidity Spread |
| Spread-Based Price Change | $-\text{Dur}(\Delta\text{Spread}) + 0.5(\text{Convexity})(\Delta\text{Spread})^2$ |
| Investment Grade Threshold | BBB– / Baa3 or higher |
| High Yield Threshold | BB+ / Ba1 or lower |

- **Government (public sector) credit analysis** assesses the ability and willingness of a national or sub-national authority to service its debt obligations.
- Two categories:
  - **Sovereign Issuers:** National governments (countries).
  - **Non-Sovereign Issuers:** Regional, municipal, agency, or supranational bodies.
- **Key distinction:** Sovereigns can control monetary policy (can print money); non-sovereigns cannot.

## Sovereign Government Debt Analysis

### 2. Primary Source of Repayment

- Derived from the government's ability to **tax economic activity**.
- Assessment focuses on the long-term stability and credibility of fiscal and monetary frameworks.

### 3. Qualitative Factors (Five Pillars of Sovereign Creditworthiness)

### 4. Quantitative Factors (Three Core Metrics)

### 5. Special Analytical Considerations for Sovereigns

- Sovereigns can refinance via:
  - Tax increases.
  - Domestic debt issuance.
  - Currency devaluation (via money creation).
- Investors must assess **ability** and **willingness** to repay.
- Political stability and institutional quality are paramount.

Exhibit 1: Qualitative Factors in Sovereign Credit Analysis

| Factor | Description and Analytical Focus |
|---|---|
| **1. Institutions & Policy Framework** | Examines governance quality, political stability, rule of law, property rights, data transparency, and policy credibility. – Economically: enforcement of contracts, culture of debt repayment, data reliability. – Politically: stable regime, peaceful foreign relations. – Willingness to pay is crucial (sovereign immunity = limited legal recourse). |
| **2. Fiscal Flexibility** | Assesses the ability to raise taxes or reduce expenditures to service debt. – Flexibility decreases with rigid spending (e.g., entitlement programs). |
| **3. Monetary Effectiveness** | Measures the independence and credibility of the central bank. – Independent banks can control inflation without political interference. – Limited monetary independence ⇒ higher inflation and currency risk. |
| **4. Economic Flexibility** | Evaluates the structure and diversity of the economy. – High GDP per capita, diversified exports, and stable growth indicate stronger creditworthiness. |
| **5. External Status** | Assesses position in global financial system and foreign reserve adequacy. – Reserve-currency nations can borrow externally in their own currency and sustain higher debt. – Geopolitical risk also affects external standing. |

Exhibit 2: Quantitative Indicators of Sovereign Credit Strength

| Factor | Typical Metrics and Interpretation |
|---|---|
| **1. Fiscal Strength** | – Debt-to-GDP and Debt-to-Revenue ratios: measure debt burden. – Interest-to-GDP or Interest-to-Revenue ratios: measure affordability. ⇓ Lower ratios ⇒ stronger fiscal capacity. |
| **2. Economic Growth and Stability** | – High real GDP growth, large and diversified economy, stable output. – Low volatility of growth ⇒ lower risk. |
| **3. External Stability** | – FX reserves to GDP or external debt ratios ↑ → strong liquidity buffer. – Low short-term external debt (due within 12 months) reduces rollover risk. – Commodity-dependent exporters face concentration risk tied to commodity prices. |

## Non-Sovereign Government Debt

### 6. Types of Non-Sovereign Issuers

### 7. Analytical Distinctions Between Sovereign and Non-Sovereign Debt

- **Monetary Policy:** – Sovereigns can create money; non-sovereigns cannot.

Exhibit 3: Categories of Non-Sovereign Government Issuers

| Issuer Type | Description and Credit Support |
|---|---|
| **Agencies** | Quasi-governmental entities for specific functions (e.g., infrastructure, housing). Backed by statutory or implicit sovereign guarantees → ratings near sovereign level. |
| **Government-Sector Banks / Financing Institutions** | Publicly sponsored intermediaries for strategic projects (e.g., green bonds). Implied sovereign backing; similar ratings to sovereign. |
| **Supranational Organizations** | Entities formed by multiple sovereigns (e.g., World Bank, IMF, IDB). Credit quality depends on support of member nations and callable capital. |
| **Regional / Local Governments** | States, provinces, municipalities. Debt known as **municipal bonds** (in the U.S.). Typically issued as:<br>• **General Obligation (GO) Bonds:** Backed by full faith, credit, and taxing power of issuer.<br>• **Revenue Bonds:** Backed by specific project revenues (airports, toll roads, hospitals). |

- **Budget Constraints:** – Regional governments usually must maintain balanced budgets.
- **Tax Base:** – Ability to service GO bonds depends on local economic strength and tax revenues.
- **Project-Specific Risk:** – Revenue bonds rely solely on project CFs (higher credit risk).

## 8. Revenue Bond Analysis (Corporate-Style Approach)

- Evaluate:
  - Stability and predictability of project revenues.
  - Operating efficiency and cost control.
  - Debt-Service Coverage Ratio (DSCR):

$$\boxed{\text{DSCR} = \frac{\text{Revenue after Operating Costs}}{\text{Interest + Principal Payments}}}$$

  - DSCR $> 1.0$ indicates adequate coverage.
- Focus is similar to corporate credit analysis:
  - Cash flow predictability.

- Leverage and liquidity ratios.
- Contingent support (e.g., state guarantees).

