

# Portfolio Management

## MODULE 20.1: Historical Risk and Return

### Learning Objective

**LOS 20.a:** Describe characteristics of the major asset classes that investors consider in forming portfolios.

#### 1. Overview: The Risk–Return Tradeoff

**Concept:** Investors face a fundamental tradeoff between **expected return** and **risk (volatility)** across asset classes.

- Higher expected returns are associated with higher risk.
- Lower-risk assets provide more stability but lower long-term returns.

**Empirical Observation (U.S. Data, 1926–2017):**

- Small-capitalization stocks had the **highest average returns** and **highest standard deviation (risk)**.
- Large-cap stocks had lower average returns and volatility than small-cap stocks.
- Long-term government bonds had moderate risk and return.
- Treasury bills (T-bills) had the **lowest risk and lowest return**.

#### 2. Comparative Summary of Major Asset Classes

Asset Class	Avg. Nominal Return	Risk (Std. Dev.)	Key Characteristics
Small-Cap Stocks	Highest ( $\approx 12\text{--}13\%$ )	Highest ( $\approx 30\%$ )	High growth potential, highly volatile.
Large-Cap Stocks	Moderate ( $\approx 10\%$ )	Moderate ( $\approx 20\%$ )	Broad diversification, lower risk than small-caps.
Long-Term Gov. Bonds	Lower ( $\approx 6\%$ )	Low-Moderate ( $\approx 10\%$ )	Fixed income, sensitive to interest rate changes.
T-Bills (Short-Term Gov. Securities)	Lowest ( $\approx 3\%$ )	Lowest ( $\approx 1\%$ )	Near risk-free, used as a proxy for the risk-free rate.

**Verbal explanation:** Across decades of data, small firms offered greater reward potential but at much higher variability. Safe assets like T-bills barely beat inflation but provide security and liquidity.

### 3. Real vs. Nominal Returns

#### Definitions:

$$\text{Real Return} = \text{Nominal Return} - \text{Inflation Rate}$$

**Example:** If the nominal return on large-cap stocks is 10% and average inflation is 2.7%,

$$\text{Real Return} = 10\% - 2.7\% = 7.3\%$$

#### Historical Evidence (U.S. 1926–2017):

- **T-Bills:** Real return 0.5%
- **Large-cap stocks:** Real return 7.3%
- Real returns are more stable than nominal returns because inflation varies significantly year to year.

#### Interpretation:

- Inflation erodes purchasing power.
- Long-term investors must focus on **real returns** to maintain wealth.
- Nominal volatility partly reflects inflation fluctuations, not just asset performance.

## 4. Risk Characteristics: Variability and Distribution of Returns

**Standard deviation ( $\sigma$ )** measures the dispersion of returns — the most common risk metric. However, historical data show returns are **not perfectly normal (Gaussian)**.

**Two important deviations:**

- **Skewness** ( $\neq 0$ ): Measures asymmetry of return distribution.
  - **Negative skew:** frequent small gains, few large losses (common in equities).
  - **Positive skew:** frequent small losses, few large gains.
- **Kurtosis** ( $\neq 3$ ): Measures “fat tails” or frequency of extreme outcomes.
  - **Excess kurtosis > 0:** More extreme outcomes than a normal distribution — greater downside and upside volatility.
  - Real-world return distributions often have **fat tails and negative skew**, indicating higher crash risk.

**Verbal Explanation:** While models often assume normally distributed returns, real data show more frequent extreme losses. Thus, relying solely on mean and variance underestimates downside risk.

## 5. Liquidity as an Additional Investment Characteristic

**Definition:** Liquidity refers to how easily and quickly an asset can be sold at its fair market value without significant price discount.

**Key points:**

- **High liquidity:** Assets like large-cap stocks and T-bills can be traded quickly with minimal price impact.
- **Low liquidity:** Assets like emerging market securities, thinly traded corporate bonds, or private equity can be difficult to sell.
- Liquidity risk is reflected in **higher required returns** for illiquid assets.

**Example (verbal):** An investor holding a rare municipal bond might have to sell it at a discount in a hurry, even if its intrinsic value hasn't changed — this is a liquidity premium.

**Effect on Expected Return:**

$$E(R_{\text{illiquid}}) = E(R_{\text{liquid}}) + \text{Liquidity Premium}$$

## 6. Key Relationships Summary Table

Concept	Key Relationship or Interpretation
Risk–Return Tradeoff	Higher risk $\Rightarrow$ higher expected return
Real Return	Nominal Return – Inflation
Negative Skew	Frequent small gains, rare large losses
Excess Kurtosis	Fat tails — more extreme events than normal distribution
Liquidity	Ease of converting to cash; illiquid assets demand higher returns

## 7. Key Takeaways

- **Major asset classes differ in risk and return:** Small-cap stocks are the riskiest and most rewarding; T-bills are the safest with the lowest return.
- **Real returns matter more than nominal:** Inflation-adjusted returns better measure long-term purchasing power.
- **Return distributions are not normal:** They are typically negatively skewed and have excess kurtosis — more frequent extreme losses.
- **Liquidity influences returns:** Illiquid assets require a premium for their trading constraints.
- Overall, investors must consider both quantitative measures (mean, variance) and qualitative factors (liquidity, distribution shape) when constructing portfolios.

## MODULE 20.2: Risk Aversion and Optimal Portfolio Selection

### Learning Objectives

- **LOS 20.b:** Explain risk aversion and its implications for portfolio selection.
- **LOS 20.c:** Explain the selection of an optimal portfolio, given an investor's utility (risk aversion) and the capital allocation line.

### 1. Risk Aversion: Definition and Investor Preferences

**Concept:** A **risk-averse investor** prefers less risk to more risk when expected returns are equal. Financial models assume all rational investors are risk averse.

#### Investor Classifications:

Investor Type	Attitude Toward Risk	Behavioral Preference
Risk-Averse	Dislikes risk; prefers certainty.	Chooses lower-risk investment for same return.
Risk-Neutral	Indifferent to risk.	Cares only about expected return; no risk preference.
Risk-Seeking (Risk-Loving)	Prefers more risk.	Chooses higher-risk investment even if expected return is equal.

#### Example (Gamble Illustration):

Gamble: 50% chance of \$100, 50% chance of \$0  $\Rightarrow E(\text{Payoff}) = \$50$

- **Risk-Averse:** Prefers a certain \$50 to the gamble.

- **Risk-Neutral:** Indifferent between gamble and certain \$50.
- **Risk-Seeking:** Prefers gamble to certain \$50.

**Implication:** For the same expected return, risk-averse investors always choose the portfolio with the lower standard deviation ( $\sigma$ ).

## 2. Investor Utility Function and Indifference Curves

**Utility Function:**

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:

- $U$  = utility (satisfaction level)
- $E(R)$  = expected return of portfolio
- $\sigma^2$  = variance of portfolio return
- $A$  = coefficient of risk aversion ( $A > 0$  for risk-averse investors)

**Interpretation:**

- $E(R)$ : provides positive utility (benefit).
- $\frac{1}{2}A\sigma^2$ : risk penalty term; higher for more risk-averse investors.

**Implications:**

- Higher  $A \rightarrow$  investor dislikes risk strongly  $\rightarrow$  steeper indifference curve.
- Lower  $A \rightarrow$  investor tolerates risk  $\rightarrow$  flatter indifference curve.

### Indifference Curves (Qualitative Description)

- Each curve shows combinations of  $E(R)$  and  $\sigma$  that provide the same utility.
- Curves slope **upward** for risk-averse investors — more risk requires more return.
- **Higher curves** represent higher utility levels.

**Example (Verbal Illustration):**

- Investor A (highly risk-averse): requires 8% return for 10% risk.
- Investor B (less risk-averse): satisfied with 6% return for same 10% risk.
- Therefore, A's indifference curve is steeper.

### 3. Combining Risk-Free and Risky Assets

**Portfolio Composition:** A portfolio may consist of:

Risk-Free Asset (return  $R_f, \sigma = 0$ ) and Risky Portfolio (return  $E(R_R), \sigma_R$ )

**Portfolio Expected Return:**

$$E(R_P) = w_R E(R_R) + (1 - w_R) R_f$$

**Portfolio Standard Deviation:**

$$\sigma_P = w_R \sigma_R$$

where:

- $w_R$  = proportion invested in the risky portfolio
- $(1 - w_R)$  = proportion invested in the risk-free asset

**Interpretation:**

- Increasing  $w_R$  increases both expected return and risk linearly.
- Relationship between  $E(R_P)$  and  $\sigma_P$  is a straight line.

**Example:** If 40% invested in risky portfolio ( $E(R_R) = 10\%, \sigma_R = 15\%$ ) and 60% in risk-free asset ( $R_f = 3\%$ ):

$$E(R_P) = 0.4(10\%) + 0.6(3\%) = 5.8\%, \quad \sigma_P = 0.4(15\%) = 6\%$$

### 4. The Capital Allocation Line (CAL)

**Equation of CAL:**

$$E(R_P) = R_f + \left( \frac{E(R_R) - R_f}{\sigma_R} \right) \sigma_P$$

**Interpretation:**

- The CAL represents all possible combinations of the risk-free asset and the risky portfolio.
- Its slope is the **Sharpe Ratio**:

$$\text{Sharpe Ratio} = \frac{E(R_R) - R_f}{\sigma_R}$$

- A higher slope implies a better risk-return tradeoff.

**Economic Meaning:**

- If investors can borrow/lend at  $R_f$ , any desired risk-return point along the CAL can be reached.

- Lending (investing in  $R_f$ ): moves left on CAL  $\rightarrow$  lower return, lower risk.
- Borrowing (leveraging): moves right on CAL  $\rightarrow$  higher return, higher risk.

**Numerical Illustration:** Suppose:

$$R_f = 2\%, \quad E(R_R) = 8\%, \quad \sigma_R = 10\%$$

$$\text{Slope of CAL} = \frac{8\% - 2\%}{10\%} = 0.6$$

For a portfolio with  $\sigma_P = 5\%$ :

$$E(R_P) = 2\% + 0.6(5\%) = 5\%$$

## 5. Two-Fund Separation Theorem

**Statement:** All investors, regardless of risk aversion, will:

1. Hold the same optimal risky portfolio (the “market portfolio”).
2. Combine it with the risk-free asset in different proportions.

**Implications:**

- Portfolio selection can be divided into two independent decisions:
  1. Identify the optimal risky portfolio.
  2. Allocate between that portfolio and the risk-free asset based on personal risk tolerance.

## 6. Selecting the Optimal Portfolio (Utility Maximization)

**Goal:** Find the combination of risky and risk-free assets that maximizes the investor’s expected utility.

**Optimization Condition:** Investor chooses the portfolio where:

$$\text{Slope of Indifference Curve} = \text{Slope of CAL}$$

i.e., the point of tangency between investor’s preferences and market opportunities.

**Mathematical Derivation:**

$$\max_{w_R} U = R_f + w_R(E(R_R) - R_f) - \frac{1}{2}A(w_R\sigma_R)^2$$

First-order condition:

$$\frac{\partial U}{\partial w_R} = 0 \Rightarrow w_R^* = \frac{E(R_R) - R_f}{A\sigma_R^2}$$

**Interpretation:**

- $w_R^*$ : optimal proportion of wealth in the risky portfolio.
- As risk aversion ( $A$ ) increases  $\rightarrow$  smaller  $w_R^*$ .
- As risk premium ( $E(R_R) - R_f$ ) increases  $\rightarrow$  larger  $w_R^*$ .
- As portfolio risk ( $\sigma_R^2$ ) increases  $\rightarrow$  smaller  $w_R^*$ .

**Example:**

$$E(R_R) = 8\%, \quad R_f = 3\%, \quad \sigma_R = 10\%, \quad A = 4$$

$$w_R^* = \frac{8 - 3}{4(10^2)} = \frac{5}{400} = 0.0125 \Rightarrow 1.25\%$$

If  $A = 1$ :

$$w_R^* = \frac{5}{100} = 0.05 \Rightarrow 5\%$$

**Interpretation:** The less risk-averse investor ( $A = 1$ ) takes a higher allocation to the risky asset.

## 7. Portfolio Choices for Investors with Different Risk Aversions

Investor Type	Risk Aversion (A)	Indifference Curve Shape	Portfolio Composition
Highly Risk-Averse	Large	Steep	Mostly risk-free assets, little risky portfolio.
Moderately Risk-Averse	Medium	Moderate slope	Balanced allocation between risky and risk-free.
Less Risk-Averse	Small	Flat	Mostly risky assets, little risk-free portion.
Risk-Neutral	0	Horizontal	100% risky portfolio (maximizes return).

## 8. Conceptual Summary (Verbal Explanation)

- Risk aversion determines how much extra return investors require to bear extra risk.
- Utility curves represent investors' satisfaction level given risk and return.
- Combining a risky portfolio with a risk-free asset yields a straight line (CAL) of possible portfolio outcomes.
- The optimal portfolio is chosen at the tangency between the CAL and the investor's highest attainable indifference curve.
- All investors hold the same optimal risky portfolio (market portfolio), differing only in how much they mix it with the risk-free asset.
- More risk-averse investors hold more in the risk-free asset; less risk-averse investors hold more of the risky portfolio.

