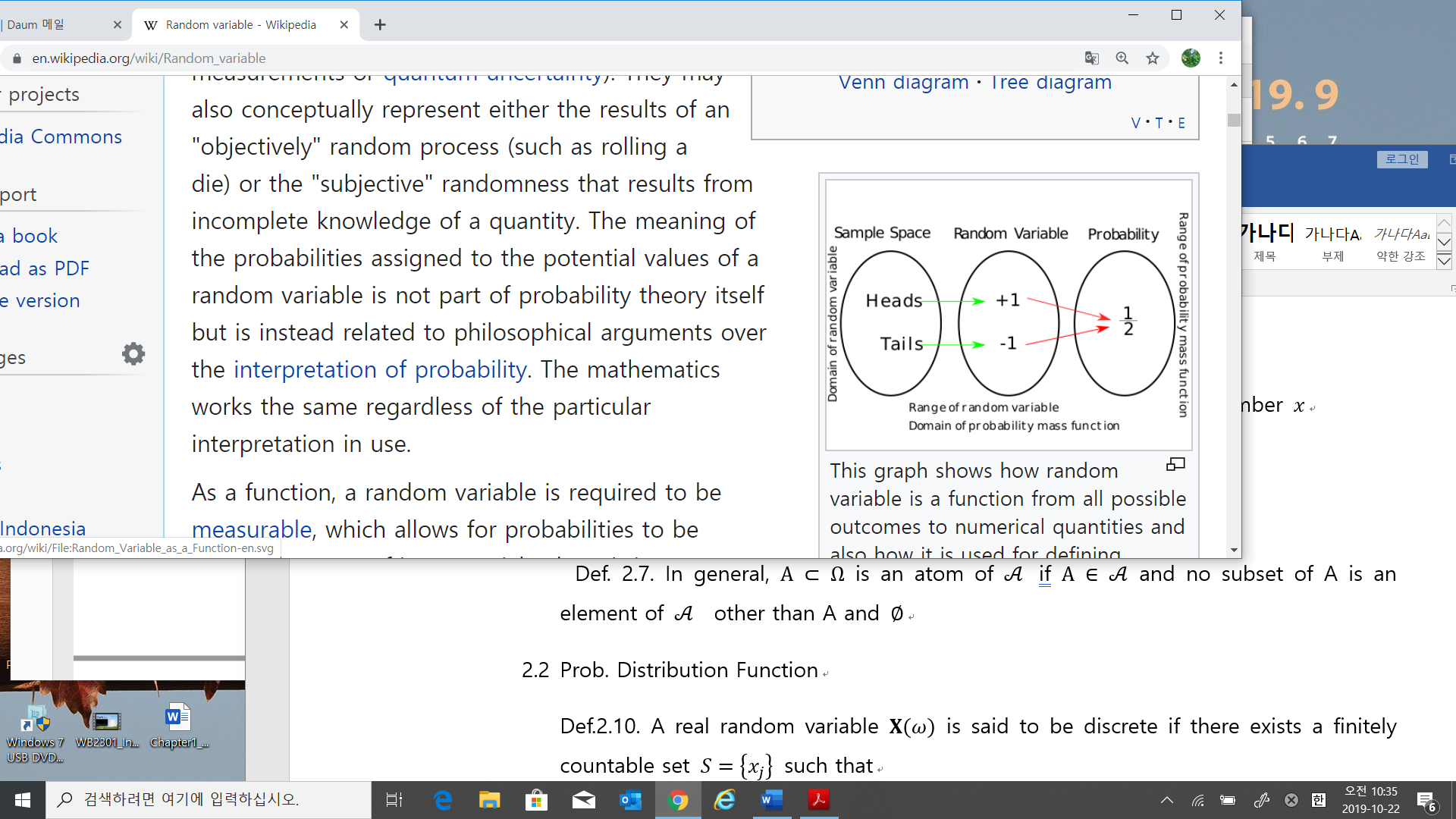
1. Random Variables and Stochastic Process
   1. Random Variables

Def. 2.1. Given a probability space,, a random variable is a real-(vector-) valued point function which carries a sample point, , into a point

in such a way that every sets, , of the form

is an element of the

* In the textbook, mis spelled. As
* Random variable
* is a function such that associates to a real number
* Wiki



Def. 2.7. In general, is an atom of if and no subset of A is an element of other than A and

* See the notation ,
* Skip from Ex.2. to the last of 2.1
  1. Prob. Distribution Function
* Probability Distribution function

Def.2.10. A real random variable is said to be discrete if there exists a finitely countable set such that

* 1. Prob. Density Function

Proposition 2.12.

