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Assignment #2

ECE 271A
Fall 2019

Computer Problem Solution

a) **Solution**:
Using the results of problem 2, the maximum likelihood estimate for the prior probabilities is following:

$$P_Y(cheetah) = N_{FG}/(N_{FG} + N_{BG}) = 0.1919$$

$$P_Y(grass) = N_{BG}/(N_{FG} + N_{BG}) = 0.8081$$

where

N_{BG} is the number of grass samples

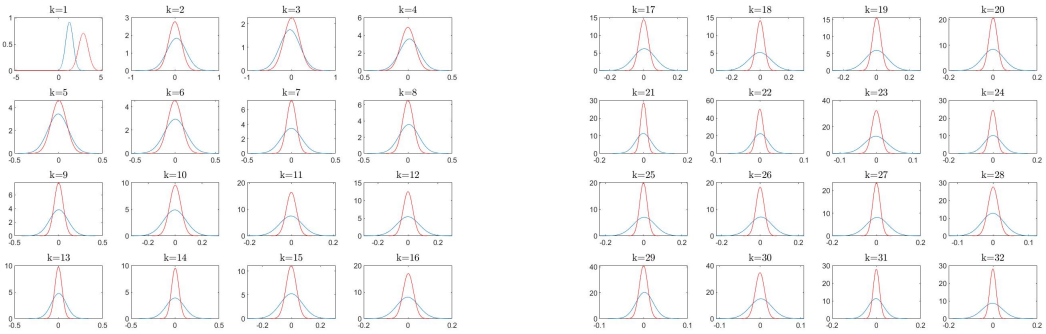
N_{FG} is the number of cheetah samples

These estimates are actually the same as the estimates that I obtained in Homework#1. It implies that the prior probabilities could be estimated by using proportion of each states in the samples if we have enough or big number of samples.

b) **Solution**:
The marginal densities for two classes

$$P_{x_k|Y}(x_k|cheetah) \text{ and } P_{x_k|Y}(x_k|grass),\ k = 1, 2, \ldots, 64$$

are shown below on Figure 1. The blue line represents $P_{x_k|Y}(x_k|cheetah)$ and the red line represents $P_{x_k|Y}(x_k|grass)$.



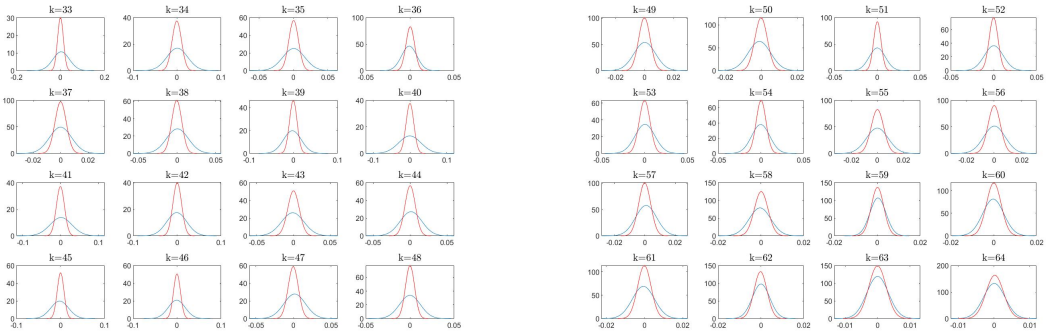


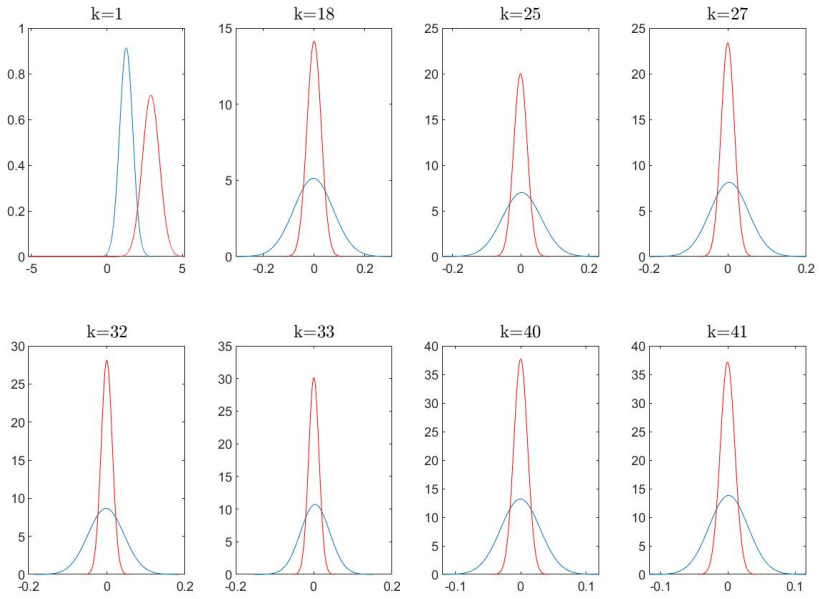
Figure 1: Marginal densities for two classes

