Permutation Invariance

December 30, 2021

Capturing invariance of $R \times P \curvearrowright S^1$

Given $x, y \in S^1$, take $||x-y||_2$ and (2) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \cdot x$, these capture the invariance under the action of the group.

Proof

All other options of rotation imply coordinates either change sign or value thus the invariance of (2) doen't hold.

Clearly distance between the two points is invariant to rotations.(As well as length along arc)