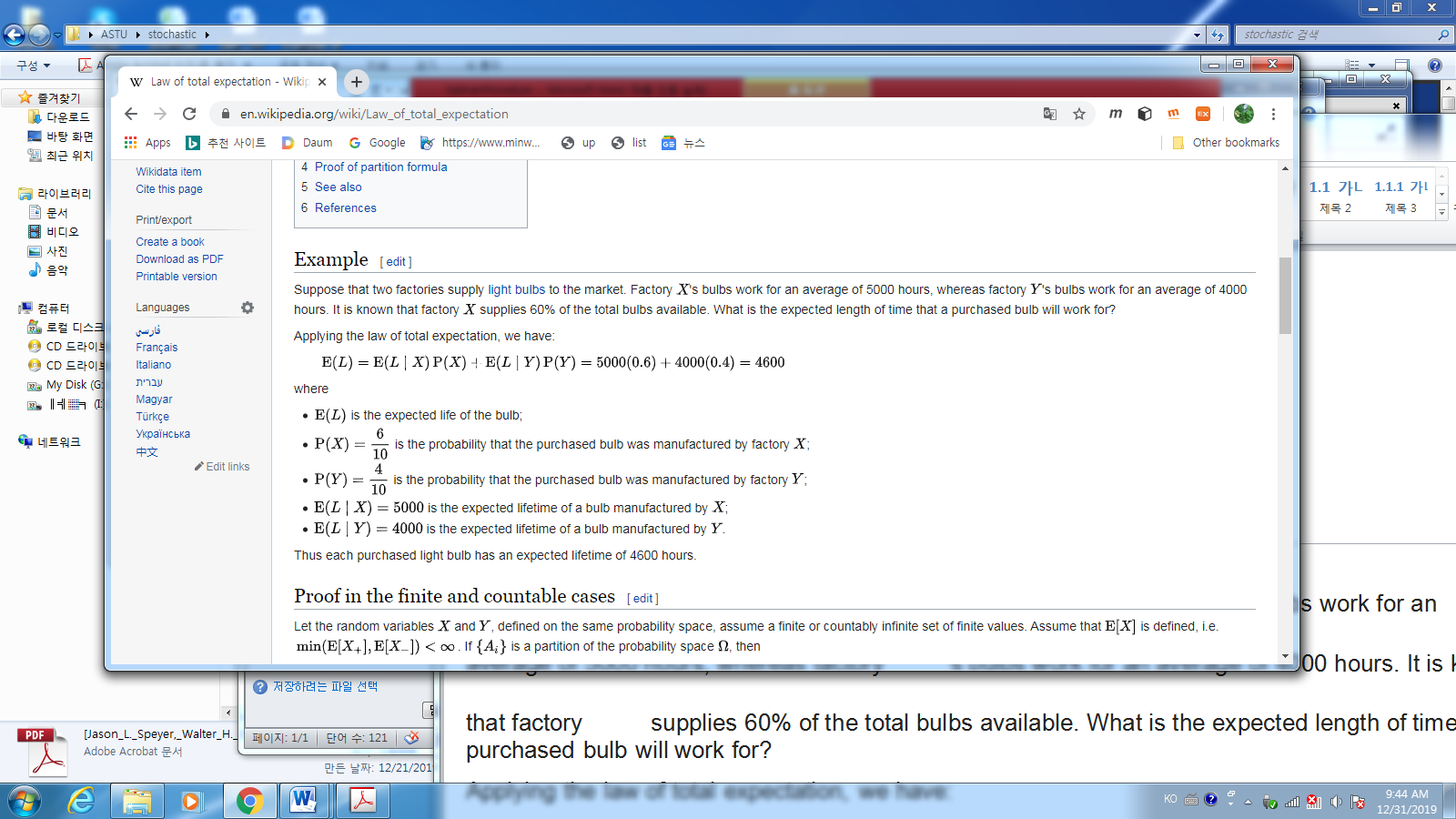
%%% Final Example.

* total expectation : proof (in the finite partitions) as <https://en.wikipedia.org/wiki/Law_of_total_expectation>

example.



1. Chapter 2. Problems 35. prove

Solution: First is considered. Using Lemma 2.34

Hence

Since

Hence (1) is

implies

i.e.,

1. Chapter 3. Problems 7

Prob. 🡪

Prob 🡨

1. Ch.3 problem 8 discrete Kalman

* Model
* initial: k=0

a) compute a priori / prediction ( and posteriori /correction covariance for

For k= 1; covariance:

+prediction Since at , there is no previous estimation, assume

+correction

For

+prediction

+correction

b) compute optimal gain

For k= 1;

+

For

+

c) compute optimate estimator with

For k= 0;

+ at t=0, the correction is assumed :

1. prediction :

For

correction :

For

+prediction:

+correction:

1. Ch.3 problem 11

Find

Solution:

First



(check whether it is a pdf.

Hence

1. Ch.4 prob 4

Prob\_1)

Now multiplying at the both sides,

which implies the column vectors of are eigen vectors of and sigma values are eigen vectors of

Prob\_2 )

No, the hence the

But!! As you know if is a square matrix, then

Hence if is a square matrix, then

1. Ch.4 prob 10

Define is an inner product, then

which implies that is an orthonormal basis in this space.

Hence the minimum mean square approximation (estimator) of is the orthogonal projection in the space of span

Hence

can be determined. For example

(a)Remember the Fourier series on ,

(b) how about

(c) consider , which is independent with .

-Find the orthogonal basis,

- Find the least square estimator of

1. Ch.4 prob 11
2. Ch.4 prob 12 LSE in static systems(batch type)

Let the matrix form

1. The least square estimator is
2. Since the estimation error

By the assumption

And the estimation error variance is

Hence the estimation error variance converges to 0 as converges to infinite

1. If we do not know , then

implies the estimator is biased. And You may show the estimation error variance converges to zero as converges to infinite.

This means even if the variance converges to zero, the mean of the estimator does not converge to the true value, implies the estimator is unbiased.

How to estimate along with and ? If we know , then

.QED

1. Ch.4 prob. 13

* is a linear combination of column vectors of %%%%%%%
* The least square estimator is

implies is a linear combination of row vectors of

1. Ch.4 prob 19
2. Least Square estimator
3. Check the bias:

Hence it is unbiased.

1. The error variance is

If , then

This is not the minimum error variance. The estimator of the minimum error variance of is the conditional expectation estimation as , and the minimum error variance is

Do you think it is true

I have not seen the formal proof, but it is true.

%%%%%% reference of prob 2(Ch.2 prob.7)

Scalar Case

* :

variance

mean

3) Summary:

is the gaussian random variable of with mean

And the variance

which is less than