Exhibit 4: Comparison of Sovereign and Non-Sovereign (Municipal) Debt

| **Feature** | **Sovereign Bonds** | **Municipal / Regional Bonds** |
|---|---|---|
| Primary Repayment Source | National tax revenues | Local tax revenues / project revenues |
| Monetary Control | Yes (can print money) | No |
| Budget Requirement | Flexible deficit financing | Often legally required to balance |
| Legal Enforcement | Limited (sovereign immunity) | Legal recourse possible within jurisdiction |
| Currency Risk | Often domestic currency; FX risk for foreign issues | Minimal if local currency |
| Common Bond Types | Treasury bills, notes, bonds | GO bonds, revenue bonds |
| Credit Support | Economic/fiscal/monetary policy credibility | Local tax base or project cash flows |
| Typical Credit Risk Level | Generally lower for reserve-currency sovereigns | Varies by local economic health |

**9. Comparative Example: Sovereign vs. Municipal**

**10. Example: Interpreting Quantitative Ratios for Sovereign Credit Quality**

- **High Credit Quality Sovereign:**
  - Low Debt-to-GDP ($< 50\%$)
  - High FX Reserves to External Debt ($> 100\%$)
  - Low Interest-to-Revenue ($< 10\%$)
  - Stable Real GDP Growth ($\sim$2–4%)
- **Low Credit Quality Sovereign:**
  - High Debt-to-GDP ($> 90\%$)
  - Low FX Reserves to External Debt ($< 30\%$)
  - Rising Inflation and Weak Currency
  - Volatile or negative real GDP growth

## Key Takeaways Summary

- **Sovereign Credit Risk Drivers:**
  1. *Qualitative:* Institutions, Fiscal Flexibility, Monetary Effectiveness, Economic Flexibility, External Status.
  2. *Quantitative:* Fiscal Strength, Economic Growth/Stability, External Stability.
- **Non-Sovereign Issuers:**
  - Agencies, government banks, supranationals, and regional governments.
  - Regional governments issue:
    - **GO Bonds:** Backed by taxing power (lower risk).
    - **Revenue Bonds:** Backed by project CFs (higher risk).
- **Key Ratios:**

$$\text{Debt Burden} = \frac{\text{Debt}}{\text{GDP}}, \quad \text{Debt Affordability} = \frac{\text{Interest}}{\text{Revenue}}, \quad \text{FX Reserve Ratio} = \frac{\text{FX Reserves}}{\text{External Debt}}$$

- Sovereign risk is influenced by both **ability** (economic/fiscal) and **willingness** (institutional/political) to repay.

Exhibit 5: Formula and Concept Summary

| **Concept** | **Formula / Definition** |
|---|---|
| Debt Burden Ratio | Debt/GDP or Debt/Revenue |
| Debt Affordability Ratio | Interest/GDP or Interest/Revenue |
| FX Reserve Ratio | FX Reserves/External Debt |
| DSCR (Revenue Bonds) | (Revenue – Opex)/(Interest + Principal) |
| GO Bond Support | Full faith and taxing power of issuer |
| Revenue Bond Support | Cash flows from specific project |

# Module 64.1: Credit Analysis for Corporate Issuers

## LOS 64.a: Qualitative and Quantitative Factors in Corporate Credit Analysis

### 1. Overview

- Corporate credit analysis evaluates a firm's **likelihood of default (POD)** and potential **loss given default (LGD)**.
- Uses both:
  - **Qualitative factors:** business model, industry risk, governance.
  - **Quantitative factors:** profitability, leverage, coverage, and liquidity ratios.

Exhibit 1: Key Qualitative Factors in Corporate Credit Analysis

| Factor | Description and Implications for Credit Quality |
|---|---|
| **Business Model** | • Predictable, recurring, and diversified cash flows → higher credit quality.<br>• Exposure to cyclical industries or customer concentration → higher risk.<br>• Long-term sustainability and adaptability of business model must be assessed. |
| **Industry Competition** | • Lower competition = stronger pricing power and margins.<br>• Evaluate structural changes, barriers to entry, and future competitive threats. |
| **Business Risk** | • Low volatility in revenue and operating margins → higher creditworthiness.<br>• Business risk sources: issuer-specific, industry-specific, or macroeconomic. |
| **Corporate Governance** | • Fair treatment of debtholders, transparency, compliance with regulations.<br>• Focus areas: – Covenant protection – Accounting quality – Historical management behavior toward creditors. |

**2. Qualitative Factors**

**3. Governance Considerations a) Covenants:**

- **Investment-grade debt:** usually includes *affirmative covenants* — maintain assets, pay taxes, comply with laws.
- **High-yield debt:** includes *negative covenants* — restrict dividend payments, new debt, or asset sales.
- Analysts assess management history for debt-financed equity actions or covenant breaches.

**b) Accounting Policies:**

- Aggressive accounting = red flag:
  - Premature revenue recognition.

  - Capitalizing expenses instead of expensing.
  - Off-balance-sheet financing (e.g., operating leases).
  - Frequent auditor or CFO changes.

**4. Quantitative Factors**

- Quantitative analysis forecasts future financials to assess:
  - Probability of Default (POD).
  - Loss Given Default (LGD).
- **Top-down analysis:** Macroeconomic and industry trends. **Bottom-up analysis:** Company-specific drivers. **Hybrid approach:** Both combined.

**Indicators of Strong Credit Quality:**

- Strong, recurring operating profits.
- Low leverage and debt reliance.
- High interest coverage.
- Ample liquidity (cash and undrawn lines of credit).

