$$
\begin{array}{ll}
Mcons_\alpha & : M\alpha \\
Mcons^{st}_\alpha & : (\sigma \to \alpha\sigma) \\
Mcons^{res}_\alpha & : (\alpha|\epsilon) \\
Mcons^{prs}_\alpha & : (\sigma \to (\sigma \to (\alpha|\epsilon) \times \sigma) \times \sigma)
\end{array}
$$

$$try^{prs}\ k\ err\ pm = lift^{st}(lift^{st}(try^{res}k(\lambda e_1.try^{res}(>>=)^{id}(\lambda e_2.fail^{prs}(e_1+e_2)))err))pm : (\alpha \to M\beta) \to (\epsilon \to M\beta) \to Mcons^{prs}_\alpha \to Mcons^{prs}_\beta$$

$$\dfrac{\dfrac{\dfrac{try^{res} : (\alpha \to M\beta) \to (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon) \qquad (>>=)^{id} : (\alpha \to M\beta)}{try^{res}(>>=)^{id} : (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon)} \qquad \dfrac{fail^{prs} : \epsilon \to Mcons^{prs}_\alpha \qquad \dfrac{e_1 : \epsilon \qquad e_2 : \epsilon}{e_1 + e_2 : \epsilon}}{fail^{prs}(e_1+e_2) : Mcons^{prs}_\alpha}}{\lambda e_2.fail^{prs}(e_1+e_2) : \epsilon \to Mcons^{prs}_\alpha}}{try^{res}(>>=)^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2)) : M(\alpha|\epsilon) \to M(\beta|\epsilon)}}{\lambda e_1.try^{res}(>>=)^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2)) : \epsilon \to M(\alpha|\epsilon) \to M(\beta|\epsilon)}$$

$$\dfrac{\dfrac{try^{res} : (\alpha \to M\beta) \to (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon) \qquad k : (\alpha \to M\beta)}{try^{res}k : (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon)} \qquad \lambda e_1.try^{res}(>>=)^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2)) : \epsilon \to M(\alpha|\epsilon) \to M(\beta|\epsilon)}{\textbf{Contradiction:}\ \text{application of } try^{res}\ k \text{ expects } (\epsilon \to M\beta) \text{ but got } (\epsilon \to M(\alpha|\epsilon) \to M(\beta|\epsilon))}$$

$$try^{prs}\ k\ err\ pm = lift^{st}(lift^{st}(try^{res}k(\lambda e_1.try^{res}return^{id}(\lambda e_2.fail^{prs}(e_1+e_2)))err))pm : (\alpha \to M\beta) \to (\epsilon \to M\beta) \to Mcons^{prs}_{\alpha} \to Mcons^{prs}_{\beta}$$

$$\dfrac{\dfrac{\dfrac{try^{res} : (\alpha \to M\beta) \to (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon) \qquad return^{id} : (\alpha \to M\beta)}{try^{res}return^{id} : (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon)} \qquad \dfrac{\dfrac{fail^{prs} : \epsilon \to Mcons^{prs}_{\alpha} \qquad \dfrac{e_1 : \epsilon \qquad e_2 : \epsilon}{e_1 + e_2 : \epsilon}}{fail^{prs}(e_1+e_2) : Mcons^{prs}_{\alpha}}}{\lambda e_2.fail^{prs}(e_1+e_2) : \epsilon \to Mcons^{prs}_{\alpha}}}{try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2)) : M(\alpha|\epsilon) \to M(\beta|\epsilon)} \qquad err : M(\alpha|\epsilon)}{try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))err : M(\beta|\epsilon)}}{\lambda e_1.try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))err : \epsilon \to M(\beta|\epsilon)}$$

$$\dfrac{\dfrac{\dfrac{try^{res} : (\alpha \to M\beta) \to (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon) \qquad k : (\alpha \to M\beta)}{try^{res}k : (\epsilon \to M\beta) \to M(\alpha|\epsilon) \to M(\beta|\epsilon)} \qquad \lambda e_1.try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))err : \epsilon \to M(\beta|\epsilon)}{try^{res}k\ (\lambda e_1.try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))err) : M(\alpha|\epsilon) \to M(\beta|\epsilon)} \qquad pm : M(\alpha|\epsilon)}{try^{res}k\ (\lambda e_1.try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))\ err)\ pm : M(\beta|\epsilon)}$$

$$\dfrac{try^{res}k\ (\lambda e_1.try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))\ err)\ pm : M(\beta|\epsilon) \qquad lift^{st} : (\alpha \to \beta) \to Mcons^{st}_{\alpha} \to Mcons^{st}_{\beta}}{lift(try^{res}k\ (\lambda e_1.try^{res}return^{id}\ (\lambda e_2.fail^{prs}(e_1+e_2))\ err)\ pm) : M(\beta|\epsilon)}$$