By visual inspection, the best 8 features and worst 8 features are:

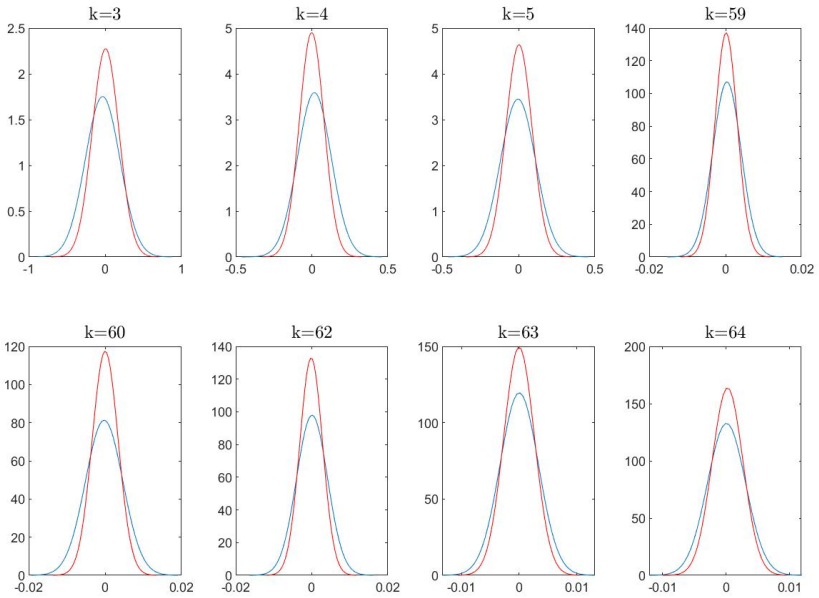
best 8 features:[1,18,25,27,32,33,40,41]

worst 8 features:[3,4,5,59,60,62,63,64]

The plots of the marginal densities for the best-8 and worst-8 features are shown below:



(a) Best 8 features



(b) Worst 8 features

Figure 2: Marginal densities for the best-8 and worst-8 features

c) **Solution**:
The classification results using 64-dimensional Gaussians and 8-dimensional Gaussians associated with the best 8 features are shown below. Compare it with the ground truth provided in image **cheetah mask.bmp** and compute the probability of error. The probability of error is also shown in the figure below.

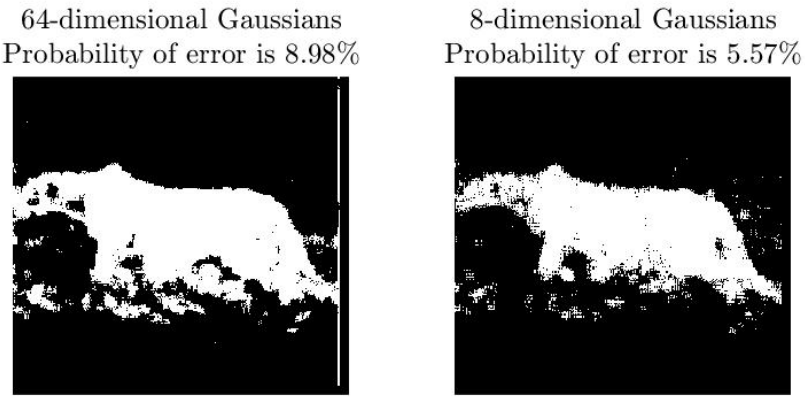


Figure 3: Classification results

Appendix

The following is the Matlab code.

