Performance Analysis of Different MIMO Transmission Modes with Link Adaptation

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***Abstract*— In this paper, link adaptation, which dynamically selects modulation level and coding rate according to the channel condition, and several multi-input multi-output (MIMO) transmission modes are studied. As we know, the subchannels' signal-to-noise radio (SNR) by singular value decomposition (SVD) are unequal, which requires different modulation and coding schemes (MCS) to maximize the throughput. However, uniform channel decomposition (UCD) greatly reduces the coding and modulation complexity by decomposing the MIMO channel into multiple subchannels with identical capacity. This paper analyzes the post-detection SINR of different MIMO transmission modes and presents their throughput performance with link adaptation under the constraint of required packet error rate (PER). We show that UCD mode is the first choice at high SNR considering both coding/modulation complexity and throughput performance, while at low SNR, Space-time Block Coding (STBC) mode outperforms other MIMO transmission modes.**

***Index Terms*— Link Adaptation, MIMO, Singular Value Decomposition (SVD), Uniform Channel Decomposition (UCD)**

I. Introduction

Mobile WiMAX is a metropolitan access technique, which provides data transfer rates that exceed those of current 3G. To meet such a great performance requirement, WiMAX adopts multiple-input multiple-output (MIMO) antenna technique, which promises significant improvements in terms of spectral efficiency and link reliability [1]. In addition, since wireless channel varies as time changes in nature, employing link adaptation which can perform the function of dynamically adjusting the modulation and coding scheme based on the experienced channel condition has been recognized as an effective way to improve the throughput performance of the cellular system [2]. Therefore, the combination of the link adaptation and MIMO technique can significantly increase the throughput of the Mobile WiMAX system.

MIMO techniques can be used to increase diversity and improve the bit error rate (BER) performance of wireless systems, increase the transmitted data rate through spatial multiplexing, or trade off both. Spatial multiplexing (SM) can increase the spectral efficiency by transmitting parallel data streams, but the multiple transmit streams interfere with each other, which degrades the system performance. While another MIMO transmission mode, Alamouti's Space-time Block Coding (STBC), can extract full diversity gain while lose multiplexing gain. The above methods assume that the channel state information (CSI) is available at the receiver side only. However, if the transmitter can obtain the CSI by feedback, e.g., Channel Coefficient Quantization, Codebook Based Quantization, and Channel Sounding, joint transceiver design can be employed. Singular value decomposition (SVD), one of the joint transceiver designs, decomposes a MIMO channel into parallel single-input single-output (SISO) subchannels. Although SVD can achieve the channel capacities, yet the capacity of the subchannels are different, which not only greatly increases the modulation and channel coding complexity but also loss capacity due to the finite constellation granularity. In [3], uniform channel decomposition (UCD) scheme decomposes a MIMO channel into multiple channels with identical capacities without losing capacity. Therefore, it can significantly reduce the channel coding and modulation complexity. This paper analyzes five MIMO transmission modes in the mobile WiMAX systems under the consideration of both achievable data rate and PER.

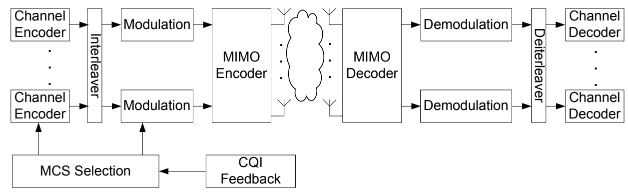

Fig. 1. The architecture of MIMO system with link adaptation

The rest of this paper is organized as follows. In Section II, a basic MIMO system is described. The characteristic of different MIMO transmission modes are studied in Section III and link adaptation is presented in Section IV. Section V shows the simulation results and conducts performance analysis. Finally conclusions are drawn in Section VI.

