1. Probability Theory
   1. Probability Theory as a Set of Outcome

* Terminology :
* **Experiment**: a probabilistic model, the output is not deterministic
* **Sample space(**: the set of all possible outcomes
* **Sample points( the elements of the sample space**
* **Events**: subsets of sample space
* Space:

Euclidean space / Linear space / Probability space / ….

* Deterministic: the output is determined, the fixed number, quantitative.

Probabilistic (Stochastic): the output is not determined.

Def. 1.1 **An event** is an outcome or a collection of outcomes. It is **a set**, and hence we use set notation to denote an event, , called set A is a subset of

* Roll a fair die ,
* Roll an odd die
* Roll a fair die: Is this 🡪 No! there is no “7” outcome of the experiment
  1. Set Theory

Def. 1.4.

1. The sets A and B are equal to sets or identical sets iff A and B have the same elements. We denote equality by writing A=B
2. A is included in B or A is a subset of B iff implies In such cases, we write

Prop. 1.6.

1. iff
2. If then

* Union --> logic as “or”
* Intersection 🡪 logic as “and”
* Complement,
* Relative complement (difference) .

Prop.1.7.

1. then

* Proof

1. By Contradiction: proof the claim is not correct.
2. and and
3. If , by assumption
4. b) is contradict to a) since 🡪 the proof is complete. QED
5. First we prove direction i.e., if

2.1) we know and .

2.2) Since if then (the first part of this proposition) , and . Hence

2.3) Now we prove direction i.e.,

2.4) implies or or

2.5) Hence

2.6) 2.5) implies (\*remember the definition of Union)

* De Morgan’s law

1. Not (A and B ) < -- > not (A) or not( B)
2. Not( A or B) < -- > not(A) and not(B)

* The null set : the set which is no elements.

1. In set theory, no definition of the null element.
2. The facts of

-If the set A and B are no common elements, then

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-Since , this implies and . Hence the

* Question: the number of subsets given a set

1) Let how many subsets of

-{1},{2},{3},{1,2},{1,3},{2,3},[1,2,3] and 🡪 7

2) Prove if # of = n, then # of subset of is

* 1. Prob. Space and the Prob. Measure

Axiom 1. Given an experiment, there exists a sample space , representing **the totality** **of possible outcome** of the experiment and a collection, of subsets, A, of called events

* **Sample space(**: the set of all possible outcomes
* **Events**: a collection of subsets of sample space **(**

Axiom 2. T each event A in there can be assigned a nonnegative number P(A) such that

Lemma 1.9.

Lemma 1.10. If A and B are two arbitrary events in the sample space , then

* 1. Algebra of Sets and Pro. Space

Def.1.12. An algebra, , is a set of sets such that the following hold:

1. implies
2. implies

Prop. 1.13. If and

2. and

Def. 1.17. A class of subsets of is a , denoted **,** if it is an algebra and if it is also closed under countable unions, i.e.,

Axiom 3. Let , of subsets of and a probability measure defined on elements of Then, if is a countable of disjoint sets, i.e., , the probability of the union is found by

Def. 1.19. If is the set containing all possible outcomes of an experiment,  **is a**  of the subsets of and is a probability measure on **,** then the triple

is called a probability space.

* 1. Key Concepts in Probability Theory

Def. 1.24. A conditional probability is the probability of the occurrence of an event subject to the hypothesis that another event has occurred