0.1 HW2_solution.m

```
clear all
%%
%Training
%Read the TrainingSamplesDCT_8.mat file
load('dataset/TrainingSamplesDCT_8.mat');
%Save TrainsampleDCT_BG and TrainsampleDCT_FG in temporary value
train_BG = TrainsampleDCT_BG;
train_FG = TrainsampleDCT_FG;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Problem (a)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
P_BG = size(train_BG,1) / (size(train_BG,1) + size(train_FG,1));
P_FG = size(train_FG,1) / (size(train_BG,1) + size(train_FG,1));
```

```
15
16 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
17 % Problem (b)
18 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
19 %Calculate the mean of every features when cheetah
20 %mean_ch is the mean
21 mean_ch = fun_mean(train_FG);
22 %Calculate the mean of every features when grass
23 %mean_gr is the mean
24 mean_gr = fun_mean(train_BG);
25
26 %Calculate the covariance matrix of cheetah
27 cov_ch = fun_cov(train_FG,mean_ch);
28 %Calculate the covariance matrix of grass
29 cov_gr = fun_cov(train_BG,mean_gr);
30
31
32 %std_gr = std(train_BG,0,1);
33
34 %Plot the 64-plots
35 for j=1:1:4
36     position = 1; %Define the subplot row index
37     for i=(j-1)*16+1:1:(j-1)*16+16
38         figure(j);
39         subplot(4,4,position);
40         x=-(mean_ch(i)+4*sqrt(cov_ch(i,i))):0.0005:(mean_ch(i)+4*sqrt(
    cov_ch(i,i)));
41         y=fun_gaussian(x,mean_ch(i),sqrt(cov_ch(i,i)));
42         %Plot the cheetah line
43         plot(x,y);
44         hold on
45         x=-(mean_gr(i)+4*sqrt(cov_gr(i,i))):0.0005:(mean_gr(i)+4*sqrt(
    cov_gr(i,i)));
46         y=fun_gaussian(x,mean_gr(i),sqrt(cov_gr(i,i)));
47         %Plot the grass line and mark it as red
48         plot(x,y,'r');
49         position = position + 1;
50         title({['k=',num2str(i)]},'Fontsize',12,'interpreter','latex');
51     end
52     set(gcf,'Position',[400,100,900,600]);
53     %Save the images
54     saveas(gcf, ['Images/subplot',num2str(j),'.jpg']);
55     close(gcf);
56 end
57
58 best = [1,18,25,27,32,33,40,41];
59 worst = [3,4,5,59,60,62,63,64];
60
61 %Plot the best 8 features
62 position = 1;
```

```
for i=best
    subplot(2,4,position);
    x=-(mean_ch(i)+4*sqrt(cov_ch(i,i))):0.0005:(mean_ch(i)+4*sqrt(cov_ch(i
    ,i)));
    y=fun_gaussian(x,mean_ch(i),sqrt(cov_ch(i,i)));
    %Plot the cheetah line
    plot(x,y);
    hold on
    x=-(mean_gr(i)+4*sqrt(cov_gr(i,i))):0.0005:(mean_gr(i)+4*sqrt(cov_gr(i
    ,i)));
    y=fun_gaussian(x,mean_gr(i),sqrt(cov_gr(i,i)));
    %Plot the grass line and mark it as red
    plot(x,y,'r');
    position = position + 1;
    title({['k=',num2str(i)]},'Fontsize',12,'interpreter','latex');
end
set(gcf,'Position',[400,100,900,600]);
%Save the images
saveas(gcf, ['Images/subplot_best8features.jpg']);
close(gcf);

%Plot the worst 8 features
position = 1;
for i=worst
    subplot(2,4,position);
    x=-(mean_ch(i)+4*sqrt(cov_ch(i,i))):0.0005:(mean_ch(i)+4*sqrt(cov_ch(i
    ,i)));
    y=fun_gaussian(x,mean_ch(i),sqrt(cov_ch(i,i)));
    %Plot the cheetah line
    plot(x,y);
    hold on
    x=-(mean_gr(i)+4*sqrt(cov_gr(i,i))):0.0005:(mean_gr(i)+4*sqrt(cov_gr(i
    ,i)));
    y=fun_gaussian(x,mean_gr(i),sqrt(cov_gr(i,i)));
    %Plot the grass line and mark it as red
    plot(x,y,'r');
    position = position + 1;
    title({['k=',num2str(i)]},'Fontsize',12,'interpreter','latex');
end
set(gcf,'Position',[400,100,900,600]);
%Save the images
saveas(gcf, ['Images/subplot_worst8features.jpg']);
close(gcf);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Problem (c)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The 64-dimensional Gaussians
