Analysis of Conditional Error Rate and Combining Schemes in HARQ

Hang Long, Wei Xiang, Shanshan Shen, Yueying Zhang, Kan Zheng, and Wenbo Wang

***Abstract*—In hybrid automatic repeat request (HARQ), maximum ratio combining (MRC) is usually employed at the receiver if the retransmission signal is the same as that in the first transmission. However, conventional performance analysis of HARQ and MRC does not consider that all previous transmissions fail. In this paper, we analyze the conditional error rate of HARQ with arbitrary combining schemes. Both the theoretical and approximate expressions of the conditional error rate are derived. Our analytical and simulation results reveal that conventional MRC is not optimum for HARQ, especially when the packet is short or the channel gain of the first transmission is much better than that of retransmission.**

***Index Terms*—Conditional error rate, hybrid automatic repeat request (HARQ), maximum ratio combining (MRC).**

I. Introduction

As a combination of both forward error correction and automatic repeat request techniques, the hybrid automatic repeat request (HARQ) technique is often employed in wireless communication systems to enable high-speed data services [1]. In the past decade, HARQ has received considerable research attention [2]–[8]. In general, conventional studies on HARQ can be divided into two broad categories, i.e., 1) what/how/when/who to retransmit [2], [3] and 2) the receiver design for retransmission including the buffer operation [4], [7], [8].

Maximum ratio combining (MRC) has been proven to be the optimum combining scheme in the single-input and multiple-output (SIMO) system [9]. The signal-to-noise ratio (SNR) after combining is the sum of the SNRs of all receive antennas. Naturally, in HARQ, if the retransmission packet is the same as the original one, MRC is usually employed by the receiver to combine multiple received packets [7], and the analysis based on the sum of the SNRs has been extended to HARQ in [5] and [6]. In [8], MRC in HARQ is extended to multiple-input and multiple-output systems, and proven to be equivalent to the optimum maximum-likelihood receiver in decoding performance.

However, there is a fundamental difference between the SIMO system and the multiple transmissions in HARQ. That is, the failure of a preceding transmission is the condition of the subsequent retransmission. This condition implies that multiple transmissions in HARQ are not independent, and thus the equivalent noise after combining is not additive white Gaussian noise. Consequently, conventional analysis of HARQ based on the independence among multiple transmissions [5]–[8] is not accurate enough. For example, in [8], the likelihood is calculated as the product of the probabilities related to multiple transmissions, and the combined noise is regarded as Gaussian distributed and is independent of the transmitted signal. This is based on the same assumptions in single-input and single-output systems [5]–[7] and thus not accurate enough. Similar inaccurate assumptions are also made in the link-to-system interface technique [10] and incremental relaying protocols [11], [12] where the failure of direct transmission is the condition of relay transmission.

The aforementioned inaccuracy in the analysis of HARQ is first pointed out in [13], where it is shown that neglecting previous transmission failure results in overoptimistic performance prediction. A simple method is proposed to predict the conditional error rate in [13]. The accurate conditional error performance of HARQ with MRC employed at the receiver is first given in [14]. However, the analytical prediction method in [13] is not accurate enough, whereas the analysis in [14] only considers MRC at the receiver. Accurate analysis of the conditional error rate of HARQ with other combining schemes at the receiver still remains an open problem.

In this paper, with arbitrary combining schemes at the receiver, the conditional error rate after combining in HARQ is analyzed. Both accurate and approximate closed-form expressions are derived. Simulation results are presented to validate the approximate expression. More importantly, we show through both analytical and simulation results that MRC is not the optimum combining scheme at the receiver for HARQ in the sense of the conditional error rate.

The remainder of this paper is organized as follows. The system model is described in Section II. The conditional error rate with arbitrary combining schemes is analyzed in Section III, where both the accurate and approximate expressions are derived. Numerical and simulation results are given in Section IV. Finally, Section V concludes this paper.

II. System Model

In the HARQ process, the received signal for the ith transmission is written as

$$y_i = h_i x + n_i \tag{1}$$

where

- x denotes the transmission signal which is assumed to be unchanged in each retransmission;
- h_i denotes the channel gain for the ith transmission;
- n_i denotes noise at the receiver.

Real signals are assumed in this study for the simplicity of expression, i.e., pulse amplitude modulation (PAM) for x and $n_i \sim \mathcal{N}(0, \sigma^2)$. We also assume that $h_i > 0$ because the error performance is not affected by the channel phase. That is, the error performance with the channel coefficient $|h_i|$ is the same as that with h_i. The analysis in this study can be easily extended to the system with complex signals since a symbol with quadrature amplitude modulation can be treated as two orthogonal symbols with PAM. For example, the error performance of a packet with N binary phase shift keying (BPSK) symbols is the same as that of $N/2$ quadrature phase shift keying (QPSK) symbols.

