Assignment #3

1.Computation Equation

Class-conditional:

$$P_{x|\mu,\Sigma} = G(x, \mu, \Sigma)$$

where

$$\Sigma = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \frac{1}{N}\sum_{i=1}^{N}x_i\right)\left(x_i - \frac{1}{N}\sum_{i=1}^{N}x_i\right)^T$$

Gaussian prior:

$$P_{\mu}(\mu) = G(\mu, \mu_0, \Sigma_0)$$

where μ_0 and Σ_0 is known. The following is the computation equations of different classification methods.

a) predictive distribution:

$$P_{x|T}(x|D) = G(x, \mu_n, \Sigma + \Sigma_n)$$

where

$$\mu_n = \Sigma_0\left(\Sigma_0 + \frac{1}{N}\Sigma\right)^{-1}\mu_{ML} + \frac{1}{N}\Sigma\left(\Sigma_0 + \frac{1}{N}\Sigma\right)^{-1}\mu_0$$

$$\Sigma_n = \Sigma_0\left(\Sigma_0 + \frac{1}{N}\Sigma\right)^{-1}\frac{1}{N}\Sigma$$

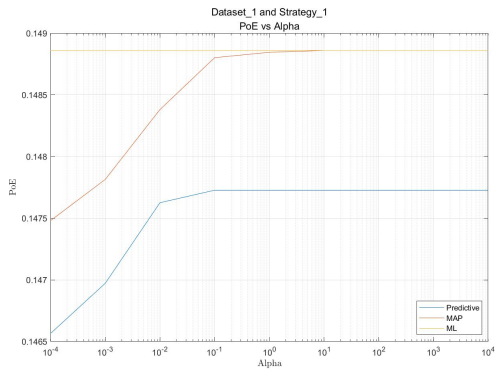
b) MAP:

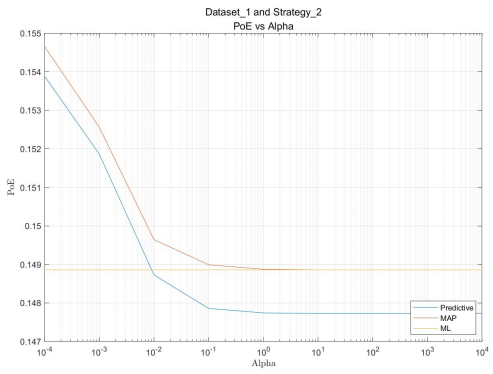
$$P_{x|T}(x|D) = G(x, \mu_n, \Sigma)$$

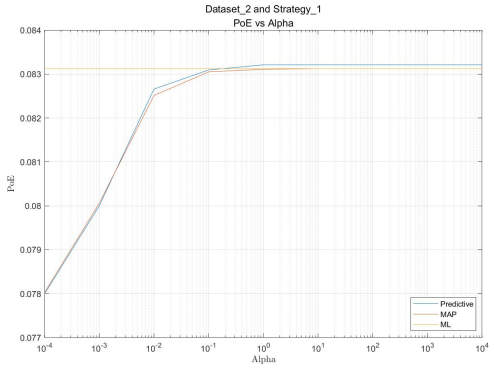
c) ML:

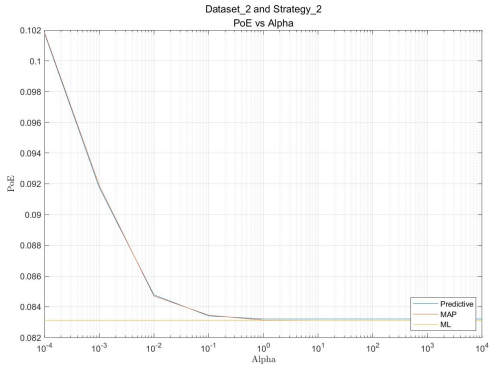
$$P_{x|T}(x|D) = G(x, \mu_{ML}, \Sigma)$$

