%% Linear Algebra: G.Strang, ” Differential Equations and Linear Algebra”, Wellesley- Cambridge Press,2014.

Chapter 4 to Chapter 8

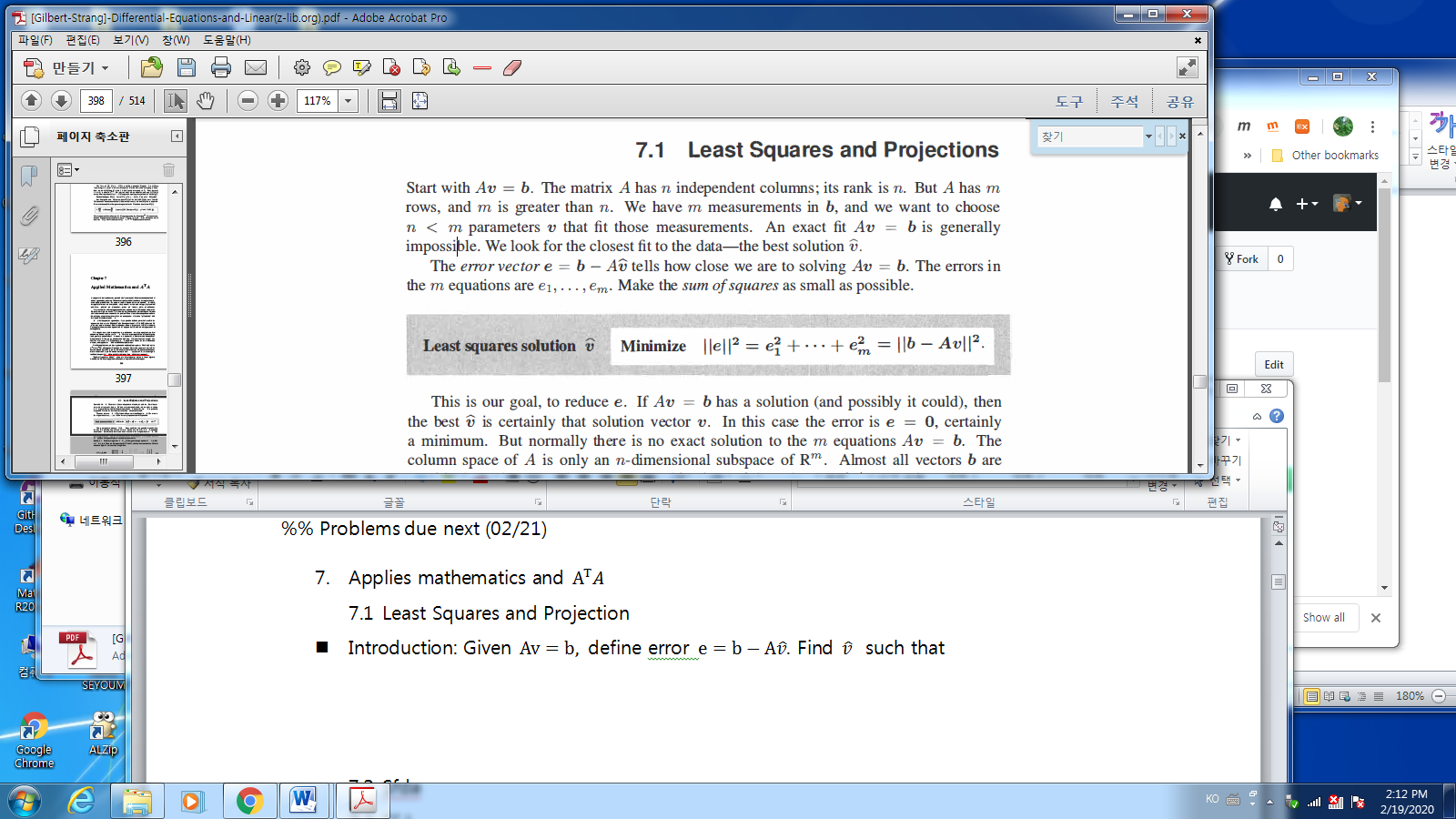
1. Linear Equations and Inverse matrices
2. Vector Spaces and Sub Spaces
3. Eigenvalues and Eigenvectors
   1. Introduction to Eigenvalues
   2. Diagonalizing a matrix
   3. Linear System
   4. The Exponential of a Matrix 🡪 skip
   5. Second order system 🡪 skip
4. Applied mathematics and ATA
5. Fourier and Laplace Transforms
6. Applies mathematics and
   1. Least Squares and Projection

* 1) Solvable:

2) unsolvable : 🡪 How can you handle it?