The SNR of the ith transmission is defined as $\gamma_i = h_i^2/\sigma^2$. For the ease of exposition, only two transmissions are considered in this study. The proposed analytical method in this paper can be extended to more generalized multiple transmissions cases at the expense of computational complexity. The combining coefficient at the receiver is defined

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as k, denoting the combining ratio between the received signals of the first and second transmissions. As a result, the combined signal at the receiver is written as

$$y' = \frac{ky_1 + y_2}{kh_1 + h_2} = x + n' \tag{2}$$

where

$$n' = \frac{kn_1 + n_2}{kh_1 + h_2}. \tag{3}$$

In the conventional studies of HARQ [5], [6], [8], it is assumed that n' is independent of x and still follows a Gaussian distribution:

$$E(|n'|^2) = \frac{(k^2+1)\sigma^2}{(kh_1 + h_2)^2} \tag{4}$$

where $E(\cdot)$ represents mathematical expectation. Based on the conventional assumption of independent retransmission in HARQ [5]–[8] and other similar techniques [10]–[12], the optimum combining coefficient can be achieved with the derivative of the equivalent noise power in (4) as follows:

$$\frac{\partial}{\partial k}\left[\frac{k^2+1}{(kh_1+h_2)^2}\right] = \frac{2k(kh_1+h_2)^2 - 2h_1(k^2+1)(kh_1+h_2)}{(kh_1+h_2)^4} = 0 \tag{5}$$

$$k_0 = \frac{h_1}{h_2} = \sqrt{\frac{\gamma_1}{\gamma_2}}. \tag{6}$$

Conventional MRC with the coefficient in (6) uses different weights for the signals of the two transmissions according to the corresponding channel gains, which is the same as that in the SIMO system. However, the failure of the first transmission is not considered in combining. As mentioned before, the assumption, on which the conventional HARQ analysis is based, that n' is independent of x (i.e., the failure of the second transmission is independent of the first transmission), is not accurate enough. Actually, n' is related to x and thus does not follow a Gaussian distribution. Additionally, MRC is not the optimum combining scheme at the receiver for HARQ. Intuitively, the failure of the first transmission implies that the received signal of the first transmission y_1 should be combined with a smaller weight than that in the SIMO system. Both the accurate analysis of the conditional error rate in Section III and the simulation results in Section IV will validate this intuition.

III. Conditional Error Rate Analysis and Approximation Expression

Before the analysis, some definitions are given. f_1 defines the event that the first transmission fails, whereas f_2 represents the event that the demodulation after combining fails, without considering the failure of the first transmission. The probabilities of f_1 and f_2 can be easily computed with the received SNRs. The length of the data packet is defined as N symbols, and $\Pr(x)_N$ denotes the probability of x with a packet length of N symbols. BPSK modulation is assumed in this study for ease of exposition. However, the analytical results derived in this study can be easily extended to the cases with higher order modulation, e.g., *M*-ary PAM.

A. Single Symbol Case

In this part, it is assumed that $N = 1$. If the first transmission fails, i.e., the only symbol is corrupted by noise ($n_1 < -h_1 \,|\, x = 1$ or $n_1 > h_1 \,|\, x = -1$), the received signal of the first transmission (y_1) is useless for combining after the second transmission. It is intuitive that the optimum combining coefficient should be $k_0 = 0$, i.e., only the signal of the second transmission is used for demodulation. The accurate conditional bit error rate (BER) analysis in the sequel will verify this conclusion.

The error probability of the first transmission is readily computed as

$$\Pr(f_1)_1 = \mathrm{Q}(h_1/\sigma) \tag{7}$$

where $\mathrm{Q}(\cdot)$ is the Q-function (eq. (2.53) in [9]). Similar to the derivation in [14], the probability that both the first and second transmissions fail can be shown as

$$\begin{aligned}\Pr(f_1, f_2)_1 &= \int\limits_{s>h_1,}\ \int\limits_{\frac{ks+t}{kh_1+h_2}>1} \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{s^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{t^2}{2\sigma^2}}\mathrm{d}t\mathrm{d}s \\ &= \mathrm{Q}(h_1/\sigma)\mathrm{Q}(h_2/\sigma) \\ &\quad + \int_{-\infty}^{h_2} \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{t^2}{2\sigma^2}}\mathrm{Q}\left(\frac{kh_1+h_2-t}{k\sigma}\right)\mathrm{d}t. \end{aligned} \tag{8}$$