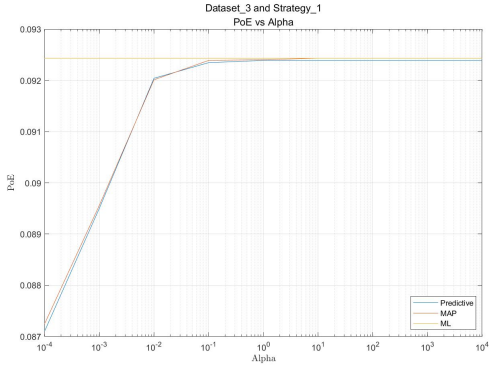
2.Curves of Classification Error vs α

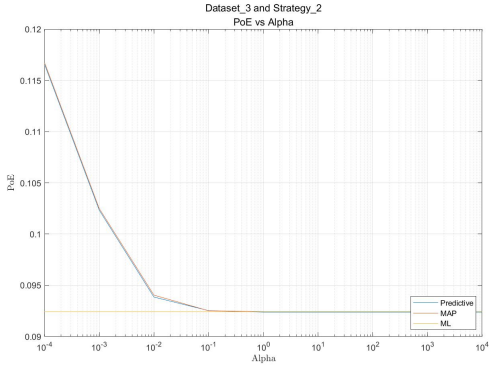


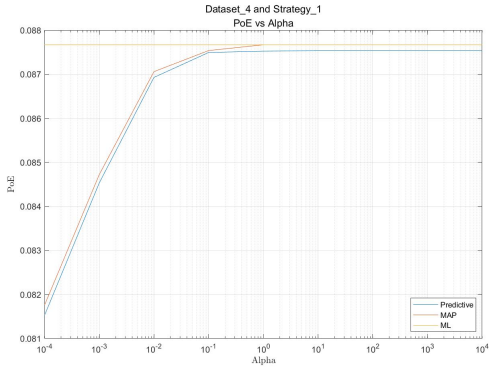


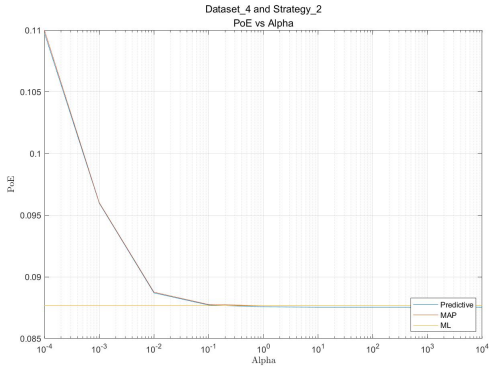












3.Observation and Explaination

a) The relative behavior of these three curves.
Observation: in strategy 1, the ML line is a constant (horizontal) line. The PoE corresponding to MAP solution increases as alpha increases, and finally converges to ML line.

The PoE corresponding to predictive solution increases as alpha increases, and finally goes to a constant value. This constant value is obviously smaller than ML in dataset1, a little bit bigger than ML in dataset2 and a little bit smaller than ML in dataset3&4. In strategy 2, things would be different, but we will discuss in section (c).

Explaination: ML line is a constant (horizontal) line because ML method has nothing to do with alpha. As alpha increases, the prior information becomes more uncertain, making the PoE of MAP and predictive solution increases. For the MAP solution, increase of alpha would push μ_n more and more closet to μ_{ML}, and finally let the MAP solution and ML solution nearly the same when alpha is large enough. For the predictive solution, increase of alpha would also push μ_n more and more closet to μ_{ML}. Meanwhile, Σ_n would approximate to $\frac{1}{N}\Sigma$, leading $\Sigma + \Sigma_n$ approximate to $\Sigma + \frac{1}{N}\Sigma$. Hence predictive solution would go to a constant value when alpha is large enough. This constant value varies depending on sample number in different dataset. That is why we can see the constant values in dataset1,2,3&4 are different.

b) How that behavior changes from dataset to dataset.
Observation: the PoE corresponding to predictive solution is obviously smaller than MAP solution in dataset1. However, in dataset2,3&4, PoE of predictive solution and MAP solution are nearly the same. In dataset2, predictive solution performs a little worse than MAP solution. But in dataset3&4, predictive solution performs a little better than MAP solution.