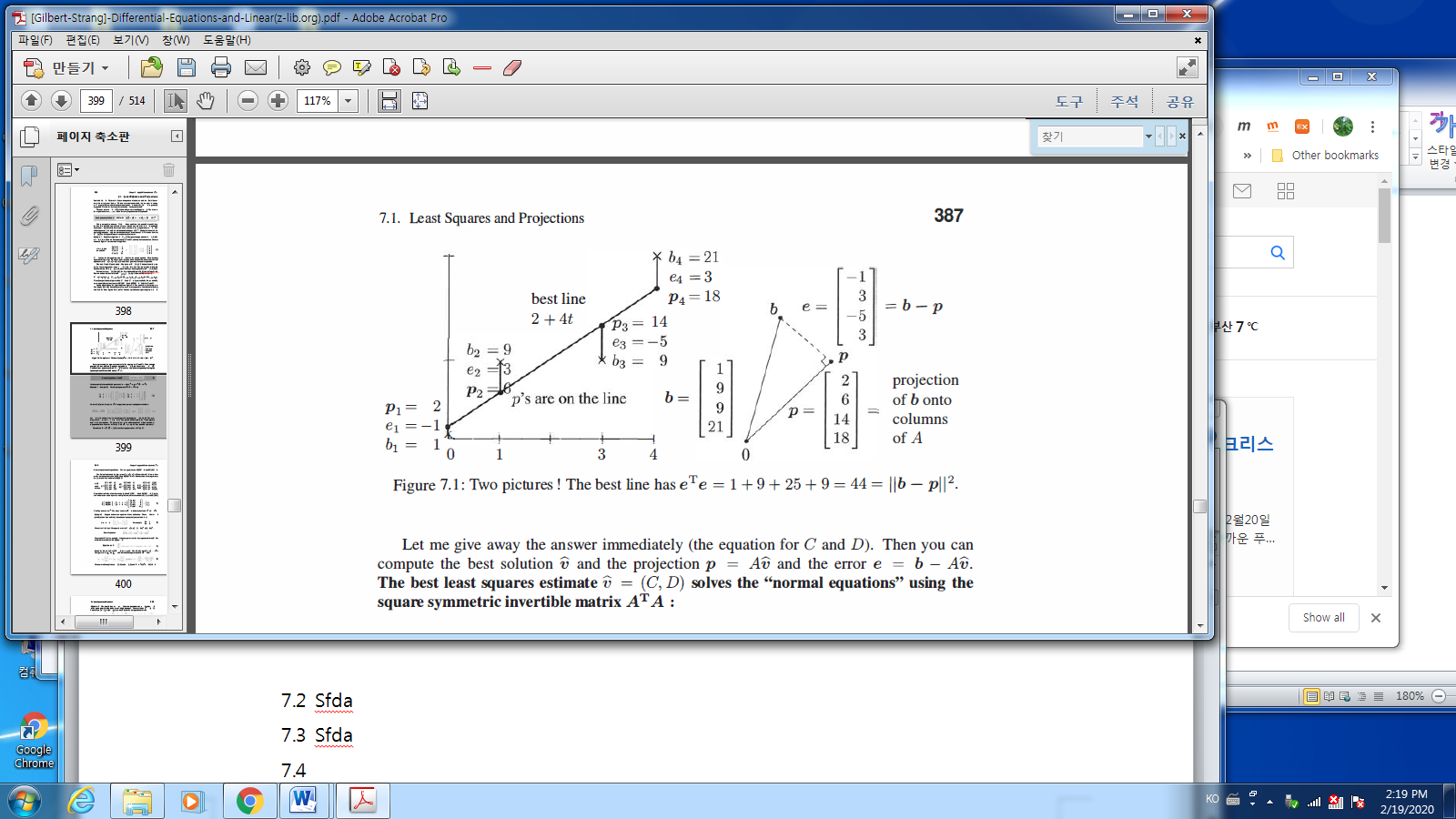
We want an estimator for v

Introduction: Given define error . Find such that