## LOS 64.b: Financial Ratios Used in Corporate Credit Analysis

Exhibit 2: Common Cash Flow Metrics

| Metric | Definition and Purpose |
|---|---|
| **EBITDA** | Operating Income + Depreciation + Amortization. Proxy for operating cash flow but ignores capital expenditures and working capital needs. |
| **CFO (Cash Flow from Operations)** | Net cash from core operations. = Net Income + Non-Cash Charges − ΔWorking Capital. Better indicator of cash available for debt service. |
| **FFO (Funds From Operations)** | Net Income + Depreciation + Amortization + Deferred Taxes + Non-Cash Items. Similar to CFO but excludes changes in working capital. |
| **FCF (Free Cash Flow)** | CFO − Capital Expenditures + Net Interest Expense. Represents discretionary cash available to debtholders and shareholders. |
| **RCF (Retained Cash Flow)** | Operating Cash Flow − Dividends. Used to assess retained liquidity after shareholder payouts. |

**1. Key Cash Flow Measures**

Exhibit 3: Key Financial Ratios in Corporate Credit Analysis

| Category | Ratio | Interpretation |
|---|---|---|
| **Profitability** | EBIT Margin = EBIT / Revenue | Higher margin = stronger profitability and higher credit quality. |
| **Coverage** | Interest Coverage = EBIT / Interest Expense | Higher coverage = stronger ability to service debt. |
| **Leverage** | Debt / EBITDA | Lower ratio = lower leverage, stronger credit profile. |
| **Leverage (Cash-based)** | RCF / Net Debt | Higher ratio = better internal funding capacity, lower credit risk. |

**2. Common Credit Ratios**

**3. Example: Comparative Credit Assessment York, Inc. vs. Zale, Inc.**

Exhibit 4: Input Data

| | **York** | **Zale** |
|---|---|---|
| Revenue | \$2,200,000 | \$11,000,000 |
| EBIT | \$550,000 | \$2,250,000 |
| Interest Expense | \$40,000 | \$160,000 |
| Depreciation/Amortization | \$220,000 | \$900,000 |
| Debt | \$1,900,000 | \$2,700,000 |
| CFO | \$300,000 | \$850,000 |
| Dividends | \$30,000 | \$200,000 |
| Cash | \$500,000 | \$1,000,000 |

**Step 1: Profitability**

$$\text{EBIT Margin (York)} = \frac{550,000}{2,200,000} = 25\%, \quad \text{EBIT Margin (Zale)} = \frac{2,250,000}{11,000,000} = 20.5\%$$

$\Rightarrow$ York is more profitable.

**Step 2: Coverage**

$$\text{Interest Coverage (York)} = \frac{550,000}{40,000} = 13.8\times, \quad \text{Interest Coverage (Zale)} = \frac{2,250,000}{160,000} = 14.1\times$$

$\Rightarrow$ Both strong; Zale slightly higher.

**Step 3: Leverage**

$$\text{Debt/EBITDA (York)} = \frac{1,900,000}{770,000} = 2.5\times, \quad \text{Debt/EBITDA (Zale)} = \frac{2,700,000}{3,150,000} = 0.9\times$$

$\Rightarrow$ York more leveraged.

**Step 4: Cash Flow Leverage**

$$\text{RCF/Net Debt (York)} = \frac{270,000}{1,400,000} = 19\%, \quad \text{RCF/Net Debt (Zale)} = \frac{650,000}{1,700,000} = 38\%$$

$\Rightarrow$ Zale generates more retained cash relative to its debt.

**Conclusion:**

- York has higher profitability but also higher leverage $\rightarrow$ slightly lower credit quality.
- Zale stronger liquidity and lower leverage $\rightarrow$ stronger credit standing.

## LOS 64.c: Debt Seniority, Security, and Priority of Claims

### 1. Concept of Seniority

- **Seniority ranking:** Determines priority of repayment in default.
- **Secured Debt:** Backed by collateral; has direct claim on pledged assets.
- **Unsecured Debt:** General claim on issuer's assets and CFs.

Exhibit 5: Priority of Claims

| Rank | Debt Type / Example |
|---|---|
| 1 | First Lien / Mortgage (specific pledged asset) |
| 2 | Senior Secured (second lien) |
| 3 | Junior Secured |
| 4 | Senior Unsecured |
| 5 | Senior Subordinated |
| 6 | Subordinated |
| 7 | Junior Subordinated |

### 2. Debt Hierarchy in Default

- All within same class rank **pari passu** (equal claim priority).
- Secured creditors have claim on collateral + general assets.
- If collateral value ¡ obligation, deficiency ranks pari passu with senior unsecured.

### 3. Recovery and Credit Ratings

- **Recovery rates:** Decline as seniority decreases.
- **Credit risk:** Increases with subordination.
- Rating agencies assign:

- **Corporate Family Rating (CFR):** Issuer-level credit quality (usually senior unsecured).
- **Corporate Credit Rating (CCR):** Issue-specific rating (depends on seniority, collateral, covenants).

**4. Notching and Structural Subordination**

- **Notching:** Adjusting an issue's rating above/below issuer rating to reflect recovery potential.
- **Structural Subordination:** Occurs when subsidiary debt has claim on subsidiary cash flows before parent's debt.
  - Parent bonds effectively subordinated to subsidiary's debt.
  - Common in holding company structures.
- **Implication:** – Highly rated issuers → minimal notching (differences in recovery less relevant). – Lower-rated issuers → more notching (recovery differences matter more).

**5. Practical Considerations in Bankruptcy**

- **Absolute priority rule:** Senior creditors paid before junior ones.
- **In practice:** May be violated if junior creditors or equity accept early settlement to expedite resolution.
- Bankruptcies are costly and slow — value of assets may deteriorate over time.

## Key Takeaways Summary

- **Qualitative Factors:** Stable business model, low competition, low business risk, strong governance.
- **Quantitative Factors:** High margins, high coverage, low leverage, high liquidity.
- **Common Ratios:**

$$\text{EBIT Margin} = \frac{\text{EBIT}}{\text{Revenue}}, \quad \text{Interest Coverage} = \frac{\text{EBIT}}{\text{Interest}}, \quad \text{Debt/EBITDA}, \quad \text{RCF/Net Debt}$$

- **Seniority Hierarchy:**

$$\text{First Lien} > \text{Senior Secured} > \text{Junior Secured} > \text{Senior Unsecured} > \text{Subordinated}$$

- **Notching:** Adjustment between issuer rating (CFR) and issue rating (CCR) based on collateral and seniority.
- **Structural Subordination:** Parent debt is subordinate to subsidiary debt if upstream transfers are restricted.