Explaination: when sample number of dataset (for example, dataset1) is relatively small, then predictive solution use more information of prior, leading predictive solution performs better than MAP solution and ML solution. However, when sample number is relatively large (for example, dataset2,3&4), $\Sigma + \Sigma_n$ would approximate to Σ, making predictive solution performs nearly similiar with MAP solution. However, when the alpha is large enough, they will converge to constant value, as discuss in section (a). The final PoE differ depending on sample number but the difference is small because at this time, the impact of alpha is predominant.

c) How all of the above change when strategy 1 is replaced by strategy 2?
Observation: In strategy 2, at a small alpha, PoE of predictive solution and MAP solution are obviously worse than ML solution. As alpha increases, PoE of predictive solution and MAP solution would decrease. As for MAP solution, it would again converges to ML line when alpha is large enough. The reason is the same as we discuss in section(a). As for predictive solution, in dataset1, predictive solution performs obviously better than MAP solution. The final constant PoE is also obviously better than MAP and ML solution. But in dataset2,3&4, performance of predictive solution is nealy similiar with MAP solution. The reason is the same as we discuss in section (b).

Explaination: The difference between strategy 1 and strategy 2 is the mean of first DCT coefficients. We plot the marginal distribution of first DCT coefficience in Homework2 and

already find the obvious difference of mean between background and foreground. Hence strategy 2 actually provides a bad prior information and play a bad guide. Predictive solution and MAP solution use information from prior, so in strategy 2, when alpha is relatively small, PoE of predictive solution and MAP solution is obvious large than ML solution. But when alpha increases, the prior becomes less important and ML estimation becomes predominant. Hence MAP solution would converges to ML line, and predictive solution would go to constant value.

Appendix

The following is the Matlab code.

0.1 HW3_solution.m

```
clear all
%Training
%Read the TrainingSamplesDCT_8.mat file
load('dataset/TrainingSamplesDCT_subsets_8.mat');
load('dataset/Alpha.mat');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Strategy 1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Use strategy 1 and train set D1
%Save D1_BG and D1_FG in temporary value
train_BG = D1_BG;
train_FG = D1_FG;
fun_general(1,train_BG,train_FG,alpha,1);
%Use strategy 1 and train set D2
%Save D1_BG and D1_FG in temporary value
train_BG = D2_BG;
train_FG = D2_FG;
fun_general(2,train_BG,train_FG,alpha,1);
%Use strategy 1 and train set D3
%Save D1_BG and D1_FG in temporary value
train_BG = D3_BG;
train_FG = D3_FG;
fun_general(3,train_BG,train_FG,alpha,1);
%Use strategy 1 and train set D4
%Save D1_BG and D1_FG in temporary value
train_BG = D4_BG;
train_FG = D4_FG;
fun_general(4,train_BG,train_FG,alpha,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Strategy 2
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Use strategy 2 and train set D1
%Save D1_BG and D1_FG in temporary value
train_BG = D1_BG;
train_FG = D1_FG;
fun_general(1,train_BG,train_FG,alpha,2);
%Use strategy 1 and train set D2
%Save D1_BG and D1_FG in temporary value
train_BG = D2_BG;
train_FG = D2_FG;
fun_general(2,train_BG,train_FG,alpha,2);
%Use strategy 1 and train set D3
%Save D1_BG and D1_FG in temporary value
train_BG = D3_BG;
train_FG = D3_FG;
fun_general(3,train_BG,train_FG,alpha,2);
%Use strategy 1 and train set D4
%Save D1_BG and D1_FG in temporary value
train_BG = D4_BG;
train_FG = D4_FG;
fun_general(4,train_BG,train_FG,alpha,2);
```