II. System Model

Consider a wireless link model with M_T transmitting antennas and M_R receiving antennas. The $M_R \times M_T$ channel matrix $\mathbf{H} = [h_{i,j}]$ describes the channel. The flat fading coefficient $h_{i,j}$ represents the complex path gain from transmit antenna j to receive antenna i . Here, we assume that the $h_{i,j}$ is independent identically distributed (i.i.d) zero mean complex

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Gaussian random variable, circularly symmetric distributed with unit variance. The received $M_R \times 1$ vector r can be written as

$$r = \mathbf{H}s + n, \tag{1}$$

where s is the $M_T \times 1$ transmitted vector and n is $M_R \times 1$ vector of additive white circularly symmetric complex Gaussian noise with each element having a variance equal to σ^2. We assume the duration of a frame is smaller than the channel coherence time, so $\mathbf{H}$ is considered constant during a frame but varies from frame to frame. There are many parameters that can be varied at the transmitter relative to the channel gain, whereas in this paper we only discuss variation of modulation constellation and coding rate. The architecture of MIMO system with link adaptation is illustrated in Fig. 1. Channel quality indicator (CQI) is feeded back to the the transmitter for modulation and coding schemes (MCS) selection.

III. MIMO Transmission Modes

In this Section, the characteristics of different MIMO transmission modes are studied, and the output signal-to-interference-and-noise radio (SINR) of those modes are given in the respective subsections.

A. SM

The problem in SM mode faced by MIMO receiver is the presence of multistream interference (MSI). The minimum mean-squared-error (MMSE) receiver balances MSI mitigation with noise enhancement and minimizes the total error. We define $\mathbf{W}_R$ as the receiver weight matrix and give the MMSE receiver weight matrix as

$$\mathbf{W}_{R_{MMSE}} = \arg\min \varepsilon\{\|\mathbf{W}_R r - s\|_F^2\}, \tag{2}$$

where $\|\cdot\|_F$ denotes Frobenius norm and ε is the expectation operator. Utilizing the orthogonality principle [4], $\mathbf{W}_{R_{MMSE}}$ is easily derived as

$$\mathbf{W}_{R_{MMSE}} = \sqrt{\frac{M_T}{E_s}} \left(\mathbf{H}^H \mathbf{H} + \frac{M_T \sigma^2}{E_s} \mathbf{I}_{M_T} \right)^{-1} \mathbf{H}^H, \tag{3}$$

where $(*)^H$ denotes Hermitian transpose, E_s is the average energy at the transmitter per symbol period and $\mathbf{I}_{M_T}$ is a $M_T \times M_T$ identity matrix. Consequently, the SINR on the kth ($k = 1, 2, \cdots, M_T$) received stream can be shown to be

$$\mathrm{SINR}_k^{SM} = \frac{1}{[(\frac{E_s}{M_T\sigma^2}\mathbf{H}^H\mathbf{H} + \mathbf{I}_{M_T})^{-1}]_{k,k}} - 1. \tag{4}$$

B. Open Loop STBC

For simplicity, in this section and next section, we restrict our attention to the case of $M_T = 2$ and $M_R = 2$. Open loop STBC encoder produces two orthogonal output blocks and each of them containing two complex symbols. We assume n_1 and n_2 are the noise at the two antennas. Then the first and the second received symbols can be expressed as

$$\begin{aligned} r_1 &= \mathbf{H}s + n_1 \\ r_2 &= \mathbf{H}\mathbf{F}s^* + n_2 \end{aligned}, \tag{5}$$

where $\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $(*)$ means conjugate. Assuming $tr(\cdot)$ denotes the trace of a matrix and (H) denotes transpose-conjugate, the receiver weight matrix is given by

$$\mathbf{W}_R = \frac{\mathbf{R}^H}{tr\,(\mathbf{R}^H\mathbf{R})}, \tag{6}$$

where $\mathbf{R} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^*\mathbf{F} \end{bmatrix}$, then the estimated s of open loop STBC is

$$\hat{s} = \mathbf{W}_R\mathbf{R}s + \mathbf{W}_R n. \tag{7}$$

After some calculations, the SINR of the two symbols can be obtained

$$\begin{aligned} \mathrm{SINR}_1^{OL-STBC} &= \frac{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{1,1}}{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{1,2} + \sum\limits_{k=1}^{4} \mathbf{W}_{R_{1k}}} \\ \mathrm{SINR}_2^{OL-STBC} &= \frac{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{2,2}}{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{2,1} + \sum\limits_{k=1}^{4} \mathbf{W}_{R_{2k}}} \end{aligned}. \tag{8}$$

