1. Random Variables and Stochastic Process
2. Conditional Expectations and Discrete-Time Kalman Filtering
   1. Minimum Variance Estimation
   2. Conditional Estimate of a Gaussian Random Vector with Additive Gaussian Noise

* Modelling
* %% I will notify means is independent.
* Problem: find
* Solution\_1 : batch algorithm

where the conditional(error) variance is

* The minimum variance estimator of

= the conditional expectation given

~ mean of x + (scale) (measurement – mean of x)

* Scaler case

* Tip.
* Solution\_2: real time(sequential)
* Extension of (3.2): Sequentially measuring of

For iteration calculation, define new variables at measurement

Or

* Important:

1. With the last measurement , the conditional mean of can be calculated by (3.14) or (3.32)



1. To get the , in the batch process you need more memory compared to the real time process
2. In general, .Hence we need recursively using (3.28)
3. Kalman gain:
   * 1. Statistical Independence(orthogonal) of the Error and the Estimate

* Orthogonal in Linear algebra

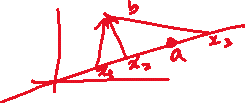
Let two vectors . Two vectors are orthogonal if the dot product of x and y,

1. Ex.

* Projection

<https://ocw.mit.edu/courses/mathematics/18-06sc-linear-algebra-fall-2011/least-squares-determinants-and-eigenvalues/projections-onto-subspaces/MIT18_06SCF11_Ses2.2sum.pdf>

If we have a vector b and a line determined by a vector a, how do we find the point on the line that is closest to b?

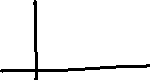


The vector from the closest point vector on the line to the vector should be perpendicular to the line

Hence

The projection vector on is defined .

* The statistical orthogonal



* Given the real , which is a random, should be in the line of perpendicular to



* 1. (Skip-But Later will review this) Maximum likelihood estimator
  2. The Discrete-Time Kalman Filter: Conditional Mean Estimator
* Problem: A dynamic system given as

1. We measure
2. The conditional estimator (the MV estimator) is
3. The propagated estimator is

* *The propagated estimator:*

= a Priori Estimator / a Predicted estimator

= measurements are previous compared to a posteriori estimator







* *In general text book notations are as follows*
* The Estimator
* The Predictor 🡪 the upper accents are different ~~

1. Propagating the conditional mean = a priori = prediction

* Remember , not

1. Propagating the variance of the a priori
2. Updating the Conditional Mean: a posteriori

* Remember not

1. Updating the covariance: a posteriori

* Define: **Residual 🡪 innovation sequence**

From (3.44)

* Define Kalman Gain: the residual gain :

Hence (3.44.1) is

* Orthogonality Properties of the Conditional Mean Estimator

1. The Innovation Sequence is independent for each

Since, the mean of ,

And,

with these facts, therefore the covariance is

which implies the gaussian are independent.

1. Error is orthogonal to ,

* Example 3.9 (estimating the speed of a car)
* Problem: estimate traveled distance by car.

1. The speed of car = 55 mi/hr, for 1 hour, implies the distance is 55 mi
2. The trip meter shown 55.3 mi

What is the best estimated distance with two information?

* Method

1. The system is modelled as
2. We may guess
3. The variance by the measurement: the quantization of the trip meter is = 0.1(mi) 🡪 The variance of the trip meter ,
4. The variance by the process: the velocity varies +/- 1 (mi/hr), the variance of the process
5. The best estimator of the distance in mean square sense,
6. Comments:  
   - the variance of a uniform RV in (a,b) is

-. Compare