0.2 fun_general.m

This function receives parameters: train data, array of alpha and strategy we want to choose, and compute error using MAP-BDR, Bayes-BDR and ML-BDR.

```
function []=fun_general(dataset,train_BG,train_FG,alpha,i)
%This function is comparasion of using different strategy, different
   dataset and
%different method plugging in BDR
%i - represent the strategy we use

%Define the strategy we use
s=i; k=1;
%Define the array to store error
error_pre = zeros(size(alpha));
error_ml = zeros(size(alpha));
error_map = zeros(size(alpha));

% for Alpha=alpha
%     error_map(1,k) = fun_mapBDR(train_BG,train_FG,Alpha,s)
%     k=k+1;
% end

for Alpha=alpha
    error_pre(1,k) = fun_bayesBDR(train_BG,train_FG,Alpha,s)
    error_map(1,k) = fun_mapBDR(train_BG,train_FG,Alpha,s)
    k=k+1;
end
error_ml(1,:)=fun_mlBDR(train_BG,train_FG);
```

```
%Plot the data
figure(1);
line_pre = plot(alpha,error_pre);
hold on;
line_map = plot(alpha,error_map);
hold on;
line_ml = plot(alpha,error_ml);
legend({'Predictive','MAP','ML'},'Location','southeast');
set(gca,'XScale','log');
grid on;
title({['Dataset_',num2str(dataset),' and Strategy_',num2str(i)]...
    ;['PoE vs Alpha']},'Fontsize',12,'interpreter','latex');
ylabel('PoE', 'interpreter', 'latex');
xlabel('Alpha', 'interpreter', 'latex');
set(gcf,'Position',[400,100,900,600]);
saveas(gcf, ['images/PoE of D',num2str(dataset),' and strategy'...
   , num2str(i),'.jpg']);
close(gcf);

end
```

0.3 fun_bayesBDR.m

This function performs Bayes-BDR.

```
function [error] = fun_bayesBDR(train_BG,train_FG,Alpha,i)
%This function is for Bayes BDR.
%Use Bayes Estimation and BDR, return the predict mask of original image.
%i - represent the strategy we use

%Load data
load(['dataset/Prior_',num2str(i),'.mat']);
%Read the mask file
I = imread('dataset/cheetah_mask.bmp');
I = im2double(I);
%Define the predict mask
mask_64 = zeros(size(I));

P_BG = size(train_BG,1) / (size(train_BG,1) + size(train_FG,1));
P_FG = size(train_FG,1) / (size(train_BG,1) + size(train_FG,1));
%Calculate the mean of every features when cheetah
%mean_ch is the mean
mean_ch = fun_mean(train_FG);
%Calculate the mean of every features when grass
%mean_gr is the mean
mean_gr = fun_mean(train_BG);
%Compute the covariance matrix of class-condition
cov_ch = fun_cov(train_FG,mean_ch);
cov_gr = fun_cov(train_BG,mean_gr);
```

```
%Compute the covariance matrix of Gaussian prior
v = Alpha * W0;
cov_prior = diag(v);
% size(cov_prior)
% size(cov_ch)
% size(cov_gr)
% size(mu0_BG)

%Compute Bayes Estimation and parameters of the predictive distribution
%Use mu_p and mu_cov as the mean and convariance matrix of the predictive
%distribution. The equation is from DHS.
%BG predictive distribution
mu_p_BG = cov_prior * inv(cov_prior + cov_gr/size(train_BG,1)) * ...
    mean_gr' + cov_gr/size(train_BG,1) * inv(cov_prior + cov_gr/size(
    train_BG,1))...
    * mu0_BG';
cov_p_BG = cov_gr + cov_prior * inv(cov_prior + cov_gr/size(train_BG,1)) *
    cov_gr/size(train_BG,1);
%FG predictive distribution
mu_p_FG = cov_prior * inv(cov_prior + cov_ch/size(train_FG,1)) * ...
    mean_ch' + cov_ch/size(train_FG,1) * inv(cov_prior + cov_ch/size(
    train_FG,1))...
    * mu0_FG';
cov_p_FG = cov_ch + cov_prior * inv(cov_prior + cov_ch/size(train_FG,1)) *
    cov_ch/size(train_FG,1);

%Load DCT file
load('DCT_coeffience.mat');
%Caculate the threshold
T = P_BG / P_FG;
%Define the loop numbers
loop_row = size(I,1) - 8 + 1;
loop_column = size(I,2) - 8 + 1;
k=1;

for i=1:1:loop_row
    for j=1:1:loop_column
        P_x_FG = fun_mvgaussian(DCT_coeffience(k,:),mu_p_FG',cov_p_FG);
        P_x_BG = fun_mvgaussian(DCT_coeffience(k,:),mu_p_BG',cov_p_BG);
        if P_x_FG/P_x_BG > T
            mask_64(i,j) = 1;
        end
        k=k+1;
    end
end
%Calculate the probability of error
error = length(find((mask_64-I)~=0)) / (size(I,1) * size(I,2));

end
```