C. Closed Loop STBC

If the transmitter knows the channel state information, we can obtain further gain by using maximum ratio transmission (MRT) technique [5], which removes part of the power from the weaker branch and reallocates it to the stronger branch. The received symbols at the two antennas of the receiver are

$$\begin{aligned} r_1 &= \mathbf{H}\mathbf{W}_T s + n_1 \\ r_2 &= \mathbf{H}\mathbf{W}_T\mathbf{F}s^* + n_2 \end{aligned}, \tag{9}$$

where $\mathbf{W}_T$ is the MRT matrix of the transmitter and is given by

$$\mathbf{W}_T = \frac{\sqrt{2}}{||\mathbf{H}||_F^2}\mathbf{H}^H. \tag{10}$$

Then the estimated symbol at the receiver can be written as

$$\hat{s} = \mathbf{W}_R\, r = \mathbf{W}_R\mathbf{R}\, s + \mathbf{W}_R\, n. \tag{11}$$

With some basic matrix computations, we derive the SINR of the two symbols

$$\begin{aligned} \mathrm{SINR}_1^{CL-STBC} &= \frac{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{1,1}}{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{1,2} + \sum\limits_{k=1}^{4} \mathbf{W}_{R_{1k}}} \\ \mathrm{SINR}_2^{CL-STBC} &= \frac{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{2,2}}{\frac{E_s}{2\sigma^2}[\mathbf{W}_R\mathbf{R}]_{2,1} + \sum\limits_{k=1}^{4} \mathbf{W}_{R_{2k}}} \end{aligned}, \tag{12}$$

where the receiver weight matrix $\mathbf{W}_R$ is the same as equation (6) and $\mathbf{R} = \begin{bmatrix} \mathbf{H}\mathbf{W}_T \\ \mathbf{H}^*\mathbf{W}_T^*\mathbf{F} \end{bmatrix}$.

In [5], it has been proved that SINR of the closed loop STBC is higher than that of the open loop STBC.

D. SVD Pre-coding

It is well known that any complex matrix $\mathbf{H}$ can be decomposed by SVD as

$$\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^H, \tag{13}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_K)$ is a real diagonal matrix of the eigenvalues, in which $K = \min(\mathbf{M}_T, \mathbf{M}_R)$. If we substitute (13) into (1) and multiply on both sides of the equation with $\mathbf{U}^H$, the MIMO channel can be treated as several parallel subchannels. And the SINR of the kth subchannel can be written as

$$\text{SINR}_k^{SVD} = \frac{E_s\lambda_k^2}{M_T\sigma^2}. \tag{14}$$

E. UCD Pre-coding

Although SVD can achieve MIMO channel capacity, it requires careful bit allocation to match the subchannel capacity and each subchannal need different channel modulator/demodulator and encoder/decoder due to the different SINR of the subchannel. In [3], UCD scheme overcomes this problem by generating decoupled subchannels with identical capacities so that equal modulation and coding scheme can be applied to each subchannel, which greatly reduces modulation and coding complexity. However, UCD has more computational complexity than SVD. Using a dirty paper decoder or combining a linear precoder with a MMSE vertical BLAST (V-BLAST) detector, UCD scheme exhibits capacity lossless. The theory of UCD is illuminated in [3], we just give a brief review of the methodology here.

According to [6], in which geometric mean decomposition (GMD) is introduced, the channel matrix $\mathbf{H}$ can be decomposed into

$$\mathbf{H} = \mathbf{Q}\mathbf{R}\mathbf{P}^H, \tag{15}$$

where $\mathbf{Q}$ and $\mathbf{P}$ have orthonormal columns, and $\mathbf{R}$ is a real upper triangular matrix with diagonal elements equal to the geometric mean of the positive singular values of $\mathbf{H}$. The UCD scheme, which is based on GMD, multiplies a precoding matrix $\mathbf{F}$ presented as follows to the transmitter:

$$\mathbf{F} = \mathbf{V}\mathbf{\Phi}^{\frac{1}{2}}\mathbf{\Omega}^H, \tag{16}$$

where $\mathbf{\Phi}$ determines the power loaded to the subchannels and is found via "water filling", and $\mathbf{\Omega}^*\mathbf{\Omega} = \mathbf{I}$.