## 9. Key Takeaways

- **Risk Aversion:** Investors prefer less risk for the same return; quantified by the coefficient  $A$ .
- **Utility Maximization:** Optimal portfolio maximizes  $U = E(R) - \frac{1}{2}A\sigma^2$ .
- **Capital Allocation Line (CAL):** Linear relationship between risk and return combining risk-free and risky assets.
- **Two-Fund Separation:** All investors hold the same risky portfolio and mix it with the risk-free asset differently.
- **Diversification Benefit:** Risk-free asset allows investors to reach their desired risk level without altering market efficiency.

## MODULE 20.3: Portfolio Standard Deviation

### Learning Objectives

- **LOS 20.d:** Calculate and interpret the mean, variance, and covariance (or correlation) of asset returns based on historical data.
- **LOS 20.e:** Calculate and interpret portfolio standard deviation.

### 1. Measures of Risk and Return for Individual Assets

In finance, the **variance** and **standard deviation** of returns are fundamental measures of risk. They quantify how much an investment's returns deviate from its expected (mean) return.

#### 1.1 Mean (Expected) Return

$$\bar{R} = \frac{1}{T} \sum_{t=1}^T R_t$$

where:

- $R_t$ : return in period  $t$
- $T$ : number of observed periods

**Interpretation:** The mean represents the central tendency (average performance) of the investment.

## 1.2 Population and Sample Variance

**Population variance:**

$$\sigma^2 = \frac{\sum_{t=1}^T (R_t - \mu)^2}{T}$$

**Sample variance:**

$$s^2 = \frac{\sum_{t=1}^T (R_t - \bar{R})^2}{T - 1}$$

**Notes:**

- In finance, we usually use **sample variance** because we work with historical samples.
- Standard deviation ( $\sigma$ ) is the square root of variance — it is expressed in the same units as returns.

**Interpretation:**

- Higher variance or standard deviation = greater dispersion around the mean = higher risk.

## 2. Covariance and Correlation Between Two Assets

Covariance and **correlation** measure how two assets move together. They are essential for understanding diversification effects in portfolios.

### 2.1 Covariance Formula

$$\text{Cov}_{1,2} = \frac{\sum_{t=1}^T (R_{t,1} - \bar{R}_1)(R_{t,2} - \bar{R}_2)}{T - 1}$$

where:

- $R_{t,1}, R_{t,2}$ : returns on assets 1 and 2 in period  $t$
- $\bar{R}_1, \bar{R}_2$ : average returns of assets 1 and 2
- $T$ : number of observations

**Interpretation:**

- $\text{Cov}_{1,2} > 0$ : assets move together (positive relationship).
- $\text{Cov}_{1,2} < 0$ : assets move in opposite directions (negative relationship).
- $\text{Cov}_{1,2} = 0$ : no linear relationship.

**Units:** Covariance is measured in *returns squared*, making it difficult to compare directly.

## 2.2 Correlation Coefficient

To standardize covariance, we divide by the product of standard deviations:

$$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2}$$

### Properties of Correlation:

- $-1 \leq \rho_{1,2} \leq +1$
- $\rho = +1$ : perfectly positively correlated — assets move exactly together.
- $\rho = -1$ : perfectly negatively correlated — assets move exactly opposite.
- $\rho = 0$ : no linear relationship.

**Interpretation:** Correlation captures both the direction and strength of co-movement between assets.

### Example: Calculating Mean, Variance, Covariance, and Correlation

**Given:** Returns (%) over 3 years:

Year	Asset A (%)	Asset B (%)
1	10	12
2	8	9
3	6	7

#### Step 1: Mean returns

$$\bar{R}_A = (10 + 8 + 6)/3 = 8\%, \quad \bar{R}_B = (12 + 9 + 7)/3 = 9.33\%$$

#### Step 2: Variance of Asset A

$$s_A^2 = \frac{(10 - 8)^2 + (8 - 8)^2 + (6 - 8)^2}{3 - 1} = \frac{8}{2} = 4 \quad \Rightarrow \sigma_A = 2\%$$

#### Variance of Asset B:

$$s_B^2 = \frac{(12 - 9.33)^2 + (9 - 9.33)^2 + (7 - 9.33)^2}{2} = 6.66 \quad \Rightarrow \sigma_B = 2.58\%$$

#### Step 3: Covariance

$$\text{Cov}_{A,B} = \frac{(10 - 8)(12 - 9.33) + (8 - 8)(9 - 9.33) + (6 - 8)(7 - 9.33)}{2} = 5.16$$

#### Step 4: Correlation

$$\rho_{A,B} = \frac{5.16}{(2)(2.58)} = 1.00$$

**Interpretation:** Assets A and B are perfectly positively correlated.

### 3. Portfolio Variance and Standard Deviation (LOS 20.e)

A portfolio's total risk depends on:

1. The risk of each individual asset ( $\sigma_1, \sigma_2$ ),
2. The proportion invested in each asset ( $w_1, w_2$ ),
3. The correlation between asset returns ( $\rho_{1,2}$ ).

#### 3.1 Formula for Portfolio Variance (Two Assets)

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}$$

and the **portfolio standard deviation**:

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}}$$

#### Interpretation:

- The third term ( $2w_1w_2\sigma_1\sigma_2\rho_{1,2}$ ) represents the **co-movement effect**.
- Portfolio risk decreases as correlation  $\rho_{1,2}$  decreases.
- Diversification benefit: combining less-than-perfectly correlated assets reduces overall portfolio risk.

#### 3.2 Special Correlation Cases

Correlation Value	Interpretation
$\rho = +1$	No diversification benefit; portfolio risk = weighted average of individual risks.
$\rho = 0$	Partial diversification; risk is lower than weighted average.
$\rho = -1$	Perfect diversification possible; risk can be eliminated with proper weights.

#### 3.3 Example: Calculating Portfolio Standard Deviation

Given:

- $w_{stocks} = 0.3, \sigma_{stocks} = 20\%$
- $w_{bonds} = 0.7, \sigma_{bonds} = 12\%$
- Correlation  $\rho = 0.60$

### Step 1: Portfolio Variance

$$\sigma_P^2 = (0.3)^2(0.20)^2 + (0.7)^2(0.12)^2 + 2(0.3)(0.7)(0.20)(0.12)(0.60)$$

$$\sigma_P^2 = 0.0036 + 0.007056 + 0.006048 = 0.016704$$

### Step 2: Portfolio Standard Deviation

$$\sigma_P = \sqrt{0.016704} = 0.1293 = 12.93\%$$

### Step 3: Perfectly Positively Correlated Case ( $\rho = +1$ )

$$\sigma_P = (0.3)(20\%) + (0.7)(12\%) = 14.4\%$$

#### Interpretation:

- When correlation = +1, no diversification benefit → linear combination of risks.
- When correlation  $\neq +1$ , portfolio risk (12.93%) is less than the weighted average (14.4%).
- Diversification reduces total risk without reducing expected return.

## 4. Conceptual Interpretation of Diversification

#### Why diversification works:

- The imperfect correlation between asset returns means their ups and downs offset each other partially.
- As long as  $\rho_{1,2} < +1$ , total portfolio risk is reduced below the weighted average of individual risks.
- The lower the correlation, the greater the benefit.

#### Extreme Case:

- $\rho = -1$ : portfolio can theoretically achieve zero standard deviation if asset weights are chosen perfectly.

**General Insight:** Diversification cannot eliminate all risk — only the portion specific to individual assets (unsystematic risk). Systematic market-wide risk remains even in well-diversified portfolios.

## 5. Summary Table of Key Formulas and Relationships

Concept	Formula / Interpretation
Mean Return	$R = \frac{1}{T} \sum R_t$
Sample Variance	$s^2 = \frac{\sum(R_t - R)^2}{T-1}$
Covariance	$\text{Cov}_{1,2} = \frac{\sum(R_{t,1} - \bar{R}_1)(R_{t,2} - \bar{R}_2)}{T-1}$
Correlation	$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2}$
Portfolio Variance	$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}$
Diversification Condition	Lower $\rho_{1,2} \Rightarrow$ Lower $\sigma_P$

## 6. Key Takeaways

- **Variance and standard deviation** measure the volatility of individual asset returns.
- **Covariance and correlation** measure how assets move together; correlation is a standardized, bounded measure.
- **Portfolio risk** depends not only on individual risks but also on how asset returns co-move.
- **Diversification** reduces portfolio risk when assets are not perfectly correlated ( $\rho < +1$ ).
- The benefit of diversification increases as correlation approaches zero or becomes negative.

## MODULE 20.4: The Efficient Frontier

### Learning Objectives

- **LOS 20.f:** Describe the effect on a portfolio's risk of investing in assets that are less than perfectly correlated.
- **LOS 20.g:** Describe and interpret the minimum-variance and efficient frontiers of risky assets and the global minimum-variance portfolio.

### 1. Effect of Correlation on Portfolio Risk (LOS 20.f)

**Concept:** The correlation ( $\rho_{12}$ ) between asset returns determines the degree of diversification benefit in a portfolio. Portfolio risk depends not only on individual asset risks but also on how returns move together.

#### 1.1 Portfolio Variance Formula (Two-Asset Case)

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

where:

- $w_1, w_2$ : portfolio weights ( $w_1 + w_2 = 1$ )
- $\sigma_1, \sigma_2$ : individual standard deviations
- $\rho_{12}$ : correlation coefficient between the two asset returns

#### Interpretation:

- The third term  $2w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$  represents the **interaction effect** — the impact of correlation.
- Portfolio variance is reduced whenever  $\rho_{12} < +1$ .
- The lower (or more negative) the correlation, the greater the diversification benefit.

## 1.2 Perfect Positive Correlation ( $\rho_{12} = +1$ )

If two assets move perfectly together:

$$\sigma_P = w_1\sigma_1 + w_2\sigma_2$$

This means portfolio risk is simply the **weighted average** of individual risks — no diversification benefit.

**Example:** 25% invested in Asset 1 ( $\sigma_1 = 20\%$ ) and 75% in Asset 2 ( $\sigma_2 = 10\%$ ):

$$\sigma_P = 0.25(20\%) + 0.75(10\%) = 12.5\%$$

There is **no reduction in risk** because returns move perfectly together.

## 1.3 Less Than Perfect Correlation ( $-1 < \rho_{12} < +1$ )

As correlation decreases:

$$\sigma_P < w_1\sigma_1 + w_2\sigma_2$$

→ Portfolio risk falls below the weighted average.

**Interpretation:**

- Correlation of +1: maximum risk (no diversification).
- Correlation of 0: moderate diversification; covariance term = 0.
- Correlation of -1: maximum diversification; covariance term becomes negative.

## 1.4 Perfect Negative Correlation ( $\rho_{12} = -1$ )

In this special case, the two assets move in exactly opposite directions. There exists a combination of weights  $(w_1, w_2)$  such that portfolio standard deviation can be reduced to zero:

$$\sigma_P = 0$$

→ A risk-free portfolio can theoretically be constructed, even using risky assets.

## 1.5 Example: Portfolio Risk as Correlation Changes

**Given:**

$$\sigma_1 = 25\%, \quad \sigma_2 = 18\%, \quad w_1 = w_2 = 0.5$$

$$\sigma_P^2 = 0.5^2(0.25^2) + 0.5^2(0.18^2) + 2(0.5)(0.5)(0.25)(0.18)\rho_{12}$$

**Compute for various correlations:**

Correlation	Portfolio Variance ( $\sigma_P^2$ )	Portfolio Std. Dev. ( $\sigma_P$ )
+1.0	0.0441	21.0%
+0.5	0.0304	17.4%
0.0	0.0235	15.3%
-0.5	0.0166	12.9%

### **Interpretation:**

- As correlation decreases, portfolio risk ( $\sigma_P$ ) falls.
- When  $\rho_{12} = -0.5$ , risk reduction is substantial.
- The lower the correlation, the greater the **diversification benefit**.

### **Key Insight:**

$$\text{Diversification benefit} \propto (1 - \rho_{12})$$

→ Holding assets with low or negative correlations provides the greatest reduction in overall risk.

## **2. Minimum-Variance and Efficient Frontiers (LOS 20.g)**

### **2.1 Concept Overview**

When combining two or more risky assets, each possible combination (set of weights) gives a distinct portfolio with:

- An expected return:  $E(R_P) = w_1E(R_1) + w_2E(R_2)$
- A risk level (standard deviation):  $\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}}$

### **Plotting all possible portfolios:**

- Creates a curve showing the tradeoff between expected return and risk.
- This curve is known as the **minimum-variance frontier**.

### **2.2 Minimum-Variance Frontier (MVF)**

**Definition:** The set of all portfolios that provide the **lowest risk** () for each possible expected return level.

### **Interpretation:**

- Every point on this curve represents an optimal portfolio for a given return target.
- Portfolios below the MVF are inefficient — they have unnecessary risk for the same return.