0.4 fun_mapBDR.m

This function performs MAP-BDR.

```
function [error] = fun_mapBDR(train_BG,train_FG,Alpha,i)
%This function is for MAP BDR.
%Use Bayes Estimation and BDR, return the predict mask of original image.
%i - represent the strategy we use

%Load data
load(['dataset/Prior_',num2str(i),'.mat']);
%Read the mask file
I = imread('dataset/cheetah_mask.bmp');
I = im2double(I);
%Define the predict mask
mask_64 = zeros(size(I));

P_BG = size(train_BG,1) / (size(train_BG,1) + size(train_FG,1));
P_FG = size(train_FG,1) / (size(train_BG,1) + size(train_FG,1));
%Calculate the mean of every features when cheetah
%mean_ch is the mean
mean_ch = fun_mean(train_FG);
%Calculate the mean of every features when grass
%mean_gr is the mean
mean_gr = fun_mean(train_BG);
%Compute the covariance matrix of class-condition
cov_ch = fun_cov(train_FG,mean_ch);
cov_gr = fun_cov(train_BG,mean_gr);
%Compute the covariance matrix of Gaussian prior
v = Alpha * W0;
cov_prior = diag(v);

%Compute MAP Estimation and parameters of the predictive distribution
%Use mu_p and mu_cov as the mean and convariance matrix of the predictive
%distribution. The equation is from DHS.
%BG predictive distribution
mu_p_BG = cov_prior * inv(cov_prior + cov_gr/size(train_BG,1)) * ...
    mean_gr' + cov_gr/size(train_BG,1) * inv(cov_prior + cov_gr/size(
   train_BG,1))...
    * mu0_BG';
cov_p_BG = cov_gr;
%FG predictive distribution
mu_p_FG = cov_prior * inv(cov_prior + cov_ch/size(train_FG,1)) * ...
    mean_ch' + cov_ch/size(train_FG,1) * inv(cov_prior + cov_ch/size(
   train_FG,1))...
    * mu0_FG';
cov_p_FG = cov_ch;

%Load DCT file
load('DCT_coeffience.mat');
%Caculate the threshold
```

```
T = P_BG / P_FG;
%Define the loop numbers
loop_row = size(I,1) - 8 + 1;
loop_column = size(I,2) - 8 + 1;
k=1;

for i=1:1:loop_row
    for j=1:1:loop_column
        P_x_FG = fun_mvgaussian(DCT_coeffience(k,:),mu_p_FG',cov_p_FG);
        P_x_BG = fun_mvgaussian(DCT_coeffience(k,:),mu_p_BG',cov_p_BG);
        if P_x_FG/P_x_BG > T
            mask_64(i,j) = 1;
        end
        k=k+1;
    end
end
%Calculate the probability of error
error = length(find((mask_64-I)~=0)) / (size(I,1) * size(I,2));

end
```