Exhibit 6: Formula and Concept Summary

| Concept | Formula / Definition |
|---|---|
| EBIT Margin | EBIT / Revenue |
| Interest Coverage | EBIT / Interest Expense |
| Debt / EBITDA | Total Debt / (EBIT + Depreciation + Amortization) |
| RCF / Net Debt | (CFO - Dividends) / (Debt - Cash) |
| Free Cash Flow | CFO - CapEx + Net Interest Expense |
| DSCR (Project Finance) | (Revenue - Opex) / (Interest + Principal) |
| Seniority Order | 1st Lien → Senior Secured → Junior Secured → Senior Unsecured → Subordinated |
| Notching | Difference between Issuer (CFR) and Issue (CCR) ratings |
| Structural Subordination | Parent debt subordinated to subsidiary debt due to cash transfer restrictions |

# Module 65.1: Fixed-Income Securitization

## LOS 65.a: Benefits of Securitization

### 1. Definition and Overview

- **Securitization:** The process of transforming illiquid financial assets (e.g., loans, receivables) into tradable securities known as **asset-backed securities (ABSs)**.
- Connects **capital providers (investors)** directly with **capital users (borrowers)**.
- Removes the originating institution (bank/corporation) from the intermediation process.

Exhibit 1: Three-Step Securitization Process

| Step | Description |
|---|---|
| **1. Origination** | A bank or corporation (the **originator**) creates a pool of loans or receivables (the **collateral**). Example: mortgages, auto loans, credit card receivables. |
| **2. Sale to SPE** | The pool of assets is sold to a **Special Purpose Entity (SPE)**—a separate legal vehicle that owns the assets and is bankruptcy remote. |
| **3. Issuance of ABS** | The SPE issues fixed-income securities (**asset-backed securities**) to investors. These are backed by cash flows from the underlying collateral. |

### 2. The Securitization Process

### 3. Key Terms and Roles

- **Originator:** The bank/corporation that creates and sells the assets (e.g., mortgage lender, auto company).
- **SPE / SPV:** The entity that buys the assets and issues ABSs. It is **bankruptcy remote**.
- **Investors:** Buy ABSs and receive periodic cash flows from the underlying pool.

### 4. Benefits of Securitization

#### (a) Benefits to Issuers (Originators)

Exhibit 2: Advantages of Securitization to Issuers

| **Benefit** | **Explanation** |
|---|---|
| **Increased Lending Capacity** | Originators can re-lend proceeds from selling loans to the SPE, allowing greater loan creation beyond balance sheet limits. |
| **Improved Profitability** | Earn fees for loan origination and for structuring/selling collateral to the SPE. |
| **Lower Regulatory Capital** | Removal of loans from balance sheet lowers capital requirements for banks (Basel capital ratios). |
| **Improved Liquidity** | Converts illiquid assets (e.g., mortgages) into cash; improves liquidity management and risk-return efficiency. |

#### (b) Benefits to Investors

Exhibit 3: Advantages of Securitization to Investors

| **Benefit** | **Explanation** |
|---|---|
| **Tailored Risk–Return Profile** | ABS structures (tranches) allow investors to select exposure suited to their risk appetite (senior vs. junior tranches). |
| **Diversified Access** | Investors gain access to diversified loan pools (e.g., auto loans, mortgages) without having to originate loans themselves. |
| **Enhanced Liquidity** | ABSs are tradable securities — more liquid than the underlying loans. Investors can exit positions more easily. |

Exhibit 4: Macroeconomic and Market Benefits

| Benefit | Explanation |
|---|---|
| **Reduced Liquidity Risk** | ABSs improve market liquidity relative to the underlying assets. |
| **Improved Market Efficiency** | Trading of ABSs allows for equilibrium pricing and better risk distribution. |
| **Lower Funding Costs** | Provides cheaper financing for originators than direct debt/equity issuance. |
| **Lower Leverage Ratios** | Growth via off-balance-sheet financing allows originators to expand without raising leverage. |

**(c) Benefits to Economies and Financial Markets**

**(d) Risks to Investors**

- **Cash flow uncertainty:** Timing and amount of cash flows vary due to prepayments or defaults (e.g., early mortgage repayments).
- **Credit risk:** Defaults or deterioration of collateral credit quality.
- **Systemic risk:** Excessive securitization can amplify systemic shocks (e.g., 2007–2009 financial crisis).

**5. Conceptual Summary**

$$\text{Securitization} = \text{Financial Intermediation by Market Mechanism}$$

$$\text{Capital Providers (Investors)} \longleftrightarrow \text{Borrowers (Collateral Pool)}$$

$$\text{Originator sells assets} \Rightarrow \text{SPE issues ABS} \Rightarrow \text{Investors fund the collateral.}$$

## LOS 65.b: Parties and Roles in Securitization

**1. Example: Fred Motor Company Securitization**

- **Scenario:** Fred Motor Company originates $1 billion in auto loans (50,000 contracts).
- **Goal:** Remove these loans from its balance sheet and free up cash for new lending.

**Structure:**

Fred (Originator/Seller) $\Rightarrow$ Sells $1bn auto loans $\Rightarrow$ Auto Loan Trust (SPE) $\Rightarrow$ Issues ABSs to Investors

**Roles:**

- **Seller / Depositor:** Fred Motor Company — originates and sells loans to SPE.
- **SPE / Issuer (Auto Loan Trust):** Buys loans and issues ABS to investors.
- **Servicer:** Fred continues servicing loans (collects payments, handles repossessions).
- **Trustee:** Independent third party ensuring proper cash flow distribution and reporting.

