1. Continuous time Kalman Filter
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* Problem

In Ch.3 (or Ch.4) in the discrete time Kalman filter, the system model is

where is a zero-mean, white Gaussian with covariance , i.e.,

* In the continuous time, white Gaussian noise:
* Definition: a random process is a zero mean with covariance if

Then is a white noise stochastic process with intensity

* except .
* White noise is the derivative of a process with uncorrelated increments(Brownian Process)
* Definition: Brownian process

1. is a Gaussian random variable
2. and
3. has independent increment

From . i.e.,

* Problem:
* Brownian motion such that

where the estimator to be derived,

* The best Mean Square Error

* Solution:

1. The best estimator
2. The Kalman Gain
3. The mean square error ,

* Ex\_1.
* is a white noise with

The solution is

* Ex\_2
* Comments:

1. Without the measurements, i.e., no estimation, the covariance of the solution of

is

1. The differential equation of 3) is called as **Riccati** equation.
2. In the discrete case, the optimal process is divided into two groups. prediction / estimation

But in continuous case, it is done simultaneously.

* 1. Properties of the Continuous Riccati Equation
* Duality: LQR - Linear Quadratic Regulator

1. Problem:

Find to minimize

1. Optimal controller

With

* Steady State properties of Riccati equation:

In order to be stable for the system (2), as time converges to infinity

* (skip):Hamiltonian matrix(6.15): (what? In optimal control it is introduced.)
* Need the unique solution.
* If (F,H) is observable and , then the unique solution.

(controllability / observability 🡪 introduced by Kalman)

* It is unknown the closed form of the solution. We need the numerical analysis.
  1. Stationarity
* Def. 6.6: A random process is stationary if
* Def. 6.7: A RP is wide-sense stationary if

And the correlation