% %Calculate the covariance matrix of cheetah
% cov_ch = fun_cov(train_FG,mean_ch);
```

```
% %Calculate the covariance matrix of grass
% cov_gr = fun_cov(train_BG,mean_gr);

%Read original image
I = imread('dataset/cheetah.bmp');
I = im2double(I);
%Define the loop numbers
loop_row = size(I,1) - 8 + 1;
loop_column = size(I,2) - 8 + 1;
%Caculate the threshold
T = P_BG / P_FG;

mask_64 = zeros(size(I));
%Read the Zig-Zag file
position_ref = load('dataset/Zig-Zag Pattern.txt');
%Define the array for saving DCT coeffiences according to Zig-Zag
DCT_coeffience = zeros([1,64]);

for i=1:1:loop_row
    for j=1:1:loop_column
        block = I(i:i+7,j:j+7);
        DCT_block = dct2(block);
        %Map DCT_block matrix to array according Zig-Zag
        for row=1:1:8
            for column=1:1:8
                DCT_coeffience(1,position_ref(row,column)+1)=DCT_block(row
   ,column);
            end
        end
        P_x_FG = fun_mvgaussian(DCT_coeffience,mean_ch,cov_ch);
        P_x_BG = fun_mvgaussian(DCT_coeffience,mean_gr,cov_gr);
        if P_x_FG/P_x_BG > T
            mask_64(i,j) = 1;
        end
    end
end

%The best 8 features
%Calculate the covariance matrix of cheetah
cov_ch = fun_cov(train_FG(:,best),mean_ch(:,best));
%Calculate the covariance matrix of grass
cov_gr = fun_cov(train_BG(:,best),mean_gr(:,best));

mask_8 = zeros(size(I));
%Define the array for saving DCT coeffiences according to Zig-Zag
DCT_coeffience = zeros([1,64]);

for i=1:1:loop_row
    for j=1:1:loop_column
        block = I(i:i+7,j:j+7);
```

```
        DCT_block = dct2(block);
        %Map DCT_block matrix to array according Zig-Zag
        for row=1:1:8
            for column=1:1:8
                DCT_coeffience(1,position_ref(row,column)+1)=DCT_block(row
    ,column);
            end
        end
        P_x_FG = fun_mvgaussian(DCT_coeffience(best),mean_ch(best),cov_ch)
    ;
        P_x_BG = fun_mvgaussian(DCT_coeffience(best),mean_gr(best),cov_gr)
    ;
        if P_x_FG/P_x_BG > T
            mask_8(i,j) = 1;
        end
    end
end

%Read the mask file
I = imread('dataset/cheetah_mask.bmp');
I = im2double(I);
subplot(1,2,1)
imshow(mask_64);
%Calculate the probability of error
error = length(find((mask_64-I)~=0)) / (size(I,1) * size(I,2));
title({['64-dimensional Gaussians'];['Probability of error is ',num2str(
    error*100,'%.2f'),'\%']},'Fontsize',12,'interpreter','latex');
subplot(1,2,2)
imshow(mask_8);
%Calculate the probability of error
error = length(find((mask_8-I)~=0)) / (size(I,1) * size(I,2));
title({['8-dimensional Gaussians'];['Probability of error is ',num2str(
    error*100,'%.2f'),'\%']},'Fontsize',12,'interpreter','latex');
%Save the image
saveas(gcf, ['Images/segmentation.jpg']);
close(gcf);
```

0.2 fun_mean.m

The function for calculating ML mean.

```
function [mean] = fun_mean(data)
%This function is the maximum likelihood estimation of mean

N = size(data,1);
A = ones(1,N);
mean = A * data ./ N;

end
```

0.3 fun_cov.m

The function for calculating covariance matrix and ML variance.

```
function [cov] = fun_cov(data,data_mean)
%This function is the maximum likelihood estimation of covariance matrix

N = size(data,1);
cov = (data-data_mean)'*(data-data_mean)./N;

end
```

0.4 fun_gaussian.m

```
function [y] = fun_gaussian(x,mean,variance)
%This function is for Gaussian

y = exp(-(x-mean).^2/2/variance^2)./variance./sqrt(2*pi);

end
```

0.5 fun_mvgaussian.m

```
function [y] = fun_mvgaussian(x,mean,cov)
%This function is for MV Gaussian

d = size(x,2);
y = 1/sqrt((2*pi)^d*det(cov))*exp(-(x-mean)*cov^-1*(x-mean)'/2);

end
```