**Mathematically:** For each target return  $E(R_T)$ , minimize portfolio variance:

$$\min_{w_1, w_2} \sigma_P^2 \quad \text{subject to } E(R_P) = E(R_T)$$

### 2.3 Efficient Frontier (EF)

**Definition:** The upper half of the minimum-variance frontier that offers:

Highest expected return for a given level of risk.

**Properties:**

- Portfolios below the efficient frontier are suboptimal.
- Portfolios above the frontier are unattainable (impossible combinations).
- Rational, risk-averse investors will always choose portfolios on the efficient frontier.

**Interpretation in Words:**

- The efficient frontier forms a curve that begins with the least-risk (global minimum-variance) portfolio and bends upward as expected return increases with risk.
- Each investor selects a point on the efficient frontier based on personal risk tolerance.

### 2.4 Global Minimum-Variance Portfolio (GMVP)

**Definition:** The single portfolio on the minimum-variance frontier that has the **lowest possible standard deviation** among all combinations of risky assets.

**Characteristics:**

- It lies at the bottom (leftmost point) of the minimum-variance frontier.
- It serves as the foundation of the efficient frontier — all efficient portfolios lie above it.

**Mathematically:**

$$\frac{d\sigma_P^2}{dw_1} = 0 \quad \Rightarrow \quad w_1^{GMVP} = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

where  $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$ .

### 2.5 Example: Efficient Frontier Construction (Verbal Explanation)

**Step 1:** Start with two risky assets with given expected returns, standard deviations, and correlations. **Step 2:** Calculate expected return and risk for multiple weight combinations (e.g., 0%, 25%, 50%, 75%, 100% in Asset 1). **Step 3:** Plot each combination — the lower boundary of all points forms the **minimum-variance frontier**. **Step 4:** Retain only the upper half (where higher returns accompany higher risks) — that is the **efficient frontier**. **Step 5:** Identify the lowest point on this curve — the **global minimum-variance portfolio (GMVP)**.

**Intuitive Meaning:**

- Portfolios on the efficient frontier dominate all portfolios below it.
- Portfolios below the frontier are inefficient because they offer lower returns for the same risk.

### 3. Diversification and the Shape of the Efficient Frontier

#### Effect of Correlation:

- $\rho_{12} = +1$ : Frontier is a straight line — no diversification.
- $0 < \rho_{12} < +1$ : Frontier becomes concave — some diversification benefit.
- $\rho_{12} < 0$ : Frontier becomes more bowed — strong diversification benefit.
- $\rho_{12} = -1$ : Possible to achieve zero-risk (risk-free) portfolio.

#### Interpretation in Words:

- The curve's "bowing" shape visually reflects the degree of diversification.
- The more negatively correlated the assets, the more the curve bends leftward (lower risk for same return).

### 4. Conceptual Summary

- Portfolio risk is not a simple average of individual asset risks — it depends on how returns co-move.
- Correlation less than  $+1$  allows risk reduction through diversification.
- The **minimum-variance frontier** represents all lowest-risk portfolios for given returns.
- The **efficient frontier** (upper part) shows portfolios offering the highest return for each risk level.
- The **global minimum-variance portfolio (GMVP)** is the least-risk combination of risky assets.
- Investors will only select portfolios on the efficient frontier since they dominate all others in risk-return space.

### 5. Key Takeaways Table

Concept	Key Insight / Formula
Portfolio Variance	$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$
Perfect Positive Correlation ( $\rho = +1$ )	No diversification; $\sigma_P = w_1\sigma_1 + w_2\sigma_2$
Zero Correlation ( $\rho = 0$ )	Partial diversification; risk reduction.
Perfect Negative Correlation ( $\rho = -1$ )	Complete diversification; possible zero-risk combination.
Minimum-Variance Frontier	Set of portfolios with lowest $\sigma$ for each $E(R_P)$ .
Efficient Frontier	Upper portion of MVF; highest $E(R_P)$ for each $\sigma_P$ .
Global Minimum-Variance Portfolio	Lowest-risk portfolio among all risky portfolios.
Diversification Benefit	Increases as $\rho_{12}$ decreases.

## MODULE 21.1: Systematic Risk and Beta

### Learning Objectives

- **LOS 21.a:** Describe the implications of combining a risk-free asset with a portfolio of risky assets.
- **LOS 21.b:** Explain the capital allocation line (CAL) and the capital market line (CML).
- **LOS 21.c:** Explain systematic and nonsystematic risk, including why an investor should not expect additional return for bearing nonsystematic risk.
- **LOS 21.d:** Explain return-generating models (including the market model) and their uses.
- **LOS 21.e:** Calculate and interpret beta.

### 1. Combining a Risk-Free Asset with a Risky Portfolio (LOS 21.a)

**Concept:** Adding a risk-free asset (with zero variance and zero correlation with risky assets) to a risky portfolio creates a **linear relationship** between portfolio risk and return.

#### 1.1 Portfolio Equations

$$E(R_P) = w_P E(R_R) + (1 - w_P) R_f$$
$$\sigma_P = w_P \sigma_R$$

where:

- $E(R_P)$ : expected return on the combined portfolio
- $E(R_R)$ : expected return on the risky portfolio
- $R_f$ : risk-free rate
- $\sigma_P$ : standard deviation of portfolio
- $w_P$ : weight invested in the risky portfolio

#### Implications:

- As  $w_P$  increases, both risk and expected return rise proportionally.
- The relationship between risk and return is a straight line through  $(R_f, 0)$  and  $(E(R_R), \sigma_R)$ .
- The line is called the **Capital Allocation Line (CAL)**.

## 2. Capital Allocation Line (CAL) and Capital Market Line (CML) (LOS 21.b)

### 2.1 The Capital Allocation Line (CAL)

**Equation:**

$$E(R_P) = R_f + \left( \frac{E(R_R) - R_f}{\sigma_R} \right) \sigma_P$$

**Interpretation:**

- $R_f$ : y-intercept (return at zero risk).
- Slope =  $\frac{E(R_R) - R_f}{\sigma_R}$  — the **Sharpe Ratio** of the risky portfolio.
- The higher the slope, the better the risk–return tradeoff.

**Borrowing and Lending:**

- If investors can borrow and lend at  $R_f$ , they can achieve portfolios beyond the risky portfolio (rightward extension of CAL).
- Lend → invest partly in risk-free asset (low risk, low return).
- Borrow → leverage risky portfolio (high risk, high return).

### 2.2 The Capital Market Line (CML)

**Concept:** If all investors have the same expectations (homogeneous expectations) about risk, return, and correlations, they all face the same efficient frontier. They will all hold the same optimal risky portfolio — the **market portfolio**.

**CML Equation:**

$$E(R_P) = R_f + \frac{E(R_M) - R_f}{\sigma_M} \sigma_P$$

where:

- $E(R_M)$ : expected return on the market portfolio
- $\sigma_M$ : standard deviation of the market portfolio
- $E(R_P), \sigma_P$ : expected return and standard deviation of any efficient portfolio

**Slope of the CML:**

$$\text{Slope} = \frac{E(R_M) - R_f}{\sigma_M} = \text{market Sharpe ratio}$$

**Interpretation:**

- All efficient portfolios lie on the CML.

- CML represents the best possible risk–return combinations.
- Individual investors differ only in their allocation between the market portfolio and the risk-free asset (risk preference).

### Passive vs. Active Strategies:

- **Passive strategy:** Hold the market portfolio and risk-free asset in some proportion (index fund + Treasury bills).
- **Active strategy:** Deviate from market weights based on perceived mispricing (overweight undervalued assets, underweight overvalued ones).

## 3. Systematic and Unsystematic Risk (LOS 21.c)

### 3.1 Risk Components of a Security

Total risk ( $\sigma_{Total}^2$ ) can be decomposed into:

$$\sigma_{Total}^2 = \sigma_{Systematic}^2 + \sigma_{Unsystematic}^2$$

where:

- **Systematic Risk:** Market-related risk that cannot be diversified away (e.g., interest rates, inflation, GDP growth).
- **Unsystematic Risk:** Firm-specific or idiosyncratic risk (e.g., product recall, management change, strike).

### 3.2 Diversification Effect

**Observation:** As the number of securities in a portfolio increases, total risk decreases — but only up to a point. Beyond approximately 30–40 securities, further diversification provides little additional risk reduction.

Risk Type	Description and Compensation
Systematic (Market) Risk	Common to all securities; cannot be eliminated. Investors are compensated for bearing it.
Unsystematic (Firm-Specific) Risk	Can be eliminated by diversification; no additional expected return.

#### Example (Conceptual):

- A biotech firm with one drug in trials has high total risk — mostly unsystematic.
- A machine tool manufacturer, exposed to business cycles, has lower total risk but higher systematic risk.
- Capital market theory: only systematic risk is priced → the manufacturer may have higher expected return despite lower total risk.

### 3.3 Key Insight

Investors are compensated only for systematic risk.

Unsystematic (diversifiable) risk offers no premium because it can be costlessly diversified away in a broad portfolio.

## 4. Return Generating Models (LOS 21.d)

**Purpose:** To estimate the expected return of a security based on its sensitivity to one or more risk factors.

### 4.1 General Multifactor Model

$$E(R_i) - R_f = \beta_{i1}E(F_1) + \beta_{i2}E(F_2) + \cdots + \beta_{ik}E(F_k)$$

where:

- $E(R_i) - R_f$ : expected excess return of Asset  $i$
- $F_1, F_2, \dots, F_k$ : systematic risk factors (e.g., GDP growth, inflation, firm size)
- $\beta_{ij}$ : sensitivity (factor loading) of Asset  $i$  to factor  $j$

**Types of Factors:**

- **Macroeconomic:** GDP growth, interest rates, inflation.
- **Fundamental:** Firm earnings, book-to-market ratio, firm size.
- **Statistical:** Factors extracted by data analysis (e.g., principal components).

**Fama–French–Carhart Model:**

$$E(R_i) - R_f = \beta_M E(R_M - R_f) + \beta_{SMB} E(\text{Size}) + \beta_{HML} E(\text{Value}) + \beta_{MOM} E(\text{Momentum})$$

- SMB = Small Minus Big (size effect)
- HML = High Minus Low (value effect)
- MOM = Momentum (Carhart's addition)

### 4.2 Single-Factor (Market) Model

$$E(R_i) - R_f = \beta_i [E(R_M) - R_f]$$

**Interpretation:**

- Market portfolio return is the single systematic factor.
- $\beta_i$  measures sensitivity to market movements.

### 4.3 Market Model (Regression Form)

$$R_i = \alpha_i + \beta_i R_M + e_i$$

where:

- $R_i$ : actual return on security  $i$
- $R_M$ : market return
- $\alpha_i$ : intercept (expected return independent of market)
- $\beta_i$ : slope coefficient (systematic risk measure)
- $e_i$ : residual (firm-specific or unsystematic return)

**Expected Return:**  $E(R_i) = \alpha_i + \beta_i E(R_M)$

**Abnormal Return:**  $e_i = R_i - (\alpha_i + \beta_i R_M)$

**Use:**

- Estimate  $\beta_i$  empirically via regression of past returns.
- Identify security performance relative to the market (alpha).

## 5. Beta: Definition, Calculation, and Interpretation (LOS 21.e)

### 5.1 Definition of Beta ( $\beta_i$ )

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)}$$

**Alternative form using correlation:**

$$\beta_i = \rho_{iM} \left( \frac{\sigma_i}{\sigma_M} \right)$$

where:

- $\rho_{iM}$ : correlation between Asset  $i$  and the market
- $\sigma_i$ : standard deviation of Asset  $i$
- $\sigma_M$ : standard deviation of market returns

### 5.2 Interpretation of Beta

Beta Value	Interpretation
$\beta = 1.0$	Asset has same systematic risk as the market.
$\beta > 1.0$	Asset is more volatile; amplifies market movements.
$\beta < 1.0$	Asset is less volatile; dampens market movements.
$\beta = 0$	No relation to market movements (e.g., risk-free asset).
$\beta < 0$	Moves opposite to the market (e.g., gold, hedges).

### 5.3 Example: Calculating Beta

**Example 1 (using correlation):**

$$\sigma_M = 20\%, \sigma_A = 30\%, \rho_{AM} = 0.8$$

$$\beta_A = 0.8 \times \frac{30\%}{20\%} = 1.2$$

→ Asset A has 20% more systematic risk than the market.

**Example 2 (using covariance):**

$$\text{Cov}(R_A, R_M) = 0.048, \text{Var}(R_M) = (0.20)^2 = 0.04$$

$$\beta_A = \frac{0.048}{0.04} = 1.20$$

→ Consistent with previous calculation.

### 5.4 Estimating Beta (Empirical Regression)

Regress excess returns on the asset against excess market returns:

$$(R_i - R_f) = \alpha_i + \beta_i(R_M - R_f) + e_i$$

**Interpretation:**

- $\beta_i$ : slope of regression line (systematic risk measure).
- $\alpha_i$ : intercept (abnormal performance, or Jensen's alpha).
- $e_i$ : residual (unsystematic, random variation).

