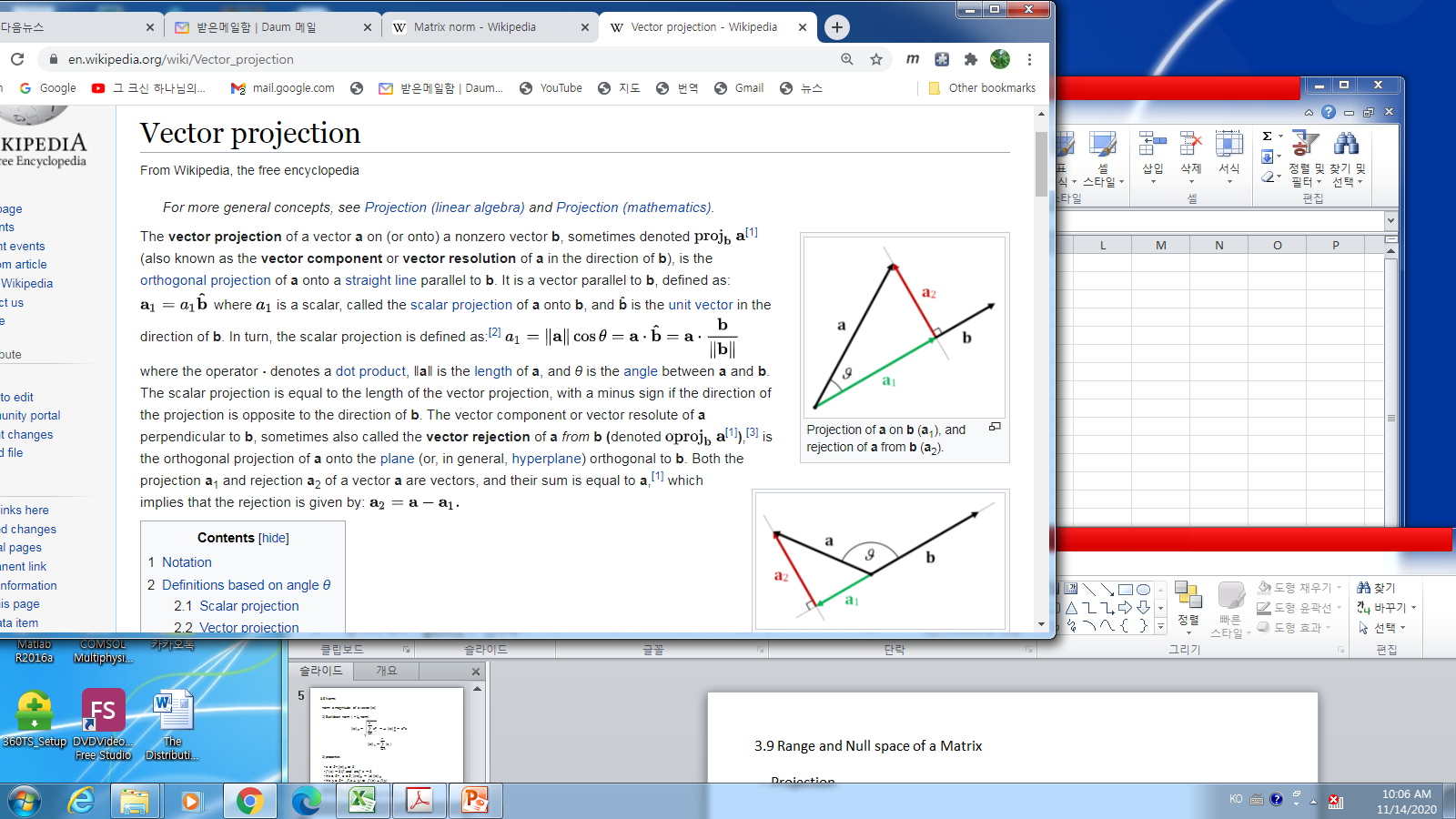
3.9 Range and Null space of a matrix

1) projection: <https://en.wikipedia.org/wiki/Vector_projection>

Example:

Let vector

Find vector such that the distance is minimum, i.e.,

: the projection to ,

Solution: from the picture, which is on the vector .

From the definition of dot product

implies

Hence the can be derived as

2) The Null space

* Def: The null space of a matrix , is a set of all vectors that solve to
* Fact
* Fact:

Every solution to has the form :

1. The particular solution:
2. The null solution(homogeneous solution): ,
3. The general solution:

* Example
* 1)
* 2)
* 3)
* Differential equation

Consider a linear differential equation as

1. The homogeneous solution:

1. The particular solution

1. The general solution

1. If initial points are given such as

To get , applying the initial conditions

Hence the solution is

3.10 The determinant

The determinant of ,

* How to calculate the determinant easily?

3.11 Quadratic Forms and Positive definite

1) quadratic forms

2) Definite matrix for a symmetric matrix

- Positive definite matrix

* Positive semi definite matrix
* Property

For any matrix , is a symmetric and positive semi definite.

3.12 Eigenvalues and Eigenvectors

* Definition: Given a square a scalar and corresponding vector satisfying the following equation,

eigenvalue of

eigenvector of

* Facts

1. Find eigenvalues

* , which is called as the characteristic equation

1. Example 1

* the determinant of is

Hence

1. Eigenvector for

🡪

1. Eigenvector for

🡪

1. Example 2

* the determinant of is

Hence

1. Eigenvector for

🡪

1. Eigenvector for

🡪

1. Example 3

Using matlab,

3.13 Eigenvalues and Eigenvectors of Symmetric Matrices

* Properties of a symmetric matrix

1. All of the eigenvalues of are real number
2. Eigenvector are orthogonal, i.e.,

* Optimal problem related to eigenvalues and eigenvector

Consider the following maximum problem, for a positive definite matrix

sol:

&& Review of Ordinary Differential Equations

<https://www.math.ubc.ca/~feldman/m267/odeReview.pdf>

1. Definition 1

* Ordinary differential equation (ODE)

Sol:

* Partial differential equation

Sol:

* The order of ODE

-The highest derivative of ODE

* The linear ODE

Ex. Non-linear ODE

* Constant coefficient linear ODE if
* Homogeneous ODE: in (1), . Otherwise it is in-homogeneous or non-homogeneous
* An initial value problem

Given (2) and given initial conditions such as

* Boundary value problem

Given (1), and given boundary conditions as

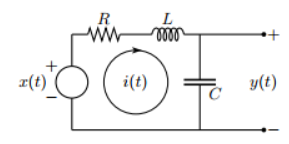
Theorem 2

1. The general solution to ODE (2) is of the form

Where

1. is the order of ODE
2. the particular solution is any solution to (2)
3. are arbitrary constants
4. are independent solutions to the homogeneous equation
5. Given any initial conditions , there is exactly one solution to (2)

Example.3



1. The homogeneous solution

Assume . Then (3) is

which yields to

1. If ,
2. If
3. The particular solution

Assume

Then (3) will be

Since

Matching coefficients yields to

Hence

1. The general solution

Example 4 Boundary Value problem

Remark: for the initial value problem, there exists a unique solution. However for the boundary value problems, it may not exit solution or the exact only one solution.

Let the ODE be

1. the homogeneous solution
2. The solution does not exist with the boundary condition

* Contradict since

-🡪 No solution satisfying this boundary conditions

1. The solution exists with the boundary condition

Hence