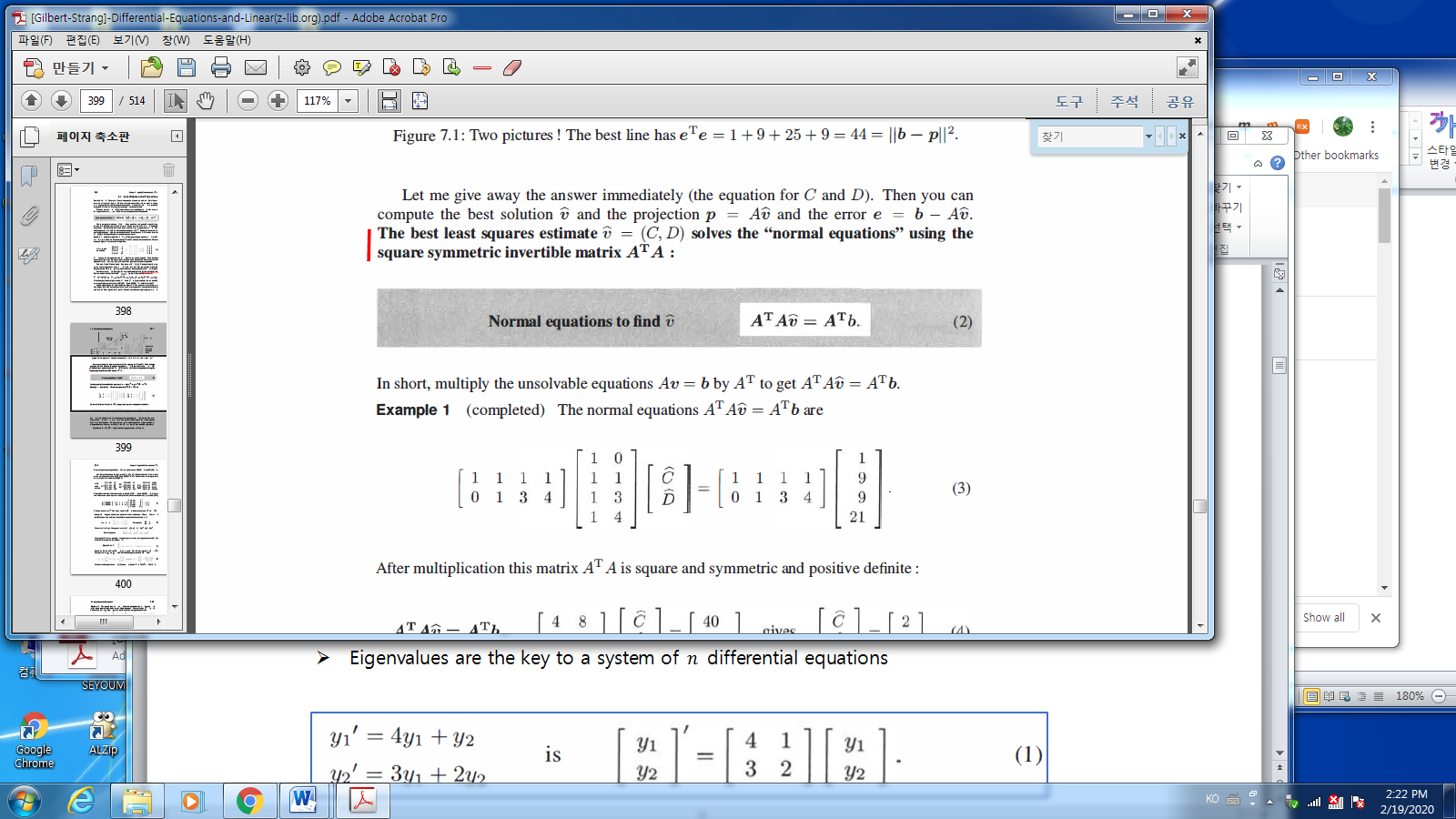


* Example: Find the straight line such that the error is minimized