**Security Characteristic Line (SCL):** This regression line graphically shows how a security's returns vary with market returns. Its slope =  $\beta_i$ ; its scatter (residuals) = unsystematic risk.

## 6. Conceptual Summary

- Adding a risk-free asset to risky portfolios forms a straight line of possible combinations (CAL).
- With homogeneous expectations, all investors hold the same market portfolio — leading to the Capital Market Line (CML).
- Portfolio risk divides into:
  - **Systematic risk:** cannot be diversified away; priced.
  - **Unsystematic risk:** can be diversified away; not priced.
- **Return generating models** estimate expected returns from exposure to systematic factors.

- **Market model** isolates the portion of return explained by market movements.
- **Beta** measures an asset's sensitivity to market risk — only this component is relevant to required returns.

## 7. Key Formula Summary

Concept	Formula
Expected Portfolio Return	$E(R_P) = w_P E(R_R) + (1 - w_P) R_f$
Portfolio Std. Dev. (Risk-Free + Risky)	$\sigma_P = w_P \sigma_R$
Capital Market Line	$E(R_P) = R_f + \frac{E(R_M) - R_f}{\sigma_M} \sigma_P$
Total Risk Decomposition	$\sigma_{Total}^2 = \sigma_{Systematic}^2 + \sigma_{Unsystematic}^2$
Market Model	$R_i = \alpha_i + \beta_i R_M + e_i$
Beta (Covariance Definition)	$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)}$
Beta (Correlation Form)	$\beta_i = \rho_{iM} \left( \frac{\sigma_i}{\sigma_M} \right)$
Expected Return (Single-Factor)	$E(R_i) - R_f = \beta_i [E(R_M) - R_f]$

## 8. Final Insights

- The **CML** describes efficient portfolios (market + risk-free asset), while the **Security Market Line (SML)** later describes individual assets.
- Beta is the essential link between individual asset risk and expected return.
- Diversification eliminates unsystematic risk, leaving only systematic risk to be rewarded.
- In equilibrium, expected returns depend only on exposure to market (systematic) risk — not on total volatility.

## MODULE 21.2: The CAPM and the SML

### Learning Objectives

- **LOS 21.f:** Explain the Capital Asset Pricing Model (CAPM), its assumptions, and the Security Market Line (SML).
- **LOS 21.g:** Calculate and interpret the expected return of an asset using the CAPM.
- **LOS 21.h:** Describe and demonstrate applications of the CAPM and the SML (valuation and mispricing analysis).
- **LOS 21.i:** Calculate and interpret the Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha.

## 1. The Capital Asset Pricing Model (CAPM)

**Core Idea:** Only **systematic risk** (market-related) is priced in equilibrium. The CAPM links expected return to systematic risk as measured by  $\beta_i$ .

$$E(R_i) = R_f + \beta_i[E(R_M) - R_f]$$

where:

- $E(R_i)$ : expected (or required) return on asset  $i$
- $R_f$ : risk-free rate of return
- $E(R_M)$ : expected return on the market portfolio
- $\beta_i$ : sensitivity of asset  $i$  to market risk
- $[E(R_M) - R_f]$ : market risk premium

### 1.1 CAPM Intuition

- Investors are only rewarded for bearing risk that cannot be diversified away.
- The greater an asset's sensitivity ( $\beta_i$ ) to market movements, the higher its required return.
- Assets with  $\beta_i = 0$  should earn  $R_f$ .
- Assets with  $\beta_i = 1$  should earn the market return  $E(R_M)$ .
- Assets with  $\beta_i > 1$  should have higher expected returns than the market.

### 1.2 Example: Expected Return via CAPM

$$\begin{aligned}R_f &= 2\%, \quad E(R_M) = 8\%, \quad \beta_A = 1.2 \\E(R_A) &= 2\% + 1.2(8\% - 2\%) = 9.2\%\end{aligned}$$

→ Stock A's expected (required) return = 9.2%, which exceeds the market return because  $\beta_A > 1$ .

## 2. Assumptions of the CAPM

1. **Risk Aversion:** Investors require higher expected returns for higher risk.
2. **Utility Maximization:** Investors select portfolios that maximize expected utility based on risk and return.
3. **Frictionless Markets:** No transaction costs, taxes, or restrictions.
4. **Single Period Horizon:** All investors plan for the same single investment period.

5. **Homogeneous Expectations:** All investors have identical expectations about returns, risk, and correlations.
6. **Divisible Assets:** All assets can be divided infinitely.
7. **Competitive Markets:** Investors are price takers; no one can influence market prices.

**Result:** Under these assumptions, all investors hold the same optimal risky portfolio — the **market portfolio**. This leads to the **Capital Market Line (CML)** for portfolios and the **Security Market Line (SML)** for individual assets.

### 3. The Security Market Line (SML)

**Equation:**

$$E(R_i) = R_f + \beta_i [E(R_M) - R_f]$$

#### 3.1 Interpretation

- The SML depicts the linear relationship between **systematic risk ( $\beta$ )** and **expected return**.
- The slope of the SML =  $[E(R_M) - R_f]$ , the market risk premium.
- All assets and portfolios (efficient or not) should lie on the SML in equilibrium.

**Key Points:**

- Risk-free asset ( $\beta = 0$ ) → return =  $R_f$ .
- Market portfolio ( $\beta = 1$ ) → return =  $E(R_M)$ .
- High- $\beta$  assets ( $\beta > 1$ ) → higher expected return.
- Low- $\beta$  assets ( $\beta < 1$ ) → lower expected return.

### 4. Comparing the CML and the SML

Feature	Capital Market Line (CML)	Security Market Line (SML)
Applicable To	Efficient portfolios only	All individual securities and portfolios
X-Axis Measure	Total risk ( $\sigma_P$ )	Systematic risk ( $\beta$ )
Y-Axis Measure	Expected return ( $E(R_P)$ )	Expected return ( $E(R_i)$ )
Slope	$\frac{E(R_M) - R_f}{\sigma_M}$	$E(R_M) - R_f$
Risk Interpretation	Combines risk-free and market portfolios	Captures beta risk of any asset
Plot Points	Efficient portfolios only	All properly priced assets

### Summary Interpretation (Verbal):

- The CML shows the best achievable combinations of risk and return using the market portfolio and risk-free asset.
- The SML shows the equilibrium expected return for any asset based on its  $\beta$ .
- Individual securities (even inefficient ones) lie on the SML if correctly priced.

## 5. CAPM Applications: Mispricing and Valuation (LOS 21.h)

**Concept:** If a stock's expected return differs from its CAPM-predicted (required) return, the stock is **mispiced**.

$$\text{Required Return} = R_f + \beta_i [E(R_M) - R_f]$$

$$\text{Alpha} = E(R_i) - [R_f + \beta_i (E(R_M) - R_f)]$$

- $\alpha > 0$ : stock is undervalued (plots above SML)
- $\alpha < 0$ : stock is overvalued (plots below SML)

### Example: Identifying Mispriced Securities

Given:

$$R_f = 7\%, \quad E(R_M) = 15\%$$

Stock	Beta	Expected Return (Forecast)
A	1.0	12.0%
B	0.8	17.5%
C	1.2	16.6%

### Step 1: Compute Required Return using CAPM

$$E(R_i) = 7\% + \beta_i (15\% - 7\%) = 7\% + 8\% \beta_i$$

Stock	Required Return (CAPM)	Comparison
A	15.0%	12.0% forecast → Overvalued
B	13.4%	17.5% forecast → Undervalued
C	16.6%	16.6% forecast → Fairly Valued

### Step 2: Trading Strategy

- **Stock A (Overvalued):** Expected return  $>$  required → Sell or short.
- **Stock B (Undervalued):** Expected return  $<$  required → Buy.
- **Stock C (Fairly Valued):** Hold or ignore.

### Graphical Explanation (Verbal):

- Stocks plotting **above** the SML have positive alpha (undervalued).
- Stocks plotting **below** the SML have negative alpha (overvalued).
- Stocks **on** the SML are fairly priced.

## 6. Risk-Adjusted Performance Measures (LOS 21.i)

Portfolio performance must be adjusted for risk to fairly evaluate managers' skill.

### 6.1 Sharpe Ratio (Total Risk-Based Measure)

$$\text{Sharpe Ratio} = \frac{R_P - R_f}{\sigma_P}$$

**Interpretation:**

- Measures excess return per unit of total risk.
- Higher Sharpe ratio  $\rightarrow$  better risk-adjusted performance.
- Can be used for diversified or concentrated portfolios.

**Comparison:** The Sharpe ratio equals the **slope of the Capital Allocation Line (CAL)** for the portfolio.

### 6.2 M-Squared ( $M^2$ ) Measure

$$M^2 = R_f + \left( \frac{R_P - R_f}{\sigma_P} \right) \sigma_M$$

where:

- $R_P, \sigma_P$ : portfolio return and standard deviation
- $R_f$ : risk-free rate
- $\sigma_M$ : market portfolio risk

**Interpretation:**

- Converts the Sharpe ratio into a percentage measure.
- $M^2$  represents the return the portfolio would have achieved if it had the same total risk as the market.
- $M_\alpha^2 = M^2 - R_M$  indicates outperformance relative to the market.

**Example:**

$$R_P = 10\%, \quad \sigma_P = 20\%, \quad R_f = 5\%, \quad R_M = 11\%, \quad \sigma_M = 30\%$$

$$\text{Sharpe Ratio} = \frac{10 - 5}{20} = 0.25$$

$$M^2 = 0.05 + 0.25(0.30) = 0.125 = 12.5\%$$

$\rightarrow M^2 - R_M = 1.5\%$ : Portfolio outperforms the market on a risk-adjusted basis.

### 6.3 Treynor Ratio (Systematic Risk-Based Measure)

$$\text{Treynor Ratio} = \frac{R_P - R_f}{\beta_P}$$

#### Interpretation:

- Measures excess return per unit of systematic risk.
- Appropriate for well-diversified portfolios (where unsystematic risk = 0).
- Slope of the SML for a portfolio.

### 6.4 Jensen's Alpha ( $\alpha_J$ )

$$\alpha_J = R_P - [R_f + \beta_P(E(R_M) - R_f)]$$

#### Interpretation:

- Measures actual return above that predicted by CAPM.
- $\alpha_J > 0$ : portfolio outperformed relative to its risk.
- $\alpha_J < 0$ : portfolio underperformed.

## 7. Choosing the Right Performance Measure

Measure	Uses	Risk Basis
Sharpe Ratio	Total performance across all risk sources	Total risk ( $\sigma_P$ )
M <sup>2</sup>	Percentage version of Sharpe ratio	Total risk ( $\sigma_P$ )
Treynor Ratio	Diversified portfolio evaluation	Systematic risk ( $\beta_P$ )
Jensen's Alpha	CAPM-based active return measure	Systematic risk ( $\beta_P$ )

#### Interpretation Guidelines:

- Use Sharpe or M<sup>2</sup> if the portfolio is not well diversified.
- Use Treynor or Jensen's alpha if the portfolio is well diversified.

## 8. Conceptual Summary

- **CAPM:** Links expected return to systematic (beta) risk only.
- **SML:** Linear relationship between  $\beta$  and expected return — all properly priced assets lie on it.
- **Mispricing:** Assets above the SML are undervalued; below it are overvalued.
- **CML vs. SML:** CML applies to efficient portfolios; SML applies to all securities.
- **Performance Measures:**

- Sharpe / M<sup>2</sup> → total risk-based (useful for single managers).
- Treynor / Jensen's Alpha → systematic risk-based (useful for multi-manager diversified funds).

## MODULE 85.1: Portfolio Management Process

### Learning Objectives

- **LOS 85.a:** Describe the portfolio approach to investing.
- **LOS 85.b:** Describe the steps in the portfolio management process.
- **LOS 85.c:** Describe types of investors and their distinctive characteristics and needs.
- **LOS 85.d:** Describe defined contribution and defined benefit pension plans.

### 1. The Portfolio Approach to Investing (LOS 85.a)

#### 1.1 Concept of Portfolio Perspective

**Definition:** The **portfolio perspective** evaluates each investment based on its *contribution* to the portfolio's total risk and return, not in isolation.

- A rational investor focuses on how a new investment affects the overall portfolio's expected return and risk (standard deviation).
- Holding a single asset (e.g., one stock) is inefficient — it exposes the investor to unnecessary unsystematic (idiosyncratic) risk.
- Diversification reduces total portfolio risk without necessarily lowering expected return.

**Contrast:** Evaluating assets in isolation ignores diversification benefits — a key mistake avoided by the portfolio approach.

#### 1.2 Modern Portfolio Theory (MPT)

- Originated from **Harry Markowitz (1950s)** — introduced mathematical modeling of diversification.
- Risk is measured by the **standard deviation of returns**.
- Portfolio risk depends on:

$$\sigma_P = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{12}}$$

- Unless asset returns are perfectly positively correlated ( $\rho = +1$ ), combining them reduces total risk.