The nulling and interference matrices of MMSE-based decision feedback equalization (DFE) can be found via the QR-decomposition [7]. Then we can write

$$\begin{bmatrix} \mathbf{H} \\ \sqrt{\sigma^2}\mathbf{I}_{M_T} \end{bmatrix} = \begin{bmatrix} \mathbf{U}\mathbf{\Lambda}\mathbf{\Phi}^{\frac{1}{2}}\mathbf{\Omega}^H \\ \sqrt{\sigma^2}\mathbf{I}_{M_T} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{M_R} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega} \end{bmatrix} \begin{bmatrix} \mathbf{U}\left[\mathbf{\Lambda}\mathbf{\Phi}^{\frac{1}{2}} \vdots \mathbf{0}_{K\times M_r}\right] \\ \sqrt{\sigma^2}\mathbf{I}_{M_T} \end{bmatrix} \mathbf{\Omega}^H. \tag{17}$$

Using GMD, the following equation is obtained

$$\begin{bmatrix} \mathbf{U}\left[\mathbf{\Lambda}\mathbf{\Phi}^{\frac{1}{2}} \vdots \mathbf{0}_{K\times M_r}\right] \\ \sqrt{\sigma^2}\ \mathbf{I}_{M_T} \end{bmatrix} = \mathbf{Q}_J\mathbf{R}_J\mathbf{P}_J^H \tag{18}$$

We define $\mathbf{\Omega} = \mathbf{P}_J$, consequently the precoding matrix $\mathbf{F}$ can be calculated according to (16). As UCD is capacity lossless, the SINR of the kth subchannel of UCD scheme can be presented as

$$\text{SINR}_k^{UCD} = \frac{E_s\bar{\lambda}^2}{M_T\sigma^2}, \tag{19}$$

where

$$\bar{\lambda}^2 = \left(\sqrt[K]{\left(1+\frac{E_s\lambda_1^2}{M_T\sigma^2}\right)\left(1+\frac{E_s\lambda_2^2}{M_T\sigma^2}\right)\cdots\left(1+\frac{E_s\lambda_K^2}{M_T\sigma^2}\right)} - 1\right)\frac{M_T\sigma^2}{E_s}. \tag{20}$$

IV. Link Adaptation

Link adaptation is used to dynamically adjust the modulation and coding to reflect characteristics of the wireless link and maximize the throughput. However, if the channel is changing faster than it can be reliably estimated and fed back to the transmitter, adaptive techniques will perform poorly, and other means of mitigating the effects of fading should be used.

In single antenna systems, link adaptation is usually based on the received SINR, as the SINR determines the packet-error rate (PER). However, in MIMO systems, individual subchannels might interfere significantly with each other, therefore post-detection SINR (P-SINR), the SINR after MIMO decoding, is implemented to choose appropriate modulation coding scheme (MCS). Using the instantaneous P-SINR, the PER of all MCS schemes are computed, then the highest rate MCS scheme achieving the target PER is selected for transmission. The PER can be defined as a function $f(\text{P-SINR}, \text{MCS})$ of P-SINR and MCS. Therefore, the optimization of the link adaptation can be expressed as

$$\begin{aligned} &max : \text{Throughput} = (1 - f\,(\text{P-SINR}, \text{MCS}))\, M_{\text{MCS}} R_{\text{MCS}} \\ &s.t. : \text{PER} = f\,(\text{P-SINR}, \text{MCS}) > \text{PER}_{target} \end{aligned} \tag{21}$$

where M_{MCS} and R_{MCS} are the modulation order and channel coding rate respectively. The detail about the MCS for the simulation is listed in Table I. Fig. 2 represents the link-level throughput performance vs. SNR for different MCS. We note from inspection of Fig. 2 that MCS should be adjusted according to the SNR to achieve the highest throughput. The result in Fig. 2 is ideal since it assumes that the modulation level and turbo coding rate are perfectly adapted to the short-term SNR. However, this assumption is not practical due to delays in the feed back path. The performance result under the consideration of the delay is presented in the next section.

V. Performance Results

The performances of different MIMO transmission modes with link adaptation are evaluated in ITU Pedestrain-B (3 km/h) channel. Considering the instantaneous channel state information cannot be available at the transmitter, we assume that the channel information feedback delay is one frame (5ms) in the simulations.