The second term in (8) is defined as

$$I_Q(h_1, h_2, \sigma^2, k) = \int_{-\infty}^{h_2} \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{t^2}{2\sigma^2}}\mathrm{Q}\left(\frac{kh_1+h_2-t}{k\sigma}\right)\mathrm{d}t. \tag{9}$$

Hence, the conditional BER can be written as

$$\Pr(f_2|f_1)_1 = \frac{\Pr(f_1,f_2)_1}{\Pr(f_1)_1} = \mathrm{Q}(h_2/\sigma) + \frac{I_Q\left(h_1,h_2,\sigma^2,k\right)}{\mathrm{Q}(h_1/\sigma)}. \tag{10}$$

It is noted that $\mathrm{Q}[(kh_1+h_2-t)/(k\sigma)]$ in (9) is a monotonic increasing function of k, so $I_Q(h_1,h_2,\sigma^2,k)$ as well as $\Pr(f_2\,|\,f_1)_1$ is also a monotonic increasing function of k. Therefore, with any assumption on the SNRs of the two transmissions (γ_1 and γ_2, or equivalently h_1 and h_2), the optimum combining coefficient at the receiver can be written as

$$k_0 = \arg\min \Pr(f_2\,|\,f_1)_1 = 0. \tag{11}$$

There is no closed-form solution for $I_Q(h_1,h_2,\sigma^2,k)$, as well as for $\Pr(f_2\,|\,f_1)_1$. Therefore, an approximate expression for the conditional BER in (10) can be derived below, where the Q-function in (9) can be approximately replaced with a closed-form exponential function as follows [14]:

$$\mathrm{Q}(\sqrt{\gamma}) \le \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\gamma}{2}}. \tag{12}$$

Integrating (12) into (9) gives rise to

$$\begin{aligned} I_Q\left(h_1,h_2,\sigma^2,k\right) &\approx \int_{-\infty}^{h_2} \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{t^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{(kh_1+h_2-t)^2}{2k^2\sigma^2}}\mathrm{d}t \\ &= \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{(kh_1+h_2)^2}{2(k^2+1)\sigma^2}}\int_{-\infty}^{h_2}\frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{(k^2+1)}{2k^2\sigma^2}\left(t-\frac{kh_1+h_2}{k^2+1}\right)^2}\mathrm{d}t. \end{aligned} \tag{13}$$

It is defined that

$$u = \sqrt{\frac{k^2+1}{k^2}}\left(t - \frac{kh_1+h_2}{k^2+1}\right). \tag{14}$$

Replacing t with u, the integral in (13) can be defined as

$$c = \int_{-\infty}^{\frac{kh_2-h_1}{\sqrt{k^2+1}}} \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{u^2}{2\sigma^2}} \mathrm{d}u = \begin{cases} 1 - \mathrm{Q}\left(\frac{kh_2-h_1}{\sigma\sqrt{k^2+1}}\right), & k > h_1/h_2 \\ \mathrm{Q}\left(\frac{h_1-kh_2}{\sigma\sqrt{k^2+1}}\right), & k \le h_1/h_2. \end{cases} \tag{15}$$

Therefore,

$$I_Q\left(h_1, h_2, \sigma^2, k\right) \approx \frac{1}{\sqrt{2\pi}} \sqrt{\frac{k^2}{k^2+1}}\, c\, \mathrm{e}^{-\frac{(kh_1+h_2)^2}{2(k^2+1)\sigma^2}}. \tag{16}$$

Then, we use $\mathrm{Q}(h_1/\sigma)$ to replace the exponential function $\mathrm{e}^{-h_1^2/2\sigma^2}/\sqrt{2\pi}$ in (16), and $I_Q(\cdot)$ can be further approximated as

$$I_Q\left(h_1, h_2, \sigma^2, k\right) \approx \mathrm{Q}(h_1/\sigma) \sqrt{\frac{k^2}{k^2+1}}\, c\, \mathrm{e}^{-\frac{h_2^2+2kh_1h_2-h_1^2}{2(k^2+1)\sigma^2}}. \tag{17}$$

In the above approximation, $I_Q(\cdot)$ is first enlarged and then reduced. At last, the approximate expression in (17) gives an upper bound of the accurate conditional error rate. The approximate expression suggests that the conditional BER is a monotonic increasing function of k, indicating that the optimum combining coefficient is $k_0 = 0$. This also demonstrates that conventional MRC is not the optimum combining scheme in terms of the conditional error rate.