- **Treynor, Sharpe, Lintner, and Mossin (1960s)** extended Markowitz's model into equilibrium pricing theory — leading to the **Capital Asset Pricing Model (CAPM)**.

### 1.3 Diversification Ratio

$$\text{Diversification Ratio} = \frac{\text{Risk of equally weighted portfolio}}{\text{Average risk of individual securities}}$$

**Example:**

- Average individual stock risk = 25%.
- Equally weighted portfolio risk = 18%.
- Diversification Ratio =  $18/25 = 0.72$ .

**Interpretation:**

- If ratio = 1 → no diversification benefit.
- If ratio  $\downarrow 1$  → portfolio risk reduced through diversification.
- Lower ratio → higher diversification benefit.

**Limitation:**

- An equal-weighted portfolio does not necessarily yield the minimum possible portfolio variance.
- Computerized **optimization models** determine optimal weights that minimize portfolio risk for a given expected return.

**Important Caveat:**

- During financial crises, asset correlations tend to increase ( $\rho \uparrow$ ), reducing diversification benefits.
- Example: During 2008 global credit contagion, correlations between equities, bonds, and real estate spiked, limiting risk reduction.

## 2. The Portfolio Management Process (LOS 85.b)

**Definition:** A structured process designed to align investment decisions with investor objectives, constraints, and market conditions.

## The Three Major Steps

### 1. Planning Step: Establishing the Investor Profile

- Analyze the investor's:
  - Risk tolerance (ability and willingness to bear risk)
  - Return objectives (required or desired returns)
  - Time horizon (short, medium, or long term)
  - Liquidity needs (cash flow or spending requirements)
  - Tax considerations (jurisdictional effects)
  - Legal or regulatory constraints
  - Unique circumstances or ethical preferences
- Result: Draft an **Investment Policy Statement (IPS)** — a document outlining investment goals, constraints, and benchmarks.
- IPS must specify:
  - **Objectives:** Risk tolerance and return requirement.
  - **Constraints:** Time horizon, taxes, liquidity, laws, unique needs.
  - **Benchmark:** Objective reference for performance evaluation (e.g., S&P 500).
- Update IPS periodically (at least every few years) or when investor's situation changes significantly.

### 2. Execution Step: Asset Allocation and Security Selection

- Analyze asset classes for risk-return tradeoffs.
- Allocate funds among major asset types (e.g., cash, fixed income, equities, real estate, private equity, hedge funds, commodities).
- Use **top-down analysis:**
  - Begin with macroeconomic outlook (GDP, inflation, interest rates).
  - Select the most attractive asset classes and regions.
- Then apply **bottom-up analysis:**
  - Identify undervalued securities within chosen asset classes using valuation models.

### 3. Feedback Step: Monitoring and Rebalancing

- Continuously monitor portfolio and market changes.
- Adjust asset allocations to restore target weights as market values fluctuate.
- Rebalance periodically (e.g., quarterly, annually, or threshold-based).
- Evaluate performance relative to the benchmark specified in the IPS.

- Performance attribution may identify sources of under/overperformance (e.g., asset allocation, security selection, timing).

**Summary Table: Steps in the Portfolio Management Process**

Step	Key Activities	Outputs
Planning	Assess investor objectives and constraints	Investment Policy Statement (IPS)
Execution	Asset allocation and security selection	Portfolio construction
Feedback	Monitor, rebalance, and evaluate performance	Rebalanced portfolio; performance report

### **3. Types of Investors and Their Characteristics (LOS 85.c)**

Investor categories differ in objectives, risk tolerance, liquidity, and horizon.

#### **3.1 Individual Investors**

- Objectives: Wealth accumulation, retirement savings, education funding, home purchase.
- Constraints: Moderate risk tolerance, liquidity for personal goals, taxable income.
- May use tax-advantaged retirement accounts (e.g., 401(k), IRA).
- Investment horizon typically long term.

#### **3.2 Institutional Investors**

Investor Type	Objective	Risk Tolerance	Horizon	Liquidity Needs
<b>Endowment</b>	Support specific program (e.g., university)	High	Long	Low (except spending needs)
<b>Foundation</b>	Fund research or charitable activities	High	Long	Low
<b>Bank</b>	Earn more on loans/investments than deposits	Low	Short to Medium	High (deposit withdrawals)
<b>Insurance – Life</b>	Fund long-term life insurance payouts	Moderate to High	Long	Low
<b>Insurance – P&amp;C</b>	Fund short-term claim obligations	Low to Moderate	Short	High
<b>Investment Company (Mutual Fund)</b>	Manage pooled investor funds within strategy	Depends on fund type	Variable	High (daily redemptions)
<b>Sovereign Wealth Fund (SWF)</b>	Manage national surplus or stabilization fund	Variable (depends on mandate)	Long	Low to Moderate

### Examples:

- **University Endowment:** Harvard or Yale endowment — aims for real growth while funding annual operations.
- **Foundation:** Gates Foundation — invests to support ongoing grants and research.
- **Life Insurer:** Prudential — invests in bonds to match long-term liabilities.
- **Property & Casualty Insurer:** Allianz — invests conservatively due to frequent claim payouts.
- **Sovereign Wealth Fund:** Abu Dhabi Investment Authority (aprox \$700B) — invests globally for intergenerational equity.

## 4. Pension Plan Types (LOS 85.d)

### 4.1 Defined Contribution (DC) Pension Plan

**Definition:** Employer contributes a fixed amount each period into an employee's retirement account. Employee decides how to invest and bears all investment risk.

- Employer's obligation ends with contributions.
- Future benefit depends on investment performance.
- Investment decisions made by employee.

- Common example: 401(k) plan.

**Formula:**

$$\text{Future Benefit} = \text{Contributions} \times (1 + r)^n$$

where  $r$  = portfolio return,  $n$  = number of compounding periods.

**Implications:**

- Investment risk → employee.
- Flexibility and portability across employers.
- Employer's cost predictable and limited.

## 4.2 Defined Benefit (DB) Pension Plan

**Definition:** Employer promises to pay a specified benefit amount to retirees, usually based on salary and years of service.

**Formula Example:**

$$\text{Annual Pension Payment} = (\text{Final Salary}) \times (\text{Benefit \% per Year}) \times (\text{Years of Service})$$

**Example:**

$$\$100,000 \times 2\% \times 20 = \$40,000 \text{ per year after retirement.}$$

**Implications:**

- Investment risk → employer (must ensure assets cover future liabilities).
- Contributions vary depending on fund performance.
- Employer manages assets through a pension fund.

### Comparison of DC vs. DB Plans

Feature	Defined Contribution (DC)	Defined Benefit (DB)
Promised Benefit	None (depends on investment returns)	Predetermined formula (salary, service years)
Investment Decision	Employee	Employer / Plan Manager
Investment Risk	Employee bears risk	Employer bears risk
Cost Predictability	Predictable for employer	Variable (depends on performance)
Portability	High (account can transfer)	Low (usually employer-tied)
Example	401(k) Plan	Traditional Company Pension

**Summary:**

- DC plans shift risk from employer to employee.
- DB plans require careful actuarial and investment management by the employer.

## 5. Conceptual Summary

- The **portfolio approach** emphasizes total portfolio risk and return, not individual asset risk.
- **Diversification** reduces risk — effectiveness depends on correlations between asset returns.
- The **portfolio management process** involves:
  1. Planning (create IPS)
  2. Execution (allocate and select)
  3. Feedback (monitor and rebalance)
- **Investor types** differ in their objectives, horizons, and liquidity needs.
- **Pension plan distinction:**
  - DC → employee risk, flexible.
  - DB → employer risk, guaranteed benefits.

## MODULE 85.2: Asset Management and Pooled Investments

### Learning Objectives

- **LOS 85.e:** Describe aspects of the asset management industry.
- **LOS 85.f:** Describe mutual funds and compare them with other pooled investment products.

## 1. Overview of the Asset Management Industry (LOS 85.e)

### 1.1 Definition and Structure

**Asset Management Industry:** Comprises firms that manage investments professionally on behalf of clients — both individuals and institutions. These firms are collectively known as the **buy-side** of the financial market, in contrast to the **sell-side** (broker-dealers, investment banks).

#### Organizational Forms:

- **Independent firms:** Stand-alone investment managers (e.g., BlackRock, Fidelity).
- **Divisions of financial conglomerates:** Asset management arms of banks or insurers (e.g., JPMorgan Asset Management, Allianz Global Investors).
- **Multi-boutique firms:** Holding companies with several specialist managers under one structure, each focusing on specific styles or asset classes.

## 1.2 Management Styles

Style	Objective	Features
Active Management	Outperform benchmark via manager skill	Uses fundamental, technical, or quantitative analysis. Higher fees.
Passive Management	Replicate benchmark performance	Index tracking or smart-beta strategies. Lower fees.

### Notes:

- Passive management 20% of total assets under management (AUM), but a smaller share of total industry revenue due to lower fees.
- **Smart Beta:** A hybrid between active and passive — tracks an index but emphasizes specific factors (e.g., value, size, momentum).

## 1.3 Traditional vs. Alternative Asset Managers

Type	Asset Focus	Typical Characteristics
Traditional Managers	Equities, fixed income, balanced funds	Lower fees, higher liquidity, widely accessible.
Alternative Managers	Private equity, hedge funds, real estate, commodities	Higher fees, lower liquidity, higher return potential.

**Trend:** Boundaries are blurring — many traditional firms are expanding into alternatives to capture higher margins.

## 1.4 Industry Trends

### 1. Rise of Passive Investing:

- Driven by low fees and skepticism about active managers' ability to consistently add alpha in efficient markets.
- Passive index funds and ETFs gaining market share globally.

### 2. Data and Technology Expansion:

- Explosion in big data and analytics tools.
- Increased use of AI, machine learning, and third-party data vendors.
- Quicker decision-making and algorithmic strategies.

### 3. Emergence of Robo-Advisors:

- Automated portfolio allocation using algorithms.
- Targeted at younger or lower-asset investors.
- Lower cost, standardized asset allocation advice.

- Examples: Betterment, Wealthfront.

### **Summary Insight:**

- Passive strategies are growing rapidly due to low costs and transparency.
- Technology and data are transforming how asset managers analyze markets and interact with clients.
- Industry convergence is reducing the distinction between traditional and alternative managers.

## **2. Mutual Funds and Pooled Investment Products (LOS 85.f)**

### **2.1 Concept of Pooled Investments**

**Definition:** A **pooled investment vehicle** combines capital from multiple investors into one fund, managed by professionals, where each investor owns shares representing proportional ownership of the total pool.

$$\text{Net Asset Value (NAV)} = \frac{\text{Total Value of Fund Assets} - \text{Liabilities}}{\text{Number of Shares Outstanding}}$$

**NAV Interpretation:** Represents the per-share market value of the fund; determines buy/sell price for investors.

### **2.2 Open-End vs. Closed-End Mutual Funds**

Feature	Open-End Fund	Closed-End Fund
<b>Share Issuance</b>	New shares created/redeemed at NAV	Fixed number of shares issued; traded on exchanges
<b>Pricing</b>	Always transacts at NAV (end of day)	Market price may differ from NAV (premium/discount)
<b>Liquidity Source</b>	Fund company (buys/sells shares)	Secondary market trading
<b>Management Fees</b>	Ongoing (usually annual expense ratio)	Similar, but trading involves brokerage costs
<b>Load Fees</b>	May have front-end or back-end loads	Typically none; trading fees apply

### **Terminology:**

- **No-load fund:** No entry or exit fees; only annual management fee.
- **Load fund:** May charge:
  - **Front-end load:** Fee when buying shares.
  - **Back-end load (redemption fee):** Fee when selling shares.

## 2.3 Types of Mutual Funds

### 1. Money Market Funds:

- Invest in short-term, high-quality debt (T-bills, commercial paper).
- Aim for stability (NAV 1.00).
- Provide interest income with minimal price volatility.

### 2. Bond Funds:

- Invest in fixed-income securities.
- Differentiated by maturity, issuer type, credit quality, and region.
- Examples: government bond funds, municipal (tax-exempt) funds, high-yield funds, global bond funds.

### 3. Stock Funds:

- Invest primarily in equities.
- **Index Funds:** Passively managed to replicate benchmark performance (e.g., S&P 500 Index Fund).
- **Actively Managed Funds:** Select securities to outperform benchmarks; higher fees and turnover.
- Active funds generally incur more taxes due to higher realized capital gains.

### 4. Balanced (Hybrid) Funds:

- Combine stocks, bonds, and sometimes cash equivalents.
- Aim for both income and capital appreciation.