**2. Bankruptcy Remoteness**

- The SPE is legally independent from the originator.
- If the originator (Fred) becomes insolvent, ABS investors still have full claim on collateral cash flows.
- This legal separation ensures the ABS structure is **bankruptcy remote**.

Exhibit 5: Legal and Contractual Protections

| **Document** | **Purpose** |
|---|---|
| **Purchase Agreement** | Defines terms of sale of collateral from originator to SPE. |
| **Prospectus / Offering Memorandum** | Outlines terms of ABS issuance, fee structure, and waterfall (cash flow allocation). |
| **Trust Agreement** | Governs duties of trustee, reporting, and safeguarding of investor interests. |

**3. Cash Flow Waterfall**

- Payments from borrowers → received by servicer → distributed by SPE:
  1. Servicing and trustee fees.
  2. Interest payments to ABS investors.
  3. Principal repayments.
- Any losses are absorbed according to tranche hierarchy (senior → mezzanine → junior).

Exhibit 6: Roles in a Typical Securitization

| **Party** | **Also Known As** | **Primary Function** |
|---|---|---|
| Originator | Seller, Depositor | Creates and sells pool of loans to SPE. |
| SPE / SPV | Issuer, Trust | Purchases collateral, issues ABS to investors. |
| Servicer | Administrator | Collects payments, enforces contracts, manages delinquencies. |
| Trustee | Custodian | Ensures proper operation of trust and payment waterfall; provides reporting. |
| Investors | Bondholders | Provide funding; receive periodic principal and interest. |

**4. Distinction Between Key Entities**

### Key Takeaways Summary

- **Definition:** Securitization converts illiquid financial assets into tradable ABSs through an SPE.
- **Benefits to Issuers:**
  - Free up balance sheet capital.
  - Generate fee income.
  - Reduce leverage and regulatory capital.
- **Benefits to Investors:**
  - Tailored risk/return profiles (via tranching).
  - Access to diverse collateral.
  - Greater liquidity.
- **Benefits to Economy:**
  - Improved liquidity and efficiency.
  - Broader credit availability and reduced funding costs.
- **Risks:** Cash flow uncertainty and credit risk of collateral.
- **Key Entities:**

$$\text{Originator (Seller)} \rightarrow \text{SPE (Issuer)} \rightarrow \text{Investors (ABS Holders)}$$

- **Bankruptcy Remote:** SPE's assets and liabilities are isolated from those of the originator.

# Module 66.1: Asset-Backed Security (ABS) Instrument and Market Features

## LOS 66.a: Covered Bonds – Characteristics, Risks & Differences from ABS

**1. Definition**

- **Covered Bonds:** Senior debt obligations of banks, backed by a segregated **cover pool** of assets (usually mortgages) that remain **on-balance-sheet** of the issuer.
- No SPE is created; the bondholders have **dual recourse**:
  1. To the **cover pool**, and
  2. To the issuer's **unencumbered assets**.

Exhibit 7: Formula and Concept Summary

| **Concept** | **Definition / Formula / Note** |
|---|---|
| ABS (Asset-Backed Security) | Security backed by loan or receivable cash flows. |
| SPE / SPV | Separate legal entity that issues ABS; bankruptcy remote. |
| Tranche | Division of ABS into senior, mezzanine, and junior risk classes. |
| Cash Flow Waterfall | Sequential payment: servicing fees → interest → principal → residuals. |
| Originator's Benefit | Liquidity, reduced capital requirements, fee income. |
| Investor's Benefit | Tailored risk exposure and improved liquidity. |
| Risks to Investor | Credit risk + prepayment (timing) risk. |
| Documents | Purchase Agreement, Prospectus, Trust Agreement. |
| Bankruptcy Remoteness | SPE assets unaffected by originator default. |

Exhibit 1: Covered Bond Structure vs. ABS

| **Feature** | **Covered Bond Characteristics** |
|---|---|
| Legal Structure | Issued directly by a bank; cover pool remains on issuer's balance sheet (no SPE). |
| Investor Recourse | Dual recourse → claims on cover pool + issuer's other assets. |
| Credit Enhancements | • **Overcollateralization:** Collateral value ¿ bond value.<br>• **Loan-to-Value (LTV) Limits:** Maximum LTV ratios on mortgages.<br>• **Third-party Monitoring:** Independent supervision of cover pool quality. |
| Capital Treatment | Assets remain on balance sheet → no capital relief (unlike ABS). |
| Collateral Maintenance | Dynamic pool → nonperforming/prepaid loans must be replaced to preserve coverage. |
| Tranching | Usually none – covered bonds are single-class instruments. |

## 2. Structure and Credit Enhancements

## 3. Types of Covered Bonds

- **Hard-bullet:** Default occurs immediately upon missed payment → accelerated repay-

ment.

- **Soft-bullet:** Maturity may be extended (up to 1 year) before default is triggered.
- **Conditional Pass-Through:** If payments still due at maturity, bond converts to pass-through mode — cash flows are passed through as recovered from collateral.

Exhibit 2: Covered Bonds vs. ABS

| **Aspect** | **Covered Bond** | **Asset-Backed Security (ABS)** |
|---|---|---|
| Issuer | Bank or financial institution | Special Purpose Entity (SPE) |
| Collateral Ownership | Remains on issuer's balance sheet | Sold to SPE (bankruptcy remote) |
| Recourse | Dual recourse (cover pool + issuer) | Limited recourse (to SPE assets only) |
| Capital Relief for Bank | No (assets stay on balance sheet) | Yes (loans removed from balance sheet) |
| Collateral Maintenance | Dynamic (replacement of non-performing loans) | Static (pool fixed at issuance) |
| Yield Level | Lower (due to dual recourse and overcollateralization) | Higher (riskier, no recourse to issuer) |
| Tranching | Rare | Common (subordination structure) |

**4. Comparison to Traditional ABS**

## LOS 66.b: Credit Enhancement Structures in Securitization

**1. Purpose** To mitigate investor exposure to credit losses in the collateral pool. Two broad categories:

- **Internal Enhancements:** Built into the ABS structure.
- **External Enhancements:** Provided by third-party guarantees (not covered in detail here).

