→ ① O/P DC Current (or) Average Current :

①

$$I_{dc} = \int_0^{\pi} \frac{I_m \sin\omega t \, d(\omega t)}{2\pi} \quad \text{——— ①}$$

$$I_m = \frac{V_m}{(R_f + R_L)}$$

$$I_{dc} = \int_0^{\pi} \frac{I_m \sin\omega t}{2\pi} d(\omega t)$$

$$I_{dc} = \frac{I_m}{2\pi} \left[-\cos\omega t \right]_0^{\pi}$$

$$I_{dc} = \frac{I_m}{2\pi} \left[-(\cos\pi - \cos 0) \right] = \frac{I_m[-(-1-1)]}{2\pi}$$

$$I_{dc} = \frac{\cancel{2} I_m}{\cancel{2}\pi}$$

$$\boxed{I_{dc} = \frac{I_m}{\pi} = \frac{V_m}{\pi(R_f + R_L)}} \quad \text{——— ②}$$

similarly

$$\boxed{V_{dc} = \frac{V_m}{\pi} = \frac{I_m(R_f + R_L)}{\pi}} \quad \text{——— ③}$$

② Voltage Regulation:

→ The variation of dc o/p voltage as a function of d.c. load current is called Regulation.

$$\% \text{ Regulation} = \frac{V_{no\,load} - V_{full\,load}}{V_{full\,load}} \times 100\%$$

$$\% \text{ Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% \quad \text{——} ④$$

$$V_{NL} = \frac{V_m}{\pi} \quad \text{——} ⑤$$

$$V_{FL} = \frac{V_m}{\pi} - I_{dc} R_f \quad \text{——} ⑥$$

$$V_{FL} = \frac{V_m}{\pi} - \frac{V_m}{\pi} \cdot \frac{R_f}{R_f + R_L}$$

$$V_{FL} = \frac{V_m}{\pi}\left[1 - \frac{R_f}{R_f + R_L}\right] = \frac{V_m}{\pi}\left[\frac{\cancel{R_f} + R_L - \cancel{R_f}}{R_f + R_L}\right]$$

$$V_{FL} = \frac{V_m}{\pi}\left[\frac{R_L}{R_f + R_L}\right] \quad \text{——} ⑦$$

Substitute eq ⑦ & eq ⑤ in eq ④.

$$\% \text{ Regulation} = \frac{\frac{V_m}{\pi} - \frac{V_m}{\pi}\left[\frac{R_L}{R_f + R_L}\right]}{\frac{V_m}{\pi}\left[\frac{R_L}{R_f + R_L}\right]}$$

$$\% \text{ Regulation} = \frac{\cancel{\frac{V_m}{\pi}}\left[1 - \frac{R_L}{R_f + R_L}\right]}{\cancel{\frac{V_m}{\pi}}\left[\frac{R_L}{R_f + R_L}\right]}$$

$$= \frac{\frac{R_f + \cancel{R_L} - \cancel{R_L}}{\cancel{R_f + R_L}}}{\frac{R_L}{\cancel{R_f + R_L}}}$$

$$\boxed{\% \text{ Regulation} = \frac{R_f}{R_L} \times 100}$$

③ Root mean square current (I_{rms}):

$$I_{rms}^2 = \frac{1}{2\pi}\int_0^{\pi} I^2 \, d\theta$$

$I = I_m \sin\theta$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta$$

$\boxed{\because \sin^2\theta = \frac{1 - \cos 2\theta}{2}}$

$$= \frac{I_m^2}{4\pi}\left[(\theta)_0^{\pi} - \left(\frac{\sin 2\theta}{2}\right)_0^{\pi}\right]$$

$$= \frac{I_m^2}{4\pi}\left[(\pi - 0) - (0 - 0)\right] = \frac{I_m^2}{4\cancel{\pi}} \times \cancel{\pi}$$

$$I_{rms}^2 = \frac{I_m^2}{4} \qquad \boxed{I_{rms} = \frac{I_m}{2}} \quad \text{— ⑧}$$