## 2.4 Comparison: Mutual Funds vs. ETFs vs. Other Pooled Products

Feature	Open-End Mutual Fund	Closed-End Fund	ETF (Exchange-Traded Fund)
Trading Venue	Fund company (end of day)	Exchange	Exchange (intraday)
Transaction Price	NAV only	Market price (premium/discount)	Close to NAV (via arbitrage)
Management Style	Active or passive	Active or passive	Mostly passive (index-based)
Liquidity	High (daily redemptions)	High (market trading)	High (intraday trading)
Capital Gains Liability	Higher (fund must sell assets for redemptions)	Moderate	Lower (in-kind redemption structure)
Dividends	Often reinvested automatically	Paid as declared	Usually paid in cash
Short Selling / Margin	Not allowed	Allowed	Allowed
Investor Costs	Management + load fees	Brokerage costs	Brokerage commissions + bid-ask spread

### Interpretation (Verbal):

- ETFs trade intraday like stocks, while open-end funds trade only once daily.
- Closed-end funds can trade at prices different from NAV depending on investor sentiment and liquidity.
- ETFs generally have lower tax liabilities due to in-kind redemptions.

## 2.5 Separately Managed Accounts (SMAs)

**Definition:** A professionally managed investment portfolio owned by a single investor, tailored to that investor's objectives and constraints.

### Characteristics:

- Direct ownership of underlying securities (no fund shares issued).
- Customized strategies and tax optimization.
- Typically for high-net-worth individuals or institutions.
- Fees based on assets under management (AUM).

## 2.6 Hedge Funds

**Definition:** Privately organized investment pools with limited investor access and less regulatory oversight than mutual funds.

### Features:

- Restricted to qualified or accredited investors (e.g., net worth or income thresholds).
- Use leverage, derivatives, and short positions.
- Wide range of strategies (long/short equity, global macro, event-driven, arbitrage, etc.).
- Typical fee structure: “2 and 20” — 2% of AUM + 20% of profits (incentive fee).

**Example:** A long/short equity hedge fund may buy undervalued stocks and short overvalued ones to generate absolute returns regardless of market direction.

## 2.7 Private Equity and Venture Capital Funds

### Private Equity:

- Invests in established firms, often taking them private via leveraged buyouts (LBOs).
- Goal: Restructure company to improve efficiency and cash flows, then exit via resale or IPO within 3–5 years.

### Venture Capital:

- Focuses on financing start-ups and early-stage firms.
- Expects that only a small fraction of investments will succeed, but with very high potential returns.
- Fund managers often take active roles in advising and mentoring management.

Feature	Private Equity	Venture Capital
Target Firms	Mature, undervalued, cash-flow positive	Start-ups, early-stage, high-growth potential
Typical Holding Period	3–7 years	5–10 years
Primary Goal	Operational improvement and exit via sale or IPO	Support innovation and achieve exponential growth
Manager Involvement	High (governance and restructuring)	Very high (advisory and mentoring role)
Expected Success Rate	Majority of deals successful	Minority succeed, but winners produce high returns

## 3. Key Insights and Comparative Overview

### 3.1 Asset Management Spectrum

Category	Examples	Risk/Return Profile
Traditional (Public Markets)	Mutual funds, ETFs, SMAs	Moderate risk, high liquidity
Alternative (Private Markets)	Hedge funds, Private equity, Venture capital	Higher risk, lower liquidity, higher return potential

### 3.2 Summary of Pooled Investment Characteristics

Investment Type	Liquidity	Regulation	Minimum Investment	Investor Base
Open-End Mutual Fund	High (daily)	High	Low	General public
Closed-End Fund	High (market trading)	High	Low to moderate	Public investors
ETF	Very High (intraday)	High	Low	Public investors
SMA	Moderate	Moderate	High	Wealthy individuals/institutions
Hedge Fund	Low to Moderate	Low	High (\$250k-\$1M)	Accredited investors
Private Equity/Venture Capital	Very Low (illiquid)	Low	Very High	Institutional / accredited investors

## 4. Conceptual Summary

- The **asset management industry** consists of firms that manage investments for clients, either actively (seeking alpha) or passively (tracking indices).
- **Active vs. Passive:** Active seeks outperformance via research; passive replicates benchmark returns with lower fees.
- **Traditional vs. Alternative:** Traditional managers focus on public equities/bonds; alternatives include private equity, hedge funds, and real assets.
- **Key Trends:**
  - Growth of passive management.
  - Expansion into alternatives.
  - Use of technology and robo-advisors.
- **Pooled Investments:**
  - Mutual funds (open/closed) → accessible, diversified.
  - ETFs → tradable, tax-efficient, low-cost.
  - SMAs → customized, single-investor portfolios.
  - Hedge funds, private equity, venture capital → high-risk, high-return, limited access.

# MODULE 86.1: Portfolio Planning and Construction

## Learning Objectives

- **LOS 86.a:** Explain the reasons for a written Investment Policy Statement (IPS).
- **LOS 86.b:** Describe the major components of an IPS.
- **LOS 86.c:** Describe risk and return objectives and how they are developed.
- **LOS 86.d:** Explain willingness vs. ability to take risk.
- **LOS 86.e:** Describe investment constraints (liquidity, time horizon, taxes, legal, unique circumstances).
- **LOS 86.f:** Explain specification of asset classes for asset allocation.
- **LOS 86.g:** Describe principles of portfolio construction and role of asset allocation.
- **LOS 86.h:** Describe integration of ESG factors into portfolio planning and construction.

### 1. Purpose of a Written Investment Policy Statement (IPS) (LOS 86.a)

**Definition:** The **Investment Policy Statement (IPS)** is a formal, written document that outlines an investor's objectives, constraints, and the policies to guide portfolio management decisions.

#### Key Reasons for a Written IPS:

- Serves as a **contract** between client and manager — clarifying expectations.
- Ensures that portfolio decisions are consistent with client's **risk tolerance** and **return goals**.
- Promotes discipline and reduces emotional decision-making.
- Provides a **benchmark** for evaluating performance.
- Helps manage long-term relationships through transparency and accountability.

**Important:** High return goals must align with acceptable risk. Unrealistically low risk with high return is incompatible — the IPS reconciles these expectations.

## 2. Major Components of the IPS (LOS 86.b)

A comprehensive IPS should address the following elements:

1. **Client Description:** Financial situation, investment objectives, constraints, risk tolerance, time horizon, and other personal factors.
2. **Purpose of IPS:** Defines the scope and intent of the document (e.g., managing family trust or pension fund).
3. **Duties and Responsibilities:** Specifies roles of client, investment manager, custodian, and advisors.
4. **Procedures for Review:** Details frequency of updates, triggers for IPS revision, and communication procedures.
5. **Investment Objectives:** States the **risk and return objectives** derived from client discussions.
6. **Investment Constraints:** Identifies liquidity needs, time horizon, taxes, legal/regulatory factors, and unique considerations (see Section 5).
7. **Investment Guidelines:** Specifies asset classes permitted, diversification standards, use of derivatives, and leverage policies.
8. **Performance Evaluation:** Defines benchmark(s) and methods for measuring and comparing results.
9. **Appendices:** Include target (strategic) asset allocation, allowable deviations, and rebalancing rules.

### Minimum IPS Requirements:

- Clear statement of client's objectives and constraints.
- Investment strategy based on these factors.
- Benchmark for performance evaluation.

## 3. Risk and Return Objectives (LOS 86.c)

### 3.1 Risk Objectives

#### Forms of Risk Objectives:

- **Absolute Risk Objective:** Expressed in absolute terms.
  - Example: “No decrease in portfolio value during any 12-month period.”
  - Example: “Not decrease in value by more than 2% over any 12-month period.”
  - Very conservative → implies near risk-free assets (e.g., T-bills).

- **Probability-Based Absolute Risk:**
  - “No greater than 5% probability of return below –5% in a year.”
  - “No greater than 4% probability of a \$20,000 loss over a year.”
- **Relative Risk Objective:** Related to performance of a benchmark.
  - Example (strict): “Returns will not be less than 12-month LIBOR in any 12-month period.”
  - Example (probabilistic): “No greater than 5% probability of underperforming MSCI World Index by more than 4%.”

### 3.2 Return Objectives

#### Absolute Return Objectives:

- “Achieve 6% per annum nominal return.”
- “Achieve 3% real return above inflation.”

#### Relative Return Objectives:

- “Exceed S&P 500 return by 2% per annum.”
- “Exceed bank deposit rate (cost of funds) by 1.5%.”

**Compatibility Rule:** Risk and return objectives must be internally consistent — higher expected return requires higher accepted risk.

**Non-Investable Benchmarks Warning:** Peer performance objectives (e.g., “top quartile endowment performance”) are not investable — less suitable for IPS benchmarks.

## 4. Ability vs. Willingness to Take Risk (LOS 86.d)

Dimension	Key Factors
Ability (Capacity)	Depends on objective financial circumstances. <ul style="list-style-type: none"> <li>• Longer time horizon → higher ability.</li> <li>• Greater wealth vs. liabilities → higher ability.</li> <li>• Stable income or insured risks → higher ability.</li> <li>• Short horizon or high liquidity needs → lower ability.</li> </ul>
Willingness (Psychological)	Depends on personal attitude and comfort with volatility. <ul style="list-style-type: none"> <li>• Determined via discussion or questionnaires.</li> <li>• Influenced by past experience, beliefs, emotions.</li> </ul>

#### Conflict Resolution:

- If **willingness & ability:** Ability dominates → portfolio risk lowered to protect capital.

- If **ability & willingness**: Educate investor, but final risk level should respect lower tolerance.

**Principle:** Construct portfolios using the **lower** of ability or willingness to bear risk to ensure client comfort and adherence.

## 5. Investment Constraints (LOS 86.e)

**Mnemonic:** RRTTLLU — Risk, Return, Time horizon, Tax, Liquidity, Legal, Unique.

### 5.1 Liquidity

- Ability to convert assets into cash without significant loss.
- Required for expenses such as tuition, healthcare, or living costs.
- Institutional example: P&C insurers need high liquidity for unpredictable claims.
- Illiquid assets (e.g., hedge funds, private equity) unsuitable for short-term needs.

### 5.2 Time Horizon

- Longer horizons allow more risk-taking and less liquidity.
- Short horizons require conservative, liquid investments.
- Example: - 20-year horizon → equities acceptable. - 1-year horizon → cash equivalents or short-term bonds.

### 5.3 Tax Considerations

- Evaluate after-tax returns; tax rates vary by income and account type.
- High-tax investors may prefer tax-exempt bonds or capital-gains-focused equities.
- Tax-deferred accounts (e.g., retirement plans) should hold high-tax-income assets like bonds.
- Taxable accounts should hold assets generating tax-preferred income (e.g., dividends, long-term capital gains).

### 5.4 Legal and Regulatory Factors

- Certain accounts (trusts, pensions, corporations) face investment restrictions.
- Compliance with fiduciary standards is essential.
- Corporate insiders face trading limits on their own firm's stock.

## 5.5 Unique Circumstances

- Ethical or religious screens (e.g., excluding tobacco, weapons, or interest-based securities).
- Employment-related diversification: avoid investments tied to investor's primary income source.
- Socially responsible or ESG-based exclusions.

## 6. Specification of Asset Classes (LOS 86.f)

**Purpose:** Define the broad categories of assets to be included in the strategic asset allocation.

### Guiding Principles:

- Assets within a class should have **high internal correlation** (similar performance characteristics).
- Asset classes should have **low correlation with each other** to provide diversification benefits.

### Traditional Asset Classes:

- Equities, Bonds, Cash, Real Estate.

### Alternative Asset Classes:

- Hedge funds, Private equity, Commodities, Infrastructure, Art, Intellectual property.

### Subdivisions:

- **Equities:** Domestic vs. Foreign, Large-cap vs. Small-cap, Developed vs. Emerging markets.
- **Bonds:** Domestic vs. International, Government vs. Corporate, Investment grade vs. High-yield, Short vs. Long maturity.

### Data Required:

- Expected returns, standard deviations, and correlations between asset classes.

## 7. Portfolio Construction and Asset Allocation (LOS 86.g)

### Step 1: Construct Efficient Frontier

- Use expected return, risk, and correlation data of asset classes.
- Identify portfolios that minimize risk for each expected return level.

### Step 2: Select Optimal Portfolio

- Combine investor's risk–return objectives (from IPS) with efficient frontier data.
- Choose the portfolio that maximizes expected utility — this becomes the **strategic asset allocation**.

### Active vs. Passive Portfolio Management:

Strategy Type	Description
Passive	Maintain strategic asset allocation using index-based investments.
Active	Deviate from strategic allocation to exploit short-term opportunities.

### Active Strategies:

- **Tactical Asset Allocation (TAA):** Temporarily adjust weights across asset classes based on market outlook.
- **Security Selection:** Over/underweight securities within asset classes.

### Risk Budgeting:

- Assign total portfolio risk across:
  1. Systematic risk (strategic allocation),
  2. Tactical risk,
  3. Security-selection risk.

### Core–Satellite Approach:

- **Core:** Passive index holdings representing most of the portfolio.
- **Satellite:** Smaller allocations to active strategies.
- Benefits: Minimizes offsetting active bets, reduces excessive trading, improves tax efficiency.