**2. Internal Credit Enhancement Methods**

**3. Example: Senior–Subordinated Structure Investor Example:**

$$\text{Tranche B Investor: Principal} = \$80 \text{ m, Coupon} = 4\% + 1.5\% = 5.5\%$$

$$\text{If losses} = \$50 \text{ m} \rightarrow \text{Tranche C absorbs } \$30 \text{ m, Tranche B absorbs } \$20 \text{ m.}$$

$$\text{New Principal} = \$60 \text{ m},$$

$$\begin{aligned}\text{Coupon Payment} &= \$550{,}000 \times \frac{60}{80} \\ &= \$412{,}500.\end{aligned}$$

Exhibit 3: Forms of Internal Credit Enhancement

| Type | Mechanism and Example |
|---|---|
| **Overcollateralization** | Collateral value > face value of ABS. Example: Collateral \$600 m vs. ABS \$500 m → \$100 m excess = 16.7% buffer. Defaults smaller or equal 16.7% absorbed without loss to ABS investors. |
| **Excess Spread** | Interest income on collateral > coupon promised to ABS investors. The difference builds up a reserve fund to cover losses. |
| **Subordination (Credit Tranching)** | ABS divided into classes (senior, mezzanine, junior). Losses allocated from bottom to top → junior tranche absorbs first losses. Provides credit protection for senior tranches. |

Exhibit 4: Credit Tranching Illustration

| Tranche | Principal (\$ millions) | Loss Absorption Order |
|---|---|---|
| A (Senior) | 400 | Protected until losses ¿ \$110 m |
| B (Subordinated) | 80 | Absorbs losses from \$30 m → \$110 m |
| C (Junior/Equity) | 30 | First to absorb losses up to \$30 m |

**4. Key Insight**

- Tranching redistributes credit risk similar to a corporate capital structure (senior → subordinated → equity).
- Senior tranches may achieve credit ratings higher than the originator due to structural protection.

## LOS 66.c: Types and Risks of Non-Mortgage ABSs

**1. Collateral Spectrum** ABSs can be backed by any cash-flow-producing assets (e.g., credit cards, auto loans, solar loans, royalties, franchise fees).

**2. Credit Card ABSs**

**3. Solar ABSs (ESG Linked)**

**4. Amortizing vs. Non-Amortizing Collateral**

- **Amortizing:** Principal reduces over time (e.g., auto loans, mortgages).
- **Non-amortizing:** Principal repayment unscheduled (e.g., credit cards).

Exhibit 5: Credit Card ABS Features

| **Feature** | **Explanation** |
|---|---|
| Collateral Type | Non-amortizing receivables (credit card balances). |
| Cash Flows | Finance charges, membership/late fees, principal repayments. |
| Interest Rate Type | Fixed or floating (often with a cap). |
| Lockout / Revolving Period | • Investors receive only interest and fees.<br>• Principal repayments used to buy new receivables.<br>• Maintains pool size and avoids prepayment risk. |
| Amortization Period | After lockout ends, principal flows through to investors. |
| Early Amortization Trigger | Activated to preserve credit quality when defaults rise. |

Exhibit 6: Characteristics of Solar ABSs

| **Feature** | **Explanation** |
|---|---|
| Collateral Type | Loans to homeowners for solar panel installation. |
| Security | Secured by solar equipment or as junior mortgage on property. |
| Investor Attraction | ESG appeal — supports renewable energy transition. |
| Credit Quality | Typically high (qualified borrowers with energy savings). |
| Credit Enhancements | Overcollateralization, excess spread, subordination. |
| Pre-Funding Period | Allows pool expansion post-issuance for diversification. |
| Key Risk | New asset class — performance not yet tested over full credit cycle. |

## LOS 66.d: Collateralized Debt Obligations (CDOs)

### 1. Definition and Structure

- **CDO:** Structured security issued by an SPE backed by a pool of debt obligations.
- Includes:
  - **CBO – Collateralized Bond Obligation:** Corporate / emerging-market bonds.
  - **CLO – Collateralized Loan Obligation:** Leveraged bank loans.
- Managed by a **collateral manager** who actively buys / sells assets to meet return objectives.

### 2. CLO Types

### 3. Collateral Quality and Investor Protection

- **Coverage Tests:** Ensure cash flows suffice to meet debt service.
- **Overcollateralization Tests:** If breached, cash flows diverted to repay senior tranches.

Exhibit 7: Types of CLO Structures

| Type | Description |
|---|---|
| **Cash-Flow CLO** | Payments to investors from interest & principal cash flows of underlying loans. |
| **Market-Value CLO** | Manager trades collateral to profit from price changes; payments depend on market values. |
| **Synthetic CLO** | Credit exposure via derivatives (e.g., credit default swaps); SPE does not own collateral directly. |

- **Diversification Tests:** Limits exposure to single borrower / industry.
- **Rating Constraints:** Caps on low-rated (CCC) assets within portfolio.