B. Multiple Symbols Case

When $N > 1$, a similar method can be proposed to derive the conditional block error rate (BLER) with an arbitrary combining coefficient. Similar to (7), the error probability of the first transmission is

$$\Pr(f_1)_N = 1 - [1 - \mathrm{Q}(h_1/\sigma)]^N. \tag{18}$$

Without considering the failure of the first transmission, the probability of correct demodulation after combining is

$$\Pr(\overline{f_2})_N = \left[1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)\right]^N. \tag{19}$$

Therefore, the probability that both transmissions fail is

$$\begin{aligned} \Pr(f_1, f_2)_N &= \Pr(\overline{f_1}, \overline{f_2})_N - 1 + \Pr(f_2)_N + \Pr(f_1)_N \\ &= \Pr(\overline{f_1}, \overline{f_2})_1^N - \left[1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)\right]^N \\ &\quad + 1 - [1 - \mathrm{Q}(h_1/\sigma)]^N \end{aligned} \tag{20}$$

where

$$\begin{aligned} \Pr(\overline{f_1}, \overline{f_2})_1 &= \Pr(f_1, f_2)_1 + 1 - \Pr(f_2)_1 - \Pr(f_1)_1 \\ &= \mathrm{Q}(h_1/\sigma)\mathrm{Q}(h_2/\sigma) + I_Q\left(h_1, h_2, \sigma^2, k\right) \\ &\quad + 1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right) - \mathrm{Q}(h_1/\sigma). \end{aligned} \tag{21}$$

The conditional BLER can be computed as

$$\begin{aligned} \Pr(f_2 \mid f_1)_N = \frac{\Pr(f_1, f_2)_N}{\Pr(f_1)_N} &= 1 - \frac{\left[1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)\right]^N}{1 - [1 - \mathrm{Q}(h_1/\sigma)]^N} \\ &\quad + \frac{\Pr(\overline{f_1}, \overline{f_2})_1^N}{1 - [1 - \mathrm{Q}(h_1/\sigma)]^N}. \end{aligned} \tag{22}$$

Integrating the approximate expression of $I_Q(h_1, h_2, \sigma^2, k)$ in (17) into (22) gives rise to the approximate expression of the conditional BLER.

Note that in the high SNR region, the denominators in (22) are close to 0. The numerical software program used in our simulation (Matlab 6.5) is not accurate enough to represent very small numbers due to the limitation in the representation of floating-point numbers, which might result in unpredictable computation errors. A simplified expression is thus derived for high SNRs in the sequel.

The following polynomial decomposition can be utilized

$$x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \cdots + xy^{n-2} + y^{n-1}). \tag{23}$$

A special function is defined as

$$g_N(x) = \sum_{i=0}^{N-1} x^i. \tag{24}$$

Therefore,

$$x^N = 1 - (1 - x)g_N(x). \tag{25}$$

According to (25), the first numerator in (22) can be written as

$$\begin{aligned} &\left[1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)\right]^N \\ &\quad = 1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right) g_N\left[1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)\right]. \end{aligned} \tag{26}$$

Similarly, the second numerator in (22) is written as

$$\Pr(\overline{f_1}, \overline{f_2})_1^N = 1 - [1 - \Pr(\overline{f_1}, \overline{f_2})_1] g_N[\Pr(\overline{f_1}, \overline{f_2})_1]. \tag{27}$$

The denominator can be written as

$$1 - [1 - \mathrm{Q}(h_1/\sigma)]^N = \mathrm{Q}(h_1/\sigma) g_N[1 - \mathrm{Q}(h_1/\sigma)]. \tag{28}$$

Substituting (26)–(28) into (22), the conditional BLER can be simplified as follows:

$$\begin{aligned} \Pr(f_2 \mid f_1)_N = 1 + a \frac{g_N\left[1 - \mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)\right]}{g_N[1 - \mathrm{Q}(h_1/\sigma)]} \\ - b \frac{g_N[\Pr(\overline{f_1}, \overline{f_2})_1]}{g_N[1 - \mathrm{Q}(h_1/\sigma)]} \end{aligned} \tag{29}$$

where

$$a = \frac{\mathrm{Q}\left(\frac{kh_1+h_2}{\sigma\sqrt{k^2+1}}\right)}{\mathrm{Q}(h_1/\sigma)} \tag{30}$$

$$\begin{aligned} b &= \frac{1 - \Pr(\overline{f_1}, \overline{f_2})_1}{\mathrm{Q}(h_1/\sigma)} \\ &= 1 - \mathrm{Q}(h_2/\sigma) + a - \frac{I_Q\left(h_1, h_2, \sigma^2, k\right)}{\mathrm{Q}(h_1/\sigma)}. \end{aligned} \tag{31}$$