## 8. ESG Integration into Portfolio Planning (LOS 86.h)

**ESG = Environmental, Social, and Governance factors.** ESG integration aligns investment decisions with ethical, sustainability, or governance considerations.

### ESG Approaches:

1. **Negative Screening:** Exclude industries (e.g., tobacco, fossil fuels, weapons).
2. **Positive Screening:** Include firms with strong ESG practices.
3. **Thematic Investing:** Target ESG themes (e.g., clean energy, gender equality).
4. **Impact Investing:** Seek measurable social/environmental impact alongside returns.

5. **Engagement / Active Ownership:** Use shareholder rights to influence ESG improvements.
6. **ESG Integration:** Systematically include ESG factors throughout security selection and asset allocation.

#### **Key Implementation Considerations:**

- Constrained investment universe → benchmark must align with ESG exclusions.
- Positive or thematic strategies require customized performance benchmarks.
- ESG may reduce diversification or incur costs but can improve long-term risk management.

#### **Example:**

- Negative screen: Exclude fossil fuel producers.
- Thematic: Invest in renewable energy firms.
- Active ownership: Vote proxies for better corporate governance.

## **9. Conceptual Summary**

- The **IPS** anchors portfolio design — defining objectives, constraints, and benchmarks.
- **Risk and return** objectives must be realistic and aligned with investor tolerance.
- **Constraints (RRTTLLU)** ensure the portfolio fits practical realities (liquidity, taxes, etc.).
- **Asset allocation** determines most of the portfolio's risk and return.
- **Risk budgeting** allocates total risk among strategic, tactical, and security-selection sources.
- **ESG integration** customizes portfolios for social and environmental alignment while managing long-term sustainability risks.

## **MODULE 87.1: Cognitive Errors vs. Emotional Biases**

### **Overview**

Traditional finance assumes that investors are rational, risk-averse, and process all available information efficiently. Behavioral finance, pioneered by **Kahneman and Tversky**, challenges this assumption by showing that humans display **systematic biases** in decision-making due to both **cognitive limitations** and **emotional influences**. These biases cause deviations from rational behavior and can result in suboptimal financial decisions.

## **LOS 87.a: Cognitive Errors vs. Emotional Biases**

- **Cognitive Errors:** Result from faulty reasoning, improper information processing, or lack of understanding of probability/statistics. They are often due to poor logic, misinterpretation, or memory errors. → **Can often be mitigated through education, awareness, or better information.**
- **Emotional Biases:** Arise from feelings, impulses, or intuition rather than conscious reasoning. → **Difficult to correct; must often be accommodated rather than eliminated.**

**Key Difference:**

<b>Cognitive Errors</b>	<b>Emotional Biases</b>
Errors in information processing or reasoning	Errors due to feelings and emotions
Can be reduced by training and awareness	Hard to control; often require behavioral adjustment
Example: Anchoring on old data	Example: Loss aversion or overconfidence

**Note:** Many biases contain both cognitive and emotional elements. When both exist, focus on mitigating the cognitive component first — it is more manageable.

## **LOS 87.b: Commonly Recognized Behavioral Biases and Their Implications**

Behavioral biases can be grouped into two categories of **cognitive errors**:

1. **Belief Perseverance Biases** – irrationally holding on to prior beliefs.
2. **Information-Processing Biases** – faulty processing or evaluation of new data.

### **1. Belief Perseverance Biases**

**Definition:** Biases that cause individuals to hold on to prior beliefs or conclusions, even when presented with new evidence.

**Main Types:**

Bias	Description and Implications
<b>Conservatism Bias</b>	Failure to update beliefs or forecasts when new information arises. Investors overweight prior probabilities and underweight new data. <b>Example:</b> An investor keeps a recession probability at 20% despite central bank tightening — ignoring updated signals. <b>Consequence:</b> Holding losing positions too long or reacting too slowly to new information.
<b>Confirmation Bias</b>	Seeking or interpreting information that confirms existing beliefs while ignoring contradictory data. <b>Example:</b> Reading only positive reviews after buying a car (or stock). <b>Consequence:</b> Overconfidence, poor decision validation, ignoring downside risk. <b>Mitigation:</b> Actively seek disconfirming evidence and opposing viewpoints.
<b>Representativeness Bias</b>	Classifying new information based on superficial similarity to existing stereotypes or small samples. Two forms: (i) <b>Base-rate neglect</b> – ignoring actual statistical frequency. (ii) <b>Sample-size neglect</b> – overgeneralizing from small samples. <b>Example:</b> Assuming a fund manager is “skilled” after three strong years without considering randomness. <b>Consequence:</b> Misclassification, trend-chasing, ignoring mean reversion.
<b>Illusion of Control Bias</b>	Belief that one can control or influence outcomes that are actually random. Linked to <b>overconfidence</b> , <b>illusion of knowledge</b> , and <b>self-attribution</b> . <b>Example:</b> Overweighting stock of one’s employer because one feels “in control.” <b>Consequence:</b> Underdiversification, excessive trading.
<b>Hindsight Bias</b>	Believing after an event that it was predictable all along. Selective memory of past predictions (“I knew it!”). <b>Example:</b> Claiming foresight after a stock rally. <b>Consequence:</b> Overconfidence, abandonment of sound analysis in favor of luck-based reasoning.

**Summary Insight:** Belief perseverance biases distort how investors update beliefs — causing underreaction, overconfidence, or inflexible decision-making.

## 2. Information-Processing Biases

**Definition:** Biases that result from incorrect information processing, weighting, or mental accounting rather than from the persistence of prior beliefs.

**Main Types:**

Bias	Description and Implications
Anchoring and Adjustment Bias	Basing decisions on an initial value (anchor) and failing to adjust sufficiently when new data emerges. <b>Example:</b> Estimating fair value of a stock relative to its current price instead of its intrinsic value. <b>Consequence:</b> Underreaction to new information; inertia in forecast updates. <b>Mitigation:</b> Re-evaluate data independently of prior anchor.
Mental Accounting Bias	Treating money differently depending on its source or purpose. <b>Example:</b> Investing a work bonus in risky stocks (“found money”) while keeping savings ultra-safe. <b>Consequence:</b> Suboptimal total portfolio, inefficient diversification, conflicting sub-portfolios. <b>Note:</b> Investors should adopt a <b>total-portfolio perspective</b> .
Framing Bias	Decision outcomes are influenced by the way choices or data are presented (gain vs. loss framing). <b>Example:</b> Prefer “saving 200 lives” (gain frame) vs. avoiding “400 deaths” (loss frame) — though outcomes identical. <b>In investments:</b> Overemphasizing short-term losses over long-term potential. <b>Consequence:</b> Risk aversion in gains, risk-seeking in losses, mis-assessed risk tolerance.
Availability Bias	Overweighting information that is recent, easily recalled, or vivid. <b>Example:</b> After a market crash, investors overestimate probability of another crash. <b>Consequence:</b> Poor diversification, overreaction to news, familiarity bias (investing only in known firms). <b>Mitigation:</b> Broaden data sources, rely on statistical evidence.

### 3. Cognitive Bias Examples Explained

#### Conservatism Bias Example

**Scenario:** An analyst keeps forecasting stable growth despite central bank tightening. **Error:** Fails to adjust recession probability given new conditions → underreaction.

**Implication:** Investor may hold outdated positions too long, ignoring relevant information.

#### Confirmation Bias Example

**Scenario:** An investor reads only bullish articles on Tesla after buying it. **Error:** Selectively processing information to confirm preexisting belief. **Implication:** Overconfidence, reduced willingness to reconsider investment thesis.

### **Representativeness Bias Example**

**Scenario:** XYZ Corp has long been a “growth” stock. Despite recent stagnation, an analyst still recommends buying because of its “growth” label. **Error:** Stereotyping based on past category — ignoring changed fundamentals. **Implication:** Misclassification and failure to recognize structural shift.

### **Anchoring and Adjustment Example**

**Scenario:** A stock once traded at \$100 but is now \$60. Analyst sets price target of \$80 simply “because it used to be 100.” **Error:** Using past price as anchor rather than intrinsic value. **Implication:** Slow reaction to new valuation realities; possible overvaluation bias.

### **Framing Bias Example**

**Scenario:** Client A told “Your portfolio gained 10% this year.” Client B told “You missed 5% vs. market.” Both have same return, but reactions differ. **Error:** Perception shifts due to framing of gain/loss. **Implication:** Emotionally inconsistent risk preferences.

## **4. Summary: Classification of Key Cognitive Biases**

Category	Biases Included
Belief Perseverance	Conservatism, Confirmation, Representativeness, Illusion of Control, Hindsight
Information Processing	Anchoring and Adjustment, Mental Accounting, Framing, Availability

## **5. Implications for Financial Decision-Making**

### **Key Effects of Biases:**

- Holding losing investments too long (conservatism, confirmation).
- Underdiversification and concentrated portfolios (illusion of control, availability).
- Overtrading and overconfidence (hindsight, illusion of control).
- Misestimation of probabilities or expected returns (representativeness, anchoring).
- Poor portfolio optimization due to segmentation of wealth (mental accounting).
- Inconsistent risk preferences depending on framing (framing bias).

### **Practical Applications:**

- Advisors must identify client biases and adapt strategy — some can be corrected, others accommodated.

- Educate investors on the effects of anchoring, overconfidence, and framing in decision-making.
- Incorporate behavioral insights into portfolio rebalancing and risk assessment.

## 6. CFA-Style Exam Pointers

- **Cognitive errors** → can be *mitigated* (education, analysis). **Emotional biases** → must often be *accommodated* (behavioral adjustment).
- **Belief perseverance** = failure to revise view; **Information-processing** = incorrect data use.
- **Framing and availability** often appear in client risk-tolerance questions.
- **Anchoring vs. conservatism:** – Anchoring = fixating on an initial number. – Conservatism = under-adjusting to new info.
- **Representativeness vs. confirmation:** – Representativeness = stereotyping new data. – Confirmation = seeking info supporting old belief.

## 7. Conceptual Summary Table

Bias Type	Practical Investment Implications
Conservatism	Fails to adjust portfolio to new data; may miss turning points.
Confirmation	Ignores disconfirming info; reinforces wrong forecasts.
Representativeness	Misclassifies investments; buys “hot” stocks or managers.
Illusion of Control	Overtrading, underdiversification, excessive risk.
Hindsight	Overconfidence, poor learning from mistakes.
Anchoring	Biased price targets, slow reaction to new events.
Mental Accounting	Inefficient asset allocation; suboptimal total return.
Framing	Inconsistent risk attitude; misjudged risk tolerance.
Availability	Overreaction to recent or vivid events; familiarity bias.

## 8. Key Takeaways

- Behavioral biases distort rational investment decisions.
- Cognitive errors stem from reasoning flaws — **can be educated away**.
- Emotional biases stem from feelings — **must be managed or accepted**.
- Advisors must diagnose which bias dominates client behavior.
- Overcoming biases leads to more consistent, long-term investment discipline.

## MODULE 87.2: Emotional Biases and Market Implications

### Overview

Traditional finance assumes that investors are rational and markets are efficient. Behavioral finance, however, recognizes that investors exhibit **emotional biases** — decisions driven by feelings, impulses, and psychological comfort rather than logic or probability. These biases often persist even when investors are aware of them and cannot be corrected through training alone.

### LOS 87.b (continued): Emotional Biases in Decision-Making

**Key Idea:** Emotional biases originate from feelings and affective reactions, not cognitive miscalculations. They are difficult to mitigate — often requiring behavioral adaptation or portfolio design rather than education.

#### Summary of Major Emotional Biases

- **Loss-Aversion Bias** — Investors feel greater pain from losses than pleasure from equal gains (“losses loom larger than gains”) and evaluate outcomes relative to a reference point (current wealth or purchase price).
  - **Example:** Prefer a certain gain (\$5) over a 50–50 chance to gain \$10, but prefer a risky loss over a certain loss.
  - **Consequences:** Selling winners too soon; holding losers too long; taking excessive risk to recover losses; trading too frequently.
  - **Distinction:** Risk aversion (preference for lower variance given equal return) vs. loss aversion (asymmetric emotional reaction to losses vs. gains).
- **Overconfidence Bias** — Excessive faith in one’s own judgment or skill.
  - **Forms:** Illusion of knowledge; self-attribution (crediting success to skill, blaming failure on external factors).
  - **Consequences:** Underestimating risk, underdiversified portfolios, excessive trading, ignoring contrary evidence.
  - **Mitigation:** Maintain performance records; compare forecasts to outcomes and track forecast accuracy.
- **Self-Control Bias** — Preference for immediate gratification over long-term benefit (hyperbolic discounting).
  - **Example:** Spending now instead of saving for retirement.
  - **Consequences:** Insufficient retirement savings; overemphasis on short-term income; risky catch-up behavior later.
  - **Mitigation:** Automatic savings plans, automatic payroll deductions, pre-commitment mechanisms.
- **Status Quo Bias** — Emotional comfort with the current situation leads to resistance to change.