Exhibit 8: ABS vs. CDO Structures

| Feature | ABS | CDO |
|---|---|---|
| Collateral Type | Consumer loans (e.g., mortgages, auto, credit cards) | Corporate / leveraged loans / bonds |
| Collateral Manager | None (static pool) | Active management of portfolio |
| Cash Flow Source | Loan repayments from borrowers | Cash flows or market value changes from managed assets |
| Risk Profile | Linked to borrower defaults and prepayments | Linked to credit spreads & manager performance |
| Complexity | Moderate | Higher (due to active management and leverage) |

**4. Comparison of ABS and CDO Cash Flow Mechanics**

## Key Concept Summary

- **Covered Bonds:** On-balance-sheet debt with dual recourse; dynamic cover pool; low yield.
- **Credit Enhancement Methods:** Overcollateralization, Excess Spread, Subordination.
- **Non-Mortgage ABSs:**
  - *Credit Card ABS:* Non-amortizing receivables with lockout period.
  - *Solar ABS:* Amortizing green loans with ESG appeal and pre-funding provision.
- **CDOs / CLOs:** SPE-issued securities backed by actively managed debt portfolios; include cash-flow, market-value, and synthetic types.

- **Key Risks:** Credit risk, prepayment risk, structural complexity.

Exhibit 9: Formula and Concept Recap

| Concept | Formula / Explanation |
|---|---|
| Overcollateralization Ratio | $\frac{\text{Collateral Value – ABS Face Value}}{\text{Collateral Value}}$ |
| Excess Spread | Interest Income on Collateral – Interest Payments to Investors |
| Tranching Hierarchy | Senior → Mezzanine → Junior (Loss Absorption Order) |
| Dual Recourse | Bondholders' claim on cover pool + unencumbered issuer assets |
| Lockout Period | Interest only; principal reinvested in new receivables |
| Amortization Trigger | Early principal repayment when credit quality deteriorates |
| CLO Tests | Coverage, Overcollateralization, Diversification, Rating Limits |

# Module 67.1: Mortgage-Backed Security (MBS) Instrument and Market Features

## LOS 67.a: Prepayment Risk and Time Tranching in Securitizations

### 1. Definition of Prepayment Risk

- **Prepayments:** Principal repayments made by borrowers earlier than scheduled (via refinancing, selling, or prepayment).
- **Prepayment Risk:** Uncertainty regarding the timing of these principal repayments — causes cash flows to differ from expected.

Exhibit 1: Components of Prepayment Risk

| Risk Type | Description |
|---|---|
| **Contraction Risk** | Risk that prepayments are faster than expected (usually when interest rates fall). *Effects:* Cash received sooner ⇒ lower reinvestment rates, smaller price appreciation (negative convexity). |
| **Extension Risk** | Risk that prepayments are slower than expected (usually when rates rise). *Effects:* Longer duration, lower PV (cash flows discounted longer at higher rate). |

### 2. Interest Rate Link and Price Behavior

- When rates ↓ → refinancing ↑ → faster prepayments → **contraction risk**.
- When rates ↑ → refinancing ↓ → slower prepayments → **extension risk**.
- **MBS exhibit negative convexity:** Price rises less when yields fall, but declines more when yields rise.

### 3. Time Tranching (Reallocation of Prepayment Risk)

- Technique to redistribute prepayment risk among multiple bond classes (tranches) with different maturities.
- **Purpose:** Align prepayment exposure with investor preference.
  - Short tranches → protect against **extension risk**.
  - Long tranches → protect against **contraction risk**.
- Example structure: **Sequential Pay CMO (see LOS 67.c)** — Tranche 1 paid first, Tranche 2 second, etc.

## LOS 67.b: Residential Mortgage Loan Characteristics

### 1. Collateral and Legal Structure

- **Collateral:** Residential real estate (house, apartment).
- **First Lien:** Lender has legal claim to property if borrower defaults (foreclosure process).

Exhibit 2: Common Features of Residential Mortgage Loans

| Feature | Description |
|---|---|
| **Prepayment Penalty** | Fee for repaying principal early (common in Europe, rare in U.S.); mitigates prepayment risk for lenders. |
| **Recourse Loan** | Lender can claim both collateral property and other borrower assets (common in Europe). |
| **Nonrecourse Loan** | Lender limited to collateral claim; borrower can walk away if underwater (common in U.S.). |

### 2. Common Loan Features

### 3. Risk Ratios

$$\text{LTV} = \frac{\text{Loan Amount}}{\text{Appraised Property Value}}, \quad \text{DTI} = \frac{\text{Monthly Debt Payments}}{\text{Monthly Pretax Income}}$$

**Example:**

$$\text{Loan} = 300{,}000, \quad \text{Value} = 400{,}000 \Rightarrow LTV = 75\%$$

$$\text{PMT} = 1{,}932.9, \quad \text{Monthly Income} = 6{,}667 \Rightarrow DTI = 29\%$$

- Lower LTV ⇒ higher borrower equity ⇒ lower default probability.
- Lower DTI ⇒ higher repayment capacity ⇒ lower credit risk.

### 4. Loan Categories

- **Prime Loans:** Low LTV, low DTI, good credit score.
- **Subprime Loans:** High LTV, high DTI, lower credit quality, higher default probability.

Exhibit 3: Agency vs. Non-Agency RMBS

| **Feature** | **Agency RMBS** | **Non-Agency RMBS** |
|---|---|---|
| Issuer | Gov't or GSE (e.g., Fannie Mae, Freddie Mac) | Private banks or institutions |
| Guarantee | Gov't or GSE guarantee | No gov't guarantee |
| Collateral Type | Conforming (meets underwriting standards) | Often nonconforming or subprime |
| Credit Enhancement | Implicit/explicit guarantee | Structural (insurance, tranching, etc.) |
| Example Risk Event | None (low risk) | 2007–2009 subprime crisis losses |

### 5. Agency vs. Non-Agency RMBS

## LOS 67.c: Residential MBS Types and Cash Flow Characteristics

### 1. Mortgage Pass-Through Securities (MPTS)

- Represent claims on cash flows from a pool of mortgages (principal + interest, minus servicing/insurance fees).
- **Pass-Through Rate (Net Coupon):** Coupon on MBS = WAC – fees.
- Cash flows include scheduled repayments + unscheduled prepayments.

