

$Mcons_\alpha$	$: M\alpha$
$Mcons_\alpha^{st}$	$: (\sigma \rightarrow \alpha\sigma)$
$Mcons_\alpha^{res}$	$: (\alpha \epsilon)$
$Mcons_\alpha^{prs}$	$: (\sigma \rightarrow (\sigma \rightarrow (\alpha \epsilon) \times \sigma) \times \sigma)$

$$try^{prs} k \text{ err } pm = lift^{st}(lift^{st}(try^{res} k(\lambda e_1. try^{res}(>>=)^{id}(\lambda e_2. fail^{prs}(e_1 + e_2)))err))pm : (\alpha \rightarrow M\beta) \rightarrow (\epsilon \rightarrow M\beta) \rightarrow Mcons_\alpha^{prs} \rightarrow Mcons_\beta^{prs}$$

$$\begin{array}{c}
\frac{\frac{try^{res} : (\alpha \rightarrow M\beta) \rightarrow (\epsilon \rightarrow M\beta) \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res} \quad (>>=)^{id} : (\alpha \rightarrow M\beta)}{try^{res}(>>=)^{id} : (\epsilon \rightarrow M\beta) \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res}} \quad \frac{\frac{fail^{prs} : \epsilon \rightarrow Mcons_\alpha^{prs} \quad \frac{e_1 : \epsilon \quad e_2 : \epsilon}{e_1 + e_2 : \epsilon}}{fail^{prs}(e_1 + e_2) : Mcons_\alpha^{prs}}}{\lambda e_2. fail^{prs}(e_1 + e_2) : \epsilon \rightarrow Mcons_\alpha^{prs}}}{\frac{try^{res}(>>=)^{id}(\lambda e_2. fail^{prs}(e_1 + e_2)) : Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res}}{\lambda e_1. try^{res}(>>=)^{id}(\lambda e_2. fail^{prs}(e_1 + e_2)) : \epsilon \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res}}} \\
\\
\frac{\frac{try^{res} : (\alpha \rightarrow M\beta) \rightarrow (\epsilon \rightarrow M\beta) \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res} \quad k : (\alpha \rightarrow M\beta)}{try^{res} k : (\epsilon \rightarrow M\beta) \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res}} \quad \lambda e_1. try^{res}(>>=)^{id}(\lambda e_2. fail^{prs}(e_1 + e_2)) : \epsilon \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res}}{\textbf{Contradiction:} \text{ application of } try^{res} k \text{ expects } (\epsilon \rightarrow M\beta) \text{ but got } (\epsilon \rightarrow Mcons_\alpha^{res} \rightarrow Mcons_\beta^{res})}
\end{array}$$

$$try^{prs} k \text{ err } pm = lift^{st}(lift^{st}(try^{res} k(\lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2)))err))pm : (\alpha \rightarrow M\beta) \rightarrow (\epsilon \rightarrow M\beta) \rightarrow Mcons_{\alpha}^{prs} \rightarrow Mcons_{\beta}^{prs}$$

$$\begin{array}{c}
\frac{\frac{try^{res} : (\alpha \rightarrow M\beta) \rightarrow (\epsilon \rightarrow M\beta) \rightarrow Mcons_{\alpha}^{res} \rightarrow Mcons_{\beta}^{res} \quad return^{id} : (\alpha \rightarrow M\beta)}{try^{res} return^{id} : (\epsilon \rightarrow M\beta) \rightarrow Mcons_{\alpha}^{res} \rightarrow Mcons_{\beta}^{res}} \quad \frac{\frac{fail^{prs} : \epsilon \rightarrow Mcons_{\alpha}^{prs} \quad \frac{e_1 : \epsilon \quad e_2 : \epsilon}{e_1 + e_2 : \epsilon}}{fail^{prs}(e_1 + e_2) : Mcons_{\alpha}^{prs}}}{\lambda e_2. fail^{prs}(e_1 + e_2) : \epsilon \rightarrow Mcons_{\alpha}^{prs}}}{\frac{try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2)) : Mcons_{\alpha}^{res} \rightarrow Mcons_{\beta}^{res} \quad err : Mcons_{\alpha}^{res}}{try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err : Mcons_{\beta}^{res}}}{\lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err : \epsilon \rightarrow Mcons_{\beta}^{res}} \\
\\
\frac{\frac{try^{res} : (\alpha \rightarrow M\beta) \rightarrow (\epsilon \rightarrow M\beta) \rightarrow Mcons_{\alpha}^{res} \rightarrow Mcons_{\beta}^{res} \quad k : (\alpha \rightarrow M\beta)}{try^{res} k : (\epsilon \rightarrow M\beta) \rightarrow Mcons_{\alpha}^{res} \rightarrow Mcons_{\beta}^{res}} \quad \lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err : \epsilon \rightarrow Mcons_{\beta}^{res}}{\frac{try^{res} k(\lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err) : Mcons_{\alpha}^{res} \rightarrow Mcons_{\beta}^{res} \quad pm : Mcons_{\alpha}^{res}}{try^{res} k(\lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err) pm : Mcons_{\beta}^{res}}} \\
\\
\frac{try^{res} k(\lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err) pm : Mcons_{\beta}^{res} \quad lift^{st} : (\alpha \rightarrow \beta) \rightarrow Mcons_{\alpha}^{st} \rightarrow Mcons_{\beta}^{st}}{lift(try^{res} k(\lambda e_1. try^{res} return^{id}(\lambda e_2. fail^{prs}(e_1 + e_2))err) pm) : Mcons_{\beta}^{res}}
\end{array}$$