- **Example:** Not updating default 401(k) allocations or failing to opt into a better fund.
- **Consequences:** Portfolios become misaligned with changing goals or risk tolerance; missed opportunities.
- **Behavioral application:** Automatic enrollment (with opt-out) increases participation (Thaler & Sunstein).
- **Endowment Bias** — Tendency to overvalue an asset simply because one owns it (ownership effect).
  - **Example:** Holding inherited stock for sentimental reasons.
  - **Consequences:** Emotional attachment leads to underdiversification and holding inappropriate or outdated assets.
  - **Test question:** “Would I buy this asset today with new money?” — helps detect the bias.
- **Regret-Aversion Bias** — Avoiding decisions to reduce potential future regret; stronger regret for actions (commission) than for inaction (omission).
  - **Example:** Not buying a promising stock to avoid feeling foolish if it declines.
  - **Consequences:** Excessive conservatism, missed opportunities, herding behavior (to share blame).
  - **Mitigation:** Pre-commitment, rules-based investing, or documented decision rules to reduce emotional paralysis.

### Behavioral Bias Comparison Table

Bias	Core Emotion	Typical Investor Behavior
Loss Aversion	Fear of loss	Hold losers; sell winners too early
Overconfidence	Pride, ego	Overtrade; underdiversify; ignore risk
Self-Control	Impatience	Under-save; short-term focus
Status Quo	Comfort	Resist portfolio changes
Endowment	Attachment	Overvalue owned assets
Regret Aversion	Fear of mistake	Follow herd; avoid action

**Exam Tip:** If bias stems from emotional attachment, fear, or self-image → classify as *emotional*. If it stems from faulty reasoning or data interpretation → classify as *cognitive*.

### LOS 87.c: Behavioral Biases and Market Characteristics

**Key Idea:** Investor biases can aggregate into market-wide phenomena — explaining anomalies and inefficiencies that traditional finance struggles to justify.

#### 1. Market Bubbles and Crashes

- **Overconfidence:** Investors overestimate skill and underweight risk, leading to overtrading and inflated valuations. – During bull markets, good results reinforce confidence (self-attribution).
- **Confirmation Bias:** Investors ignore signals that prices are too high; dismiss warnings as noise.

- **Anchoring:** Investors treat recent high prices as “normal,” refusing to revise valuations downward.
- **Regret Aversion:** Fear of missing out keeps investors in markets even when they believe prices are excessive.
- **Outcome:** Excess demand inflates bubbles; when sentiment reverses, sharp corrections follow.

**Example:** Dot-com bubble (late 1990s) — investors extrapolated short-term growth, ignored valuations, and herded into tech stocks.

**Behavioral Explanation:** Market participants as a group exhibit **collective cognitive dissonance** — refusing to accept contradictory evidence until forced by large losses.

## 2. The Value vs. Growth Anomaly

- Traditional models found that **value stocks** (low P/E, high book-to-market) outperform **growth stocks**.
- **Fama-French (1992):** Explained excess returns by additional risk factors (size, value).
- **Behavioral View:** – Investors overpay for “glamour” growth stocks due to **representativeness** and the **halo effect**. – They underreact to positive information about unpopular (value) stocks.
- **Halo Effect:** Extending one positive attribute (e.g., rapid growth) into unjustified general optimism about quality and future performance.

**Implication:** Behavioral mispricing persists until fundamentals reassert, generating excess value-stock returns.

## 3. Home Bias and Familiarity Effects

- **Home Bias:** Investors overweight domestic or regional securities despite the diversification benefits of global investing.
- **Reasons:**
  - Emotional comfort and familiarity.
  - Illusion of information advantage (“I know these firms better”).
  - Preference for proximity and cultural similarity.
- **Consequences:**
  - Reduced diversification.
  - Underexposure to international opportunities.

**Behavioral Drivers:** Familiarity bias (availability) and overconfidence in local knowledge.

## 4. Behavioral Finance and Market Inefficiency

**Traditional View:** Anomalies are due to model misspecification or random chance.

**Behavioral View:** Anomalies reflect predictable patterns of human bias and emotion, especially:

- Overreaction (overconfidence, representativeness).
- Underreaction (conservatism, anchoring).
- Excessive volatility (herding, loss aversion).
- Momentum and reversal patterns (due to delayed or emotional adjustments).

**Limitations:** Behavioral finance explains tendencies, not precise market equilibria. Biases help interpret deviations but not always forecast them.

## 5. Summary of Behavioral Market Effects

Bias / Emotion	Market-Level Effect
Overconfidence	Overtrading, bubbles, excess volatility.
Confirmation	Ignoring negative news; persistent overvaluation.
Anchoring	Slow adjustment to fundamentals after shocks.
Regret Aversion	Herding; delayed exits from bubbles.
Representativeness / Halo	Overpricing of growth stocks; value premium.
Home / Familiarity	Underdiversification; regional concentration.
Loss Aversion	Selling winners too early; holding losers too long.

## 6. Conceptual Summary

- **Emotional biases** are rooted in feelings — difficult to eliminate.
- They lead to predictable investor errors (holding losers, underdiversifying, herding).
- When biases aggregate across many participants, markets display anomalies such as bubbles, crashes, and the value premium.
- Recognizing these behavioral patterns improves both portfolio management and client advisory quality.

## MODULE 88.1: Introduction to Risk Management

### LOS 88.a: Defining Risk Management

- **Risk management** is the process of:
  1. Identifying the organization's **risk tolerance**.

- 2. Identifying and measuring the **risks faced**.
- 3. Modifying and monitoring these risks over time.
- The goal is not to eliminate risk but to achieve the **optimal bundle of risks** consistent with the organization's objectives and capacity.
- **Rationale:**
  - Risk cannot and should not be completely avoided — returns above the risk-free rate require accepting risk.
  - Management controls the amount and types of risk, not actual realized returns.
- **In essence:**

Risk Management = Identify + Measure + Modify + Monitor  
while aligning all exposures with the firm's risk tolerance.

## **LOS 88.b: Features of a Risk Management Framework**

- A comprehensive framework must include:
  1. **Risk Governance:** Senior management oversight and policy setting.
  2. **Risk Tolerance Determination:** Define acceptable total exposure.
  3. **Risk Identification & Measurement:** Map all existing risks.
  4. **Risk Management & Mitigation:** Adjust exposures toward the optimal mix.
  5. **Risk Monitoring:** Continuous tracking and reporting.
  6. **Communication:** Ensure cross-departmental awareness.
  7. **Strategic Risk Analysis:** Link risk outcomes to business objectives.
- **Key Objective:** Align organizational risk exposures with its strategic goals and stakeholder interests.

## **LOS 88.c: Risk Governance and Effective Oversight**

- **Risk Governance:** Senior management's responsibility for:
  - Determining firmwide **risk tolerance**.
  - Designing the **risk management structure**.
  - Overseeing execution and compliance.
- **Objective:** Manage risk to support the firm's strategic objectives — pursue efficient risks, limit excessive ones, avoid those outside tolerance.
- **Implementation:**
  - Create a **Risk Management Committee** to integrate firmwide perspectives.
  - Provide clear reporting lines and independent risk oversight.

## **LOS 88.d: Risk Tolerance**

- **Definition:** The total amount and types of risk an organization is willing and able to accept.
- **Factors Influencing Risk Tolerance:**
  1. Firm's expertise in its business lines.
  2. Financial strength and ability to absorb losses.
  3. Regulatory environment and external constraints.
  4. Management's ability to respond to negative events.
- The objective is to **match risk exposure with capacity and goals**.

## **LOS 88.e: Risk Budgeting**

- **Definition:** Allocation of total acceptable risk (the “risk budget”) across business units, asset classes, or risk factors.
- **Purpose:** Achieve maximum expected return for a given total risk level.
- **Approaches:**
  - **Single-measure basis:** e.g., total portfolio variance, beta, duration, or VaR.
  - **Asset-class basis:** e.g., domestic equity, bonds, international equity, etc.
  - **Risk-factor basis:** e.g., interest rate, FX, equity market, and credit risks.
- **Goal:** Combine these exposures to fit within the organization’s risk tolerance.

## **LOS 88.f: Financial vs. Non-Financial Risks**

Risk Type	Definition and Examples
<b>Financial Risks</b>	Arise from exposure to financial markets.
Credit Risk	Counterparty fails to fulfill contractual obligations.
Liquidity Risk	Loss from inability to sell assets quickly at fair value.
Market Risk	Loss due to changes in market prices or rates (stocks, commodities, FX, interest rates).
<b>Non-Financial Risks</b>	Arise from operations or external environment.
Operational Risk	Loss from human error, system failure, or fraud (includes <i>cyber risk</i> ).
Solvency Risk	Organization cannot continue operations due to lack of cash.
Regulatory Risk	Unfavorable change in regulation increases costs or limits activities.
Government/Political Risk	Policy or tax changes impose costs outside the regulatory framework.
Legal Risk	Exposure to litigation or contractual uncertainty.
Model Risk	Analytical or valuation models are incorrect.
Tail Risk	Extreme outcomes more probable than predicted by normal models.
Accounting Risk	Accounting choices or estimates are judged incorrect.

### Special cases for individuals:

- **Mortality risk:** Death before providing for dependents (mitigated by life insurance).
- **Longevity risk:** Living longer than assets last (mitigated by annuities).
- **Health-care risk:** Managed with health insurance.

### Interaction of Risks:

- Risks are interrelated, not additive.
- Example: Market decline  $\Rightarrow$  counterparty defaults  $\Rightarrow$  legal disputes  $\Rightarrow$  liquidity strain  $\Rightarrow$  solvency risk.
- During crises, correlations among risks increase — amplifying total exposure.

## LOS 88.g: Measuring and Modifying Risk Exposures

### (1) Common Measures of Risk

Measure	Description / Use
Standard Deviation	Measures total volatility (stand-alone risk). Inadequate for non-normal distributions (fat tails).
Beta	Measures systematic (market) risk for equities within diversified portfolios.
Duration	Measures price sensitivity of bonds to interest-rate changes.
<b>Option Greeks</b>	Sensitivities for derivative positions:
Delta	Change in derivative value per unit change in underlying price.
Gamma	Change in delta per unit change in underlying price.
Vega	Change in derivative value per unit change in volatility.
Rho	Change in derivative value per unit change in risk-free rate.

### (2) Tail Risk Metrics

- **Value at Risk (VaR):** Minimum expected loss over a time horizon at a given probability level. Example: One-month 5% VaR of \$1 million → 5% chance of losing at least \$1 million in one month.
- **Conditional VaR (CVaR):** Expected loss *given* the loss exceeds VaR (average of the tail losses). Useful complement to VaR, similar to “loss given default”.

**Limitations:** VaR is sensitive to model assumptions and distributions — it does not give the maximum possible loss.

### (3) Subjective and Market-Based Risk Assessment

- **Stress Testing:** Examine effect of extreme but plausible shocks in one variable (e.g., interest-rate spike).
- **Scenario Analysis:** Multi-variable “what-if” testing — combined macro and market changes (e.g., rate hike + oil shock + FX move).
- **Market-Implied Risk:** Use option prices, insurance premiums, or credit spreads to infer the market’s aggregate perception of risk.
- **Operational / Political Risks:** Often require subjective estimates when quantitative data are unavailable.

#### (4) Modifying Risk Exposures

Method	Description and Example
<b>Avoidance</b>	Do not engage in the activity. Example: Do not invest in high-political-risk country.
<b>Prevention / Reduction</b>	Implement controls to reduce probability or impact. Example: Cybersecurity measures to prevent data breaches.
<b>Retention (Self-Insurance)</b>	Accept the risk internally; may create reserves for potential losses. Example: Firm keeps exposure to routine operational losses.
<b>Transfer</b>	Shift risk to another party through contracts. Example: Buy insurance, surety bonds, or fidelity bonds. – <i>Insurance</i> : Pay premium to transfer risk of fire, theft, etc. – <i>Surety bond</i> : Pays if third party fails contractual obligation. – <i>Fidelity bond</i> : Covers employee theft or fraud.
<b>Shifting (via Derivatives)</b>	Alter distribution of outcomes through derivative instruments. Examples: – Buy put options (establish floor value). – Sell calls (reduce upside, collect premium). – Use swaps or forwards to hedge FX or interest-rate risk.

#### Choosing Among Methods:

- Always weigh **costs vs. benefits**.
- Combine tools — e.g., diversify, insure, and hedge simultaneously.
- Aim for a final risk profile that matches the organization's risk tolerance and strategic objectives.

#### Key Takeaways

- **Goal:** Not to eliminate risk, but to manage it efficiently.
- **Framework:** Governance → Measurement → Modification → Monitoring.
- **Effective Risk Governance:** Aligns all risk decisions with corporate strategy.
- **Risk Budgeting:** Distributes risk to maximize expected return per unit of accepted risk.
- **Risk Interactions:** Must be analyzed jointly — crises often reveal hidden correlations.
- **Modification Tools:** Avoid, prevent, retain, transfer, or shift — depending on cost-benefit trade-off.