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4. Ripple factor :

$$r = \frac{\text{rms value of a.c. Components of o/p wave}}{\text{dc value of the o/p wave}}$$

$$r = \frac{I_{ac}}{I_{dc}} \quad \text{—— (9)}$$

The effective value of the pulsating o/p current

$$I_{rms}^2 = I_{ac}^2 + I_{dc}^2$$

$$\sqrt{I_{rms}^2 - I_{dc}^2} = I_{ac}$$

$$r = \sqrt{\frac{I_{rms}^2 - I_{dc}^2}{I_{dc}^2}}$$

$$r = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - \frac{\cancel{I_{dc}^2}}{\cancel{I_{dc}^2}}}$$

$$r = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} \quad \text{—— (10)}$$

eq (2) & eq (8) in eq (10)

$$r = \sqrt{\left(I_m/2\right)^2 / \left(I_m/\pi\right)^2 - 1}$$

$$r = \sqrt{\frac{\cancel{I_m^2}}{4} \times \frac{\pi^2}{\cancel{I_m^2}} - 1} = r = \sqrt{\frac{\pi^2}{4} - 1}$$

$$r = 1.21$$

5. Efficiency of Half wave rectifier :

$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\text{o/p DC power delivered to the Load}}{\text{i/p AC from the secondary winding}} \quad \text{— (11)}$$

$$P_{dc} = I_{dc}^2 R_L = \left(\frac{I_m}{\pi}\right)^2 R_L$$

$$P_{dc} = \frac{I_m^2}{\pi^2} R_L \quad \text{— (12)}$$

$$P_{ac} = I_{rms}^2 (R_f + R_L)$$

$$= \frac{I_m^2}{4} (R_f + R_L) \quad \text{— (13)}$$

eq (12) & eq (13) in eq (11)

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{I_m^2/\pi^2 \, R_L}{\frac{I_m^2}{4}(R_f + R_L)}$$

$$= \frac{\cancel{I_m^2}}{\pi^2} R_L \times \frac{4}{\cancel{I_m^2}(R_f + R_L)}$$

$$= \frac{4}{\pi^2} \frac{R_L}{R_f + R_L}$$

$$= \frac{4}{\pi^2} \frac{1}{\left(\frac{R_f}{R_L} + \frac{\cancel{R_L}}{\cancel{R_L}}\right)} = \frac{4}{\pi^2} \frac{1}{\left(1 + \frac{R_f}{R_L}\right)}$$

If $R_L \gg R_f$

$$\eta = \frac{4}{\pi^2} = 0.406$$

$$\% \eta = 40.6\%$$ → of the a.c. power is converted into d.c.

(6) Transformer utilization factor (TUF):

$$TUF = \frac{\text{d.c. power delivered to the Load}}{\text{a.c. power rating of the transformer secondary}} \quad \text{--- (14)}$$

$$TUF = \frac{P_{dc}}{P_{ac\,(rated)}} \quad \text{--- (15)}$$

$$P_{ac\,(rated)} = V_{ac\,(rms)} \times I_{rms} \quad \text{--- (16)}$$

$$V_{ac\,(rms)} = \frac{V_m}{\sqrt{2}} \quad \text{--- (17)} \qquad I_{rms} = \frac{I_m}{2} \quad \text{--- eq (8)}$$

eq (8) & eq (17) in eq (16)

$$P_{ac\,(rated)} = \frac{V_m}{\sqrt{2}} \frac{I_m}{2} \quad \text{--- (18)}$$

$$P_{dc} = I_{dc}^2 R_L = \left(\frac{I_m}{\pi}\right)^2 R_L \quad \text{--- (19)}$$

eq (18) & eq (19) in eq (15).

(*)

$$TUF = \frac{\left(\frac{I_m}{\pi}\right)^2 R_L}{\frac{V_m}{\sqrt{2}} \frac{I_m}{2}}$$

$$V_m = I_m (R_f + R_L) \qquad \left[I_m = \frac{V_m}{R_f + R_L} \right]$$

$$TUF = \frac{\frac{I_m^2}{\pi^2} R_L}{\frac{I_m (R_f + R_L)}{\sqrt{2}} \frac{I_m}{2}}$$

$$TUF = \frac{\frac{I_m^2}{\pi^2} R_L}{\frac{I_m^2 (R_f + R_L)}{2\sqrt{2}}} = \frac{\cancel{I_m^2}}{\pi^2} R_L \times \frac{2\sqrt{2}}{\cancel{I_m^2} (R_f + R_L)}$$

$$TUF = \frac{2\sqrt{2}}{\pi^2} \frac{R_L}{R_f + R_L} = \frac{2\sqrt{2}}{\pi^2} \frac{1}{\left(\frac{R_f + R_L}{R_L}\right)}$$

$$TUF = \frac{2\sqrt{2}}{\pi^2} \frac{1}{\left(\frac{R_f}{R_L} + 1\right)}$$

if $R_L >> R_f$

$$TUF = \frac{2\sqrt{2}}{\pi^2}$$

$$TUF = 0.287$$

$$\% \, TUF = 28.7 \%$$

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7. Form factor (FF):

$$FF = \frac{\text{rms value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi}$$

$$= 1.57$$

8. peak factor (PF):

$$PF = \frac{\text{peak value}}{\text{Rms value}} = \frac{I_m}{I_m/2} = 2$$

→ Centre-tap full wave Rectifier :