The capacity improvement offered by link adaptation over non-adaptive systems is remarkable, as illustrated by Fig. 3.

TABLE I
MCS SCHEME

MCS scheme	1	2	3	4	5	6	7	8
Modulation	QPSK	QPSK	16QAM	16QAM	64QAM	64QAM	64QAM	64QAM
CTC Rate†	1/2	3/4	1/2	3/4	1/2	2/3	3/4	5/6

†CTC is Convolutional Turbo Coding.

In this figure, we represent the link level throughput vs. SNR using SVD. The blue curve and red curve are throughput of each MCS for the two streams of SVD, while the black curve is the throughput with link adaptation. As the selected MCS schemes of the two steams are different, two sets of encoder/decoder and modulator/demodulator are needed.

Fig. 4 proves that the closed loop STBC transmission mode with link adaptation has the highest throughput at low SNR, but it has a ceiling value because this mode can only transmit one stream with two antennas. And Open loop STBC has low performance than the closed loop STBC. But, practically, its performance is affected little by the accuracy of the feedback because there is no need of channel information for the space-time encoder, which makes open loop STBC adapted for fast fading channel. Furthermore, SM mode has low throughput compared to SVD mode due to the significant MSI of SM mode. Finally, we find that the UCD mode outperforms the other MIMO modes at high SNR. Hence, UCD mode is the first choice at high SNR for the reason of both high performance and low modulation and coding complexity.

VI. CONCLUSION

In this paper we describe a link level performance analysis of MIMO transmission modes with link adaptation. We consider eight different MCS for link adaptation and find great throughput improvement after combining MIMO technique with link adaptation. Under the assumption of one frame of channel state information feedback delay at the transmitter, we find that the closed loop STBC outperforms other MIMO modes at low SNR and UCD can achieve the highest throughput at high SNR with only one set of encoder/decoder and modulator/demodulator.

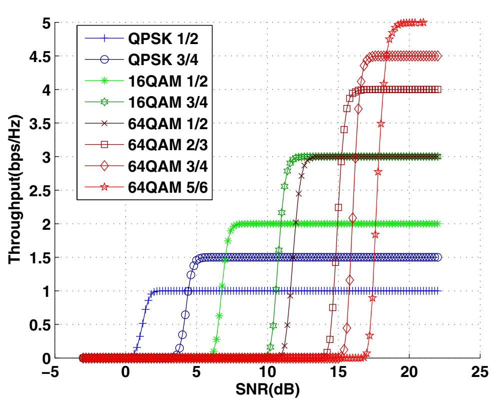

Fig. 2. Throughtput vs. SNR for different MCS.

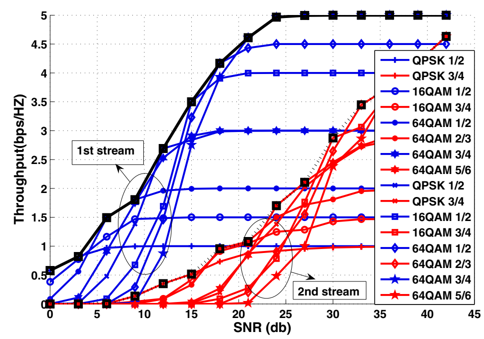

Fig. 3. Throughtput vs. SNR for different MCS in SVD with 2×2 antennas.

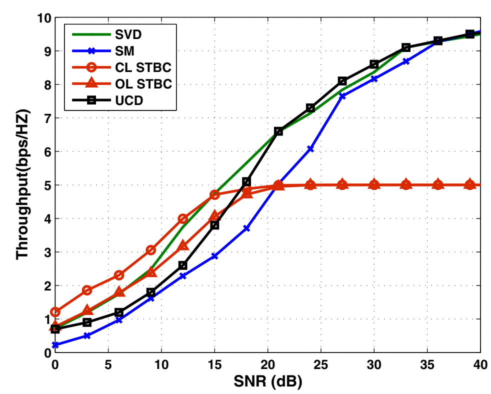

Fig. 4. Throughtput vs. SNR in different MIMO transmission modes with link adaptation ($M_T = 2$, $M_R = 2$).

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