### 2. Key Metrics

$$\text{WAM} = \frac{\sum(w_i \times \text{Mortgage Maturity})}{\sum w_i}, \quad \text{WAC} = \frac{\sum(w_i \times \text{Mortgage Rate})}{\sum w_i}$$

where $w_i = \frac{\text{Outstanding Balance of Loan } i}{\text{Total Balance of Pool}}$.

### 3. Collateralized Mortgage Obligations (CMOs)

- Structured securities backed by RMBS or mortgage pools.
- Redistribute prepayment risk among tranches.
- Expand investor base by matching duration and risk preferences.

### 4. Example: Sequential Pay Tranches

$$\text{Tranche A} \rightarrow \text{Principal Paid First}; \quad \text{Tranche B} \rightarrow \text{Next in Line}$$

- Tranche A: Lower extension risk.
- Tranche B: Lower contraction risk.

Exhibit 4: Common CMO Tranche Types and Features

| **Tranche Type** | **Description / Risk Exposure** |
|---|---|
| **Z-Tranche (Accrual)** | No interest paid initially; interest accrues to principal. Paid after accrual period ends. Lowest priority. |
| **Principal-Only (PO)** | Receives only principal. Benefits from faster prepayments (higher returns). |
| **Interest-Only (IO)** | Receives only interest. Harmed by faster prepayments (fewer coupon payments). |
| **Floating-Rate / Inverse Floaters** | Coupon linked to reference rate (LIBOR or SOFR). Inverse floaters move opposite to rate changes. |
| **Residual (Equity) Tranche** | Last in payment priority; absorbs residual risk and cash flows. |
| **Planned Amortization Class (PAC)** | Receives predictable payments within defined prepayment speed range; supported by support tranche. |
| **Support Tranche (Companion)** | Absorbs excess or shortfall in prepayments to stabilize PAC payments; higher prepayment risk. |

### 5. Other CMO Tranche Types

## LOS 67.d: Commercial Mortgage-Backed Securities (CMBS)

### 1. Definition and Collateral

- CMBS backed by commercial mortgages on income-producing real estate: apartments, offices, shopping centers, hotels, warehouses, etc.
- Collateral often less diversified (few or single large property loans).

**2. Key Ratios for Credit Analysis**

$$\text{DSCR} = \frac{\text{Net Operating Income (NOI)}}{\text{Debt Service Payment}}, \quad \text{LTV} = \frac{\text{Loan Amount}}{\text{Property Value}}$$

- **Higher DSCR** → better ability to service debt.
- **Lower LTV** → higher equity cushion, lower credit risk.

Exhibit 5: CMBS Call Protection Mechanisms

| Type | Description |
|---|---|
| **Prepayment Lock-out** | Prohibits prepayment for 2–5 years. |
| **Prepayment Penalty Points** | Fee (as %) charged on prepaid principal (e.g., 2 points = 2%). |
| **Defeasance** | Borrower replaces loan with government securities producing identical payments — allows sale of property but preserves lender cash flow. |
| **Structural Call Protection** | Sequential-pay tranching at CMBS level ensures priority-based repayment order. |

**3. Loan-Level and Structural Call Protection**

**4. Balloon Maturity and Balloon Risk**

- **Partially amortizing loans:** Large principal balance due at maturity = **Balloon payment.**
- **Balloon risk:** Borrower may fail to refinance → term extended (**extension risk**).
- Lender may enter "workout period" with modified terms.

**5. CMBS vs. RMBS Comparison**

## Key Concept Summary

- **Prepayment Risk:** Timing uncertainty of principal repayments.

  Contraction Risk (faster prepayment) vs. Extension Risk (slower prepayment)

- **Time Tranching:** Distributes prepayment risk across tranches (short vs. long).
- **Residential Loans:** Characterized by LTV, DTI, recourse type, prepayment terms.
- **Agency RMBS:** Gov't or GSE guaranteed; conforming standards.

Exhibit 6: RMBS vs. CMBS Comparison

| **Feature** | **RMBS** | **CMBS** |
|---|---|---|
| Collateral Type | Residential homes | Commercial income-producing property |
| Borrower Type | Individuals | Real estate investors / companies |
| Cash Flow Source | Mortgage payments from homeowners | Rental income (via tenants) |
| Loan Type | Fully amortizing | Often partially amortizing (balloon maturity) |
| Risk Focus | Borrower credit quality | Property NOI, LTV, DSCR |
| Prepayment Risk | Contraction and extension | Limited via call protection |
| Balloon Risk | Rare | Common (extension form) |

- **Non-Agency RMBS:** Private issue; structural credit enhancements.
- **MBS Structures:**
  - **Pass-Throughs:** Direct flow of principal + interest.
  - **CMOs:** Structured tranches redistributing prepayment risk.
  - **PAC/Support Tranches:** Stabilize payments across prepayment speed ranges.
- **CMBS:** Backed by commercial properties; key ratios = DSCR, LTV; risks = call protection, balloon risk.

Exhibit 7: Formula Recap

| **Concept** | **Formula / Definition** |
|---|---|
| Loan-to-Value (LTV) | $\frac{\text{Loan Amount}}{\text{Property Value}}$ |
| Debt-to-Income (DTI) | $\frac{\text{Monthly Debt Payments}}{\text{Monthly Gross Income}}$ |
| Debt Service Coverage Ratio (DSCR) | $\frac{\text{Net Operating Income}}{\text{Debt Service Payment}}$ |
| Weighted Average Maturity (WAM) | Weighted avg. of mortgage maturities by balance |
| Weighted Average Coupon (WAC) | Weighted avg. of mortgage rates by balance |
| Pass-Through Rate | WAC – servicing/insurance fees |