(i) DC o/p current & Average current (I_{dc}):

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i \, d\theta$$

$$\left[\because \cos\pi = -1, \quad \cos 2\pi = 1, \quad \cos 0 = 1 \right]$$

$$I_{dc} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta \, d\theta + \frac{1}{2\pi}\int_{\pi}^{2\pi} -I_m \sin\theta \, d\theta$$

$$= \frac{1}{2\pi}\left[I_m(-\cos\theta)_0^{\pi} - I_m(-\cos\theta)_{\pi}^{2\pi} \right]$$

$$= \frac{-I_m}{2\pi}\left[(-1-1) - (1+1) \right] = \frac{\cancel{4}^{2} I_m}{\cancel{2}\pi}$$

$$I_m = \frac{V_m}{(R_f + R_L)} \qquad \therefore \boxed{I_{dc} = \frac{2I_m}{\pi}} \text{ ——— (1)}$$

Similarly

$$\boxed{I_{dc} = \frac{2V_m}{\pi(R_f+R_L)}} \qquad \boxed{V_{dc} = \frac{2V_m}{\pi}} \text{ ——— (2)}$$

(ii) Root mean square current (I_{rms}):

$$I_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} I^2 \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$= \frac{I_m^2}{4\pi}\left[(\theta)_0^{2\pi} - \left(\frac{\sin 2\theta}{2}\right)_0^{2\pi} \right]$$

$$\frac{I_m^2}{4\pi}\left[(2\pi-0)-(0-0)\right]=\frac{I_m^2}{2}$$

$$\therefore \boxed{I_{rms}=\frac{I_m}{\sqrt{2}}} \quad \text{—— (3)}$$

(iii) Voltage Regulation :

→ The variation of dc o/p voltage as a function of d.c. load current is called Regulation.

$$\%\ \text{Regulation} = \frac{V_{no\,load}-V_{Full\,load}}{V_{full\,load}}\times 100\%$$

$$\%\ \text{Regulation} = \frac{V_{NL}-V_{FL}}{V_{FL}}\times 100 \quad \text{—— (4)}$$

$$V_{NL}=\frac{2V_m}{\pi} \quad \text{—— (5)}$$

$$V_{FL}=\frac{2V_m}{\pi}-I_{dc}R_f \quad \text{—— (6)}$$

$$V_{FL}=\frac{2V_m}{\pi}-\frac{2V_m}{\pi}\frac{R_f}{R_f+R_L} \qquad \left[\because I_{dc}=\frac{2V_m}{\pi(R_f+R_L)}\right]$$

$$V_{FL}=\frac{2V_m}{\pi}\left[1-\frac{R_f}{R_f+R_L}\right]$$

$$V_{FL}=\frac{2V_m}{\pi}\left[\frac{\cancel{R_f}+R_L-\cancel{R_f}}{R_f+R_L}\right]$$

$$V_{FL}=\frac{2V_m}{\pi}\frac{R_L}{R_f+R_L} \quad \text{—— (7)}$$

eq (5) & eq (7) in eq (4)

$$\% \text{ Regulation} = \frac{\frac{2V_m}{\pi} - \frac{2V_m}{\pi}\frac{R_L}{R_f+R_L}}{\frac{2V_m}{\pi}\frac{R_L}{R_f+R_L}}$$

$$= \frac{\cancel{\frac{2V_m}{\pi}}\left[1 - \frac{R_L}{R_f+R_L}\right]}{\cancel{\frac{2V_m}{\pi}}\frac{R_L}{R_f+R_L}}$$

$$= \frac{\left[\frac{R_f \cancel{+R_L} - \cancel{R_L}}{\cancel{R_f+R_L}}\right]}{\frac{R_L}{\cancel{R_f+R_L}}}$$

$$\boxed{\% \text{ Regulation} = \frac{R_f}{R_L} \times 100} \quad \text{—— (8)}$$

(iv) Ripple factor :

$$r = \frac{\text{rms value of a.c. Components of o/p wave}}{\text{dc value of the o/p wave}}$$

$$r = \frac{I_{ac}}{I_{dc}} \quad \text{—— (9)}$$

effective value of the pulsating o/p current

$$I_{rms}^2 = I_{ac}^2 + I_{dc}^2$$

$$I_{ac} = \sqrt{I_{rms}^2 - I_{dc}^2}$$

$$r = \sqrt{\frac{I_{rms}^2 - I_{dc}^2}{I_{dc}^2}}$$

$$r = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - \frac{\cancel{I_{dc}^2}}{\cancel{I_{dc}^2}}}$$

$$r = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} \quad \text{——— (10)}$$

$$\left[\because I_{dc} = \frac{2I_m}{\pi} \qquad \because I_{rms} = \frac{I_m}{\sqrt{2}}\right]$$

$$r = \sqrt{\left(\frac{I_m/\sqrt{2}}{2I_m/\pi}\right)^2 - 1}$$

$$r = \sqrt{\frac{\cancel{I_m^2}}{2} \times \frac{\pi^2}{4\cancel{I_m^2}} - 1}$$

$$r = \sqrt{\frac{\pi^2}{8} - 1} = 0.482$$

$$\boxed{r = 0.482}$$

v) Efficiency of Fullwave rectifier:

The ratio of d.c. o/p power to a.c. i/p power

$$\eta = \frac{\text{d.c. o/p power}}{\text{a.c. i/p power}}$$

$$\eta = \frac{P_{dc}}{P_{ac}} \quad \text{—— (11)}$$

$$P_{dc} = \frac{V_{dc}^2}{R_L} \quad \text{—— (12)}$$

$$P_{ac} = \frac{V_{rms}^2}{R_L} \quad \text{—— (13)}$$

eq (12) & eq (13) in eq (11)

$$\eta = \frac{V_{dc}^2/\cancel{R_L}}{V_{rms}^2/\cancel{R_L}}$$

$$\eta = \frac{V_{dc}^2}{V_{rms}^2} \qquad \left[\begin{array}{l} \because V_{dc} = \frac{2V_m}{\pi} \\ \because V_{rms} = \frac{V_m}{\sqrt{2}} \end{array}\right]$$

$$\eta = \frac{\left(\frac{2V_m}{\pi}\right)^2}{\left(\frac{V_m}{\sqrt{2}}\right)^2} = \frac{4\cancel{V_m^2}}{\pi^2} \times \frac{2}{\cancel{V_m^2}}$$

$$\eta = \frac{8}{\pi^2} = 0.812$$

$$\boxed{\%\eta = 81.2\%}$$

→ Transformer utilization factor (TUF) in full wave Rectifier:

→ A FWR has '2' windings across the transformer secondary

therefore,

$$TUF_{secondary} = 2 \times TUF_{secondary}(HWR)$$

$$= 2 \times 0.287 = 0.574$$

→ TUF primary is defined as the ratio of dc o/p delivered to the load to the ac rating of the transformer primary.

$$TUF = \frac{P_{dc}}{P_{ac\,(rated)}} = \frac{I_{dc}^2 R_L}{I_{rms} \times V_{rms}\,(rated)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}, \quad I_{dc} = \frac{2I_m}{\pi}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{I_m(R_f+R_L)}{\sqrt{2}} = \frac{I_m R_L}{\sqrt{2}}$$

$$TUF = \frac{\left(\frac{2I_m}{\pi}\right)^2 R_L}{\frac{I_m}{\sqrt{2}} \times \frac{I_m R_L}{\sqrt{2}}} = \frac{\frac{4I_m^2}{\pi^2} R_L}{\frac{I_m^2 R_L}{2}}$$

$$= \frac{4\cancel{I_m^2}\cancel{R_L}}{\pi^2} \times \frac{2}{\cancel{I_m^2}\cancel{R_L}} = \frac{8}{\pi^2} = 0.812$$

therefore average $TUF = \frac{0.574 + 0.812}{2} = 0.693$

→ form factor :

$$\text{form factor} = \frac{\text{rms value of the o/p voltage}}{\text{average value of the o/p voltage}}$$

$$= \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

→ peak factor :

$$\text{peak factor} = \frac{\text{peak value of the o/p voltage}}{\text{rms value of the o/p voltage}}$$

$$= \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

→ Comparison b/w Half wave, Center tap & Bridge Rectifiers:

Characteristics	Half wave Rectifiers	Full wave Rectifiers	
		Center tap	Bridge
No. of Diodes	1	2	4
Transformer requirement	Not essential	essential	not essential
I_{dc}	I_m/π	$2I_m/\pi$	$2I_m/\pi$
I_{rms}	$\frac{I_m}{2}$	$\frac{I_m}{\sqrt{2}}$	$\frac{I_m}{\sqrt{2}}$
I_m	$\frac{V_m}{R_f+R_L}$	$\frac{V_m}{R_f+R_L}$	$\frac{V_m}{R_L+2R_f}$
V_{dc}	$\frac{I_m R_L}{\pi} = \frac{V_m}{\pi}$	$\frac{2I_m R_L}{\pi} = \frac{2V_m}{\pi}$	$\frac{2I_m R_L}{\pi}$
η	40.6%	81.2%	81.2%
(TUF)	0.286	0.693	0.812
fundamental frq. of ripple	f	2f	2f
Ripple factor (r)	1.21	0.482	0.482
Voltage Regulation	Good	Better	Good