0.5 fun_mlBDR.m

This function performs ML-BDR.

```
function [error] = fun_bayesBDR(train_BG,train_FG)
%This function is for ML BDR.

%Read the mask file
I = imread('dataset/cheetah_mask.bmp');
I = im2double(I);
%Define the predict mask
mask_64 = zeros(size(I));

P_BG = size(train_BG,1) / (size(train_BG,1) + size(train_FG,1));
P_FG = size(train_FG,1) / (size(train_BG,1) + size(train_FG,1));
%Calculate the mean of every features when cheetah
%mean_ch is the mean
mean_ch = fun_mean(train_FG);
%Calculate the mean of every features when grass
%mean_gr is the mean
mean_gr = fun_mean(train_BG);

%Calculate the covariance matrix of cheetah
cov_ch = fun_cov(train_FG,mean_ch);
%Calculate the covariance matrix of grass
cov_gr = fun_cov(train_BG,mean_gr);

%Load DCT file
```

```
load('DCT_coeffience.mat');
%Caculate the threshold
T = P_BG / P_FG;
%Define the loop numbers
loop_row = size(I,1) - 8 + 1;
loop_column = size(I,2) - 8 + 1;
k=1;

for z=1:1:loop_row
    for j=1:1:loop_column
        P_x_FG = fun_mvgaussian(DCT_coeffience(k,:),mean_ch,cov_ch);
        P_x_BG = fun_mvgaussian(DCT_coeffience(k,:),mean_gr,cov_gr);
        if P_x_FG/P_x_BG > T
            mask_64(z,j) = 1;
        end
        k=k+1;
    end
end
%Calculate the probability of error
error = length(find((mask_64-I)~=0)) / (size(I,1) * size(I,2));

end
```

0.6 fun_zigzag.m

This function compute DCT coefficients and save it as DCT_coefficience.mat.

```
function [] = fun_zigzag()
%This function is for computing the DCT coe?cients of original image.
%Decomposition into 8 x 8 image blocks, compute the DCT of each block, and
%zig-zag scan.

%Read original image
I = imread('dataset/cheetah.bmp');
I = im2double(I);
%Define the loop numbers
loop_row = size(I,1) - 8 + 1;
loop_column = size(I,2) - 8 + 1;
%Read the Zig-Zag file
position_ref = load('dataset/Zig-Zag Pattern.txt');
%Define the array for saving DCT coeffiences according to Zig-Zag
DCT_coeffience = zeros([loop_row*loop_column,64]);
k=1;

for i=1:1:loop_row
    for j=1:1:loop_column
        block = I(i:i+7,j:j+7);
        DCT_block = dct2(block);
        %Map DCT_block matrix to array according Zig-Zag
```

```
        for row=1:1:8
            for column=1:1:8
                DCT_coeffience(k,position_ref(row,column)+1)=DCT_block(row
   ,column);
            end
        end
        k=k+1;
    end
end
save('../DCT_coeffience.mat','DCT_coeffience');
end
```

0.7 fun_mvgaussian.m

This function perfroms multi-gaussian.

```
function [y] = fun_mvgaussian(x,mean,cov)
%This function is for MV Gaussian

d = size(x,2);
y = 1/sqrt((2*pi)^d*det(cov))*exp(-(x-mean)*cov^-1*(x-mean)'/2);

end
```

0.8 fun_cov.m

```
function [cov] = fun_cov(data,data_mean)
%This function is the maximum likelihood estimation of covariance matrix

N = size(data,1);
cov = (data-data_mean)'*(data-data_mean)./N;

end
```

0.9 fun_mean.m

```
function [mean] = fun_mean(data)
%This function is the maximum likelihood estimation of mean

N = size(data,1);
A = ones(1,N);
mean = A * data ./ N;

end
```