If $\gamma_1 \gg 1$ ($h_1 \gg 1$), $\mathrm{Q}(h_1/\sigma) \to 0$ and $g_N[1 - \mathrm{Q}(h_1/\sigma)] \to N$. The simplified expression in (29) alleviates the problem of inaccurate

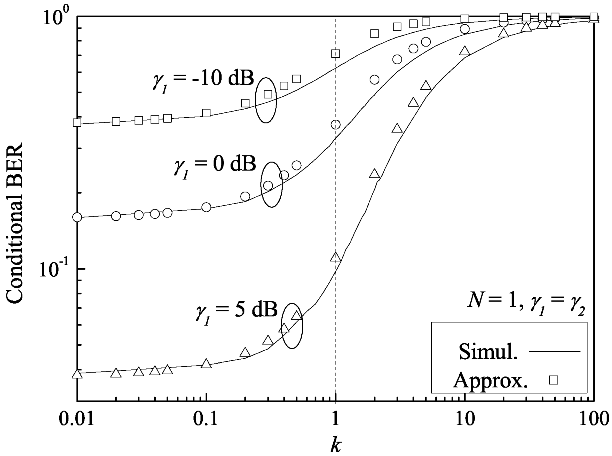

Fig. 1. Conditional BER when $N = 1$ and $\gamma_1 = \gamma_2$.

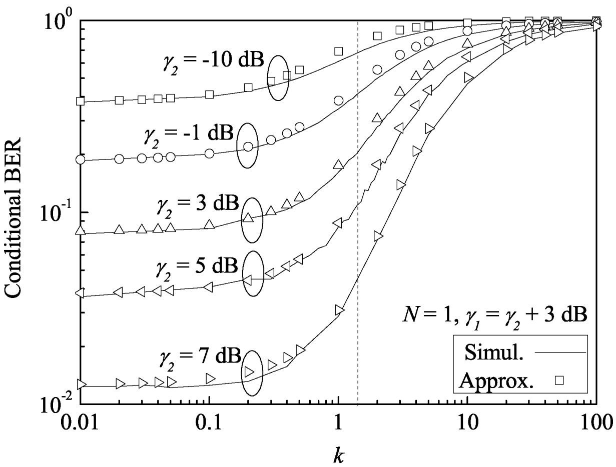

Fig. 2. Conditional BER when $N = 1$ and $\gamma_1 = \gamma_2 +$ 3 dB.

floating-point number representation in the numerical software program we used. Integrating (17) into (31), the approximate expression can also have a simplified form.

IV. Simulation Results and Validation of Approximation

Both Monte Carlo simulation results of the conditional error rate and the numerical computation results based on the derived expressions are given in this section. The accurate values of $I_Q(h_1, h_2, \sigma^2, k)$ can be computed with numerical computation. It is noted that Monte Carlo simulation results are nearly identical to the numerical results. Therefore, only Monte Carlo simulation results (abbreviated as "Simul.") are plotted in the following figures as denoted by the real lines, where the results of approximate expressions are denoted by the hollow symbols (abbreviated as "Approx.").

A. Single Symbol Case

The conditional BER results when $N = 1$ are plotted in Figs. 1–3. The conventional "optimum" MRC coefficient ($k = h_1/h_2 = \sqrt{\gamma_1/\gamma_2}$) is also plotted in the figures with the vertical dash lines as a comparison baseline. As can be observed from Fig. 1, the MRC coefficient of $k = 1$ is obviously not a good choice for the combination at the receiver. The optimum combining coefficient should be $k_0 = 0$ as proven in Section III-A, which is also demonstrated by the figures. Besides, the results with the proposed approximate expression are very close to the accurate values of the conditional BER, especially when k is close to 0.

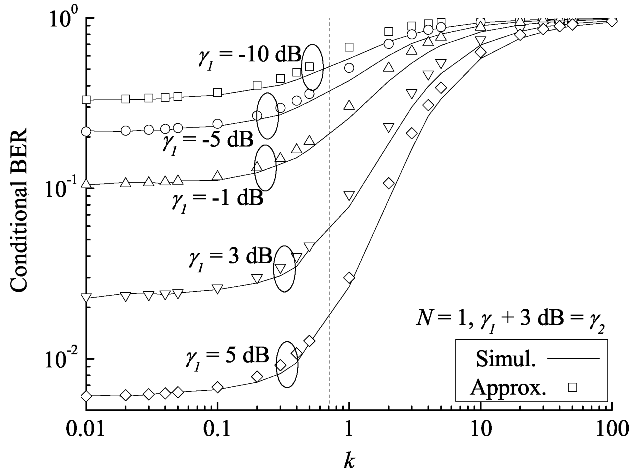

Fig. 3. Conditional BER when $N = 1$ and $\gamma_1 + 3$ dB $= \gamma_2$.