* 1. Probabilistic Concepts Applied to Random Variables

Def 2.16. Two random variables and are called independent if any event of the form id independent of any event of the form where are sets in

* 1. Functions of a Random Variable

Prop. 2.17 Given and and a vector with density function , the n-vector

has the density function

Where stands for the absolute value of the determinant of the matrix

* 1. Expectations and Moments of a Random Variable
  2. Characteristic Functions

Lemma 2.27

Prop.2.28 If is a Gaussian random vector with mean, m, and covariance matrix P, then its characteristic function is

Prop. 2.29. Uncorrelated Gaussian random variables are independent

Theorem 2.30. If is a Gaussian random vector with mean , and covariance, , and if , where is a Gaussian random vector with zero mean and covariance, , then is a Gaussian random vector with mean, , and covariance, .

* The central limit theorem

Theorem 2.31. Let be i.i.d. random variables with finite mean and variance,

and denote their sum as . Then the distribution of the normalized sum

is a Gaussian distribution with mean 0 and variance 1 in the limit as

* Proof : textbook P.52
* Remarks:

1. See, the condition, that means   
   the mean and the variance is constant, which implies the random process is the sample probability space, but the experiment is many time processing. For example,
2. A die, which is fair or not, you roll the same die many times. Then the mean of the sum () is a Gaussian if .
3. Some RV has no mean, then it will not be applicable.
   1. Conditional Expectations and Conditional Probabilities

* The conditional expectation
* Remarks
* is a constant, means it is not random variable.
* if is a constant, then is a constant
* if is a RV, then is a **Random Variable** of y
* **Iterated expectation** (See the proof at p.57 and remember)
* Remark

Even if we do not know .

* Ex. , 🡪

Lemma 2.34.

* 1. Stochastic Process

Def. 2.36. A stochastic process is a family of random variables, , indexed by a real parameter and defined on a common probability space .

Ex. 2.37

Def. 2.38.

1. A stochastic process is said to be continuous in probability at t if

for all

1. A stochastic process is said to be separable if there exists a countable, dense set such that for any closed set

differ by a set such that

Theorem 2.40. The rational numbers in provide a separating set S.

Def. 2.42. Let X be a random process defined on the time interval, T. Let

be a partition of the time interval, T. If the increments, are mutually independent for any partition of T, then X is said to be a process with **independent increments**.

Def. 2.43 We say that a random process, X, is a Gaussian process if for every finite collection, the corresponding density function,

is a Gaussian density function.

Def. 2.44 We say that a random process X is a Gaussian process if every finite linear combination of the form

is a Gaussian random variable

Def 2.45. A random process, where T is a subset of the real line, is said to be a **Markov process** if for any increasing collection

or, equivalently

* 1. Gauss-Markov Processes – **The fundamental**

1. Dynamics

* State , is a known matrix, is a Gaussian Random sequence.

1. Given Conditions
2. Noise

where

1. The states

1. The correlation

which implies

1. The mean and covariance

* The mean
* The covariance
* The **transition** matrix – notation abuse
* We will discuss in the next Tuesday. However in discrete linear system(or Markov process) there is a definition of the transition matrix.
  1. Non-linear Stochastic Difference Equations 🡪 skip

1. Conditional Expectations and Discrete-Time Kalman Filtering
   1. Minimum Variance Estimation

* Problem statement – static parameter estimation

where .

1. The maximum a posterior estimate , , ~ the peak value of
2. The minimax estimate,, ~ the median or the midpoint of
3. The minimum variance estimate,, ~ the mean or the center of mass of

* The minimum variance estimator
* Remark 3.1.argmin
* Define convex distance function

Is a non-negative and convex distance function.

-. Convex function: a function is convex if for any points, and some scalar such that

-. Example of a convex function

* Define a loss function , such that

-.

-.

Theorem 3.2(Sherman’s Theorem). Let x be a random vector with mean, , and density,. Let be a loss function as defined above. If . Is symmetric about and unimodal(i.e., has only one peak), then minimizes .

* Without observation, the minimum variance estimator.

Sol:

* Remark Sherman’s theorem is generalized of the above estimator.

Theorem 3.6. Given the equation(3.1). if the estimate is a function of , then the minimum variance estimate is the conditional mean.

Proof: let be an estimator of ( that means given is a constant).

Then for

Since the cross term is

.

Then

Since are independent of and is positive semidefinite,

1. is equivalent to

* Remarks:

1. It means
2. It is unbiased

* Ex.3.8: conditional mean and variance of the sum of two RV’s

Let measurement , , are independent. Find the the minimum variance estimate

* a priori information,
* a noise,

1. Before finding ,calculate (in the text book, characteristic function used)

We will use the convolution integral

* ,
* ,
* ,
* ,
* What is

1. Calculate . Hence first find .

Since ,

Hence