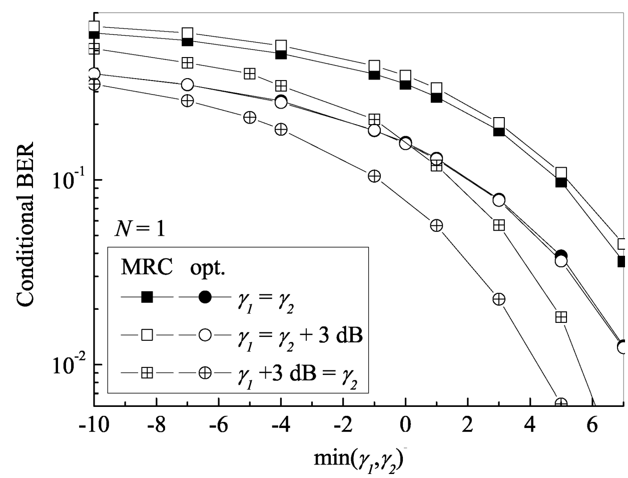

Fig. 4. Conditional BER comparison between conventional MRC and the accurate optimum combining scheme when $N = 1$.

Similar observations can be seen from Figs. 2 and 3. The prediction error of the approximate expression is almost negligible when $k < 1$. The conditional BER $\Pr(f_2 \,|\, f_1)_1$ is close to 1 when $k \gg 1$, so the prediction error is not obvious either. The approximation method proposed in Section III-A is thus validated.

Based on the analysis in Section III-A and the results in Figs. 1–3, the optimum combining scheme after retransmission should adopt $k_0 = 0$. The conditional BER comparison between conventional MRC and the optimum combining scheme is illustrated in Fig. 4, where the results of the optimum combining scheme are denoted by "opt.". With $k_0 = 0$, the conditional BER is only related to the second transmission, so that the curves of "opt." for the cases of $\gamma_1 = \gamma_2$ and $\gamma_1 = \gamma_2 +$ 3 dB are the same. The SNR gap between MRC and the optimum combining scheme is over 2.3 dB for all the cases considered in Fig. 4.

B. Multiple Symbols Case

When $N > 1$, the conditional BLER curves with Monte Carlo simulation and the proposed approximate expression are given in Figs. 5–7. Different from the observation in Figs. 1–3, the conditional BLER is not a monotonic increasing function of k. Apparently, each conditional BLER curve has a minimum that is smaller than the conventional MRC coefficient denoted by the vertical dash line. This observation clearly demonstrates that conventional MRC is not optimum for HARQ.

As shown in Fig. 6, when $\gamma_1 \ll \gamma_2$, conventional MRC performs closely to the scheme with the optimum combining coefficient. Un-

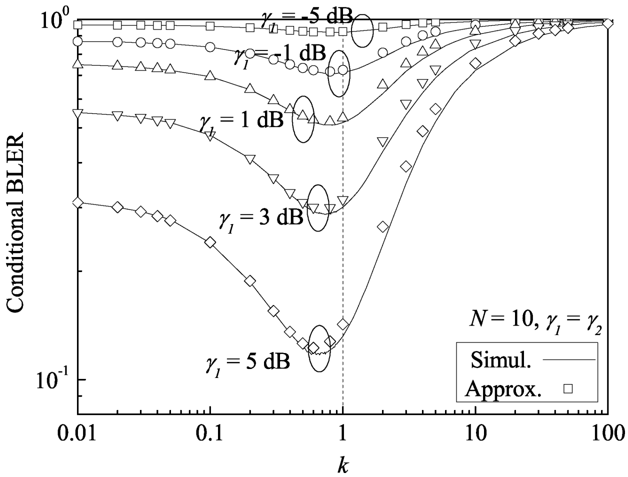

Fig. 5. Conditional BLER when $N = 10$ and $\gamma_1 = \gamma_2$.

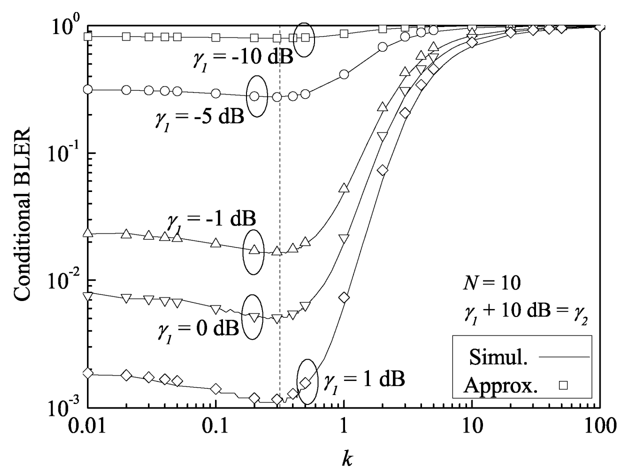

Fig. 6. Conditional BLER when $N = 10$ and $\gamma_1 + 10$ dB $= \gamma_2$.

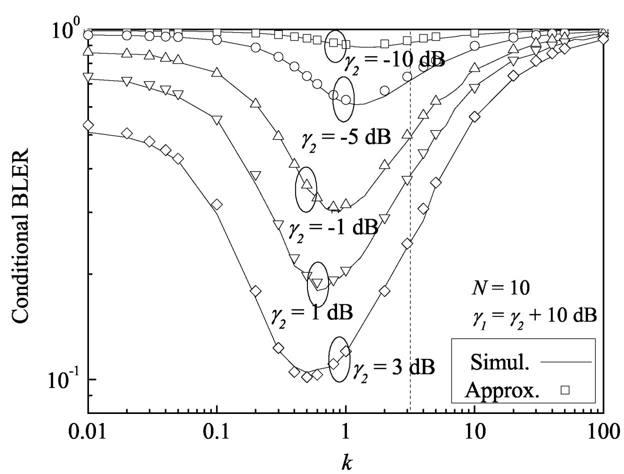

Fig. 7. Conditional BLER when $N = 10$ and $\gamma_1 = \gamma_2 +$ 10 dB.

fortunately, for other scenarios, e.g., $N = 1$, or $\gamma_1 \geq \gamma_2$, the conditional BLER with conventional MRC is far from minimum as shown in Figs. 5 and 7. Additionally, the proposed approximate expression gives almost identical results as given by Monte Carlo simulation and numerical computation. Based on the accurate and approximate results of the conditional BLER after combining, the optimum combining coefficient can be obtained with numerical computation.

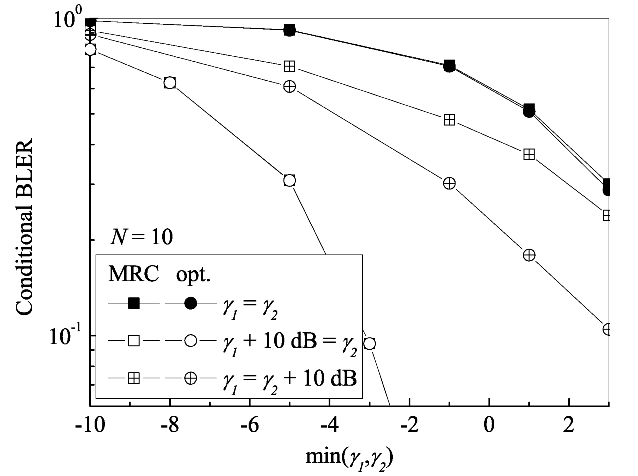

Fig. 8. Conditional BER comparison between conventional MRC and the accurate optimum combining scheme when $N = 10$.

Similarly, the results of the optimum combining scheme for $N = 10$ can also be obtained according to Figs. 5–7. The comparison between MRC and the optimum combining performance can be seen from Fig. 8. For $\gamma_1 \leq \gamma_2$, the conditional BLERs of MRC and the optimum combining scheme are similar, which can also be observed in Figs. 5–6. If the channel of the second transmission is weak, the SNR gap between MRC and the optimum combining scheme is larger than 3 dB when $\gamma_2 \leq 3$ dB as shown in Fig. 8.

As mentioned previously, the analysis in this study can be easily extended to the case of complex signal model. Results in this part is equivalent to the case of $N = 5$ and QPSK modulation.

V. Conclusion

The conditional error rate performance of HARQ with arbitrary combining coefficient at the receiver is analyzed in this paper. The accurate and approximate closed-form expressions are both presented. It is demonstrated through our analytical as well as simulation results that conventional MRC is not the optimum combining scheme for HARQ. When $N = 1$, the optimum combining coefficient is proven to be 0 with BPSK modulation. When $N > 1$, the optimum combining coefficient can be found according to the derived expressions which is smaller than the conventional MRC coefficient. Especially if $\gamma_1 \geq \gamma_2$ $(h_1 \geq h_2)$, the optimum combining coefficient is considerably smaller than the one used by conventional MRC.

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Cooperative Multiband Joint Detection With Correlated Spectral Occupancy in Cognitive Radio Networks

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***Abstract*—In this paper, a frequency-coupled optimum linear energy combiner (OLEC) structure, recently proposed for single user scenarios, is generalized to multiple users and integrated into the spatial-spectral joint detection framework introduced by Quan *et al.*, to take further advantage of subband occupancy correlation in cooperative wideband spectrum sensing. In particular, the design of the detection thresholds and fusion weights used by a bank of subband multiuser OLECs is formulated as a joint optimization problem, i.e. maximization of aggregate opportunistic throughput under interference constraints (or vice versa). Through numerical experiments with a Markov model of subband occupancy, the proposed scheme is shown to significantly enhance CR network performance in terms of these global metrics.**

***Index Terms*—Cognitive radio, cooperative processing, multiband joint detection, spectrum sensing.**

I. Introduction

Cognitive radios (CRs), which maintain awareness of their environment through spectrum sensing, have emerged as a key enabling technology for dynamic spectrum access [1], [2]. Specifically, CRs must have the capability to detect and opportunistically use momentarily silent portions of the licensed frequency spectrum, called *spectrum holes* [3]. In this scenario, one refers to a user of the wireless system to which the frequency band has been licensed (e.g., WLAN or broadcast television) as a primary user (PU), and to the CR of interest as a secondary user (SU). Spectrum sensing has gained further importance as CR is an integral component of the IEEE 802.22 standard and the focus of other emerging applications [4]. In practice, spectrum sensing must be fast and accurate in order to maximize the opportunistic throughput without adding unacceptable level of interference to the PUs. Until now, several approaches have been considered for this application, including matched filtering, cyclo-stationary feature extraction, and energy detection [2]. The extension of spectrum sensing techniques to multiple antenna CR is considered in [5], [6].

Many studies, such as [7] and [8], advocate the use of energy detection for spectrum sensing since it can meet the basic requirements of CR systems while offering flexibility and robustness in implementation. In this approach, the energy of the received signal is computed and used in a binary hypothesis test to decide the occupancy state of the frequency band under probing [9]. Energy detection can be applied in both narrowband and wideband settings, where in the latter case, it is usually performed by dividing the wideband spectrum into smaller component subbands and carrying out narrowband detection in each subband independently [10]. Recently, *joint* multiband energy detection, where the thresholds used in individual subbands are determined from wideband considerations, has shown great promises for spectrum sensing in CR networks [11]. In this scheme, the cost of interfering with the PUs and the Shannon theoretic capacity for each subband are applied to define global measures of *aggregate interference* and *aggregate opportunistic throughput*, respectively. Subsequently, an optimum set of detection thresholds is designed that maximizes the opportunistic throughput aggregated over all the subbands while keeping the aggregate interference under a critical value.

Alongside these developments, compromised detection due to multipath fading and shadowing has oriented research efforts towards *cooperative* spectrum sensing. In this formulation, sensing information from multiple CRs experiencing different channel conditions is combined at a fusion center to provide more reliable detection of the PU signals by exploiting the built-in spatial diversity of the CR network [12], [13]. The extension of joint multiband energy detection to cooperative spectrum sensing is considered in [11], where further performance enhancements are demonstrated. In [14], cooperative spectrum sensing is performed using channel gain (CG) maps, which enable tracking the location and transmit powers of multiple PUs. In [15], the effect of time dispersive reporting channels on cooperative spectrum sensing is investigated, where the authors propose a widely linear fusion scheme to exploit the multipath nature of the reporting channels.

Existing approaches for wideband spectrum sensing employ a decoupled processing structure in which hypothesis-testing in any given subband is carried out on the energy computed from the observed data in that particular subband only, i.e., independently of other subbands' data. This is so even for the joint multiband energy detection schemes in [11], where only the detection thresholds used in individual subbands are optimized from a wideband perspective. While the frequency-decoupled structure is optimal when the occupancies of the frequency subbands are independent of each other, this condition is generally not satisfied, especially in the presence of wideband PU signals, such as broadcast television or WLAN systems [16]. As a result, more recently, the topic of spectrum sensing in the presence of correlation between the frequency subband occupancies has been gaining notable attention [17], [18]. To exploit such correlation and thus improve sensing performance, [19] proposes a novel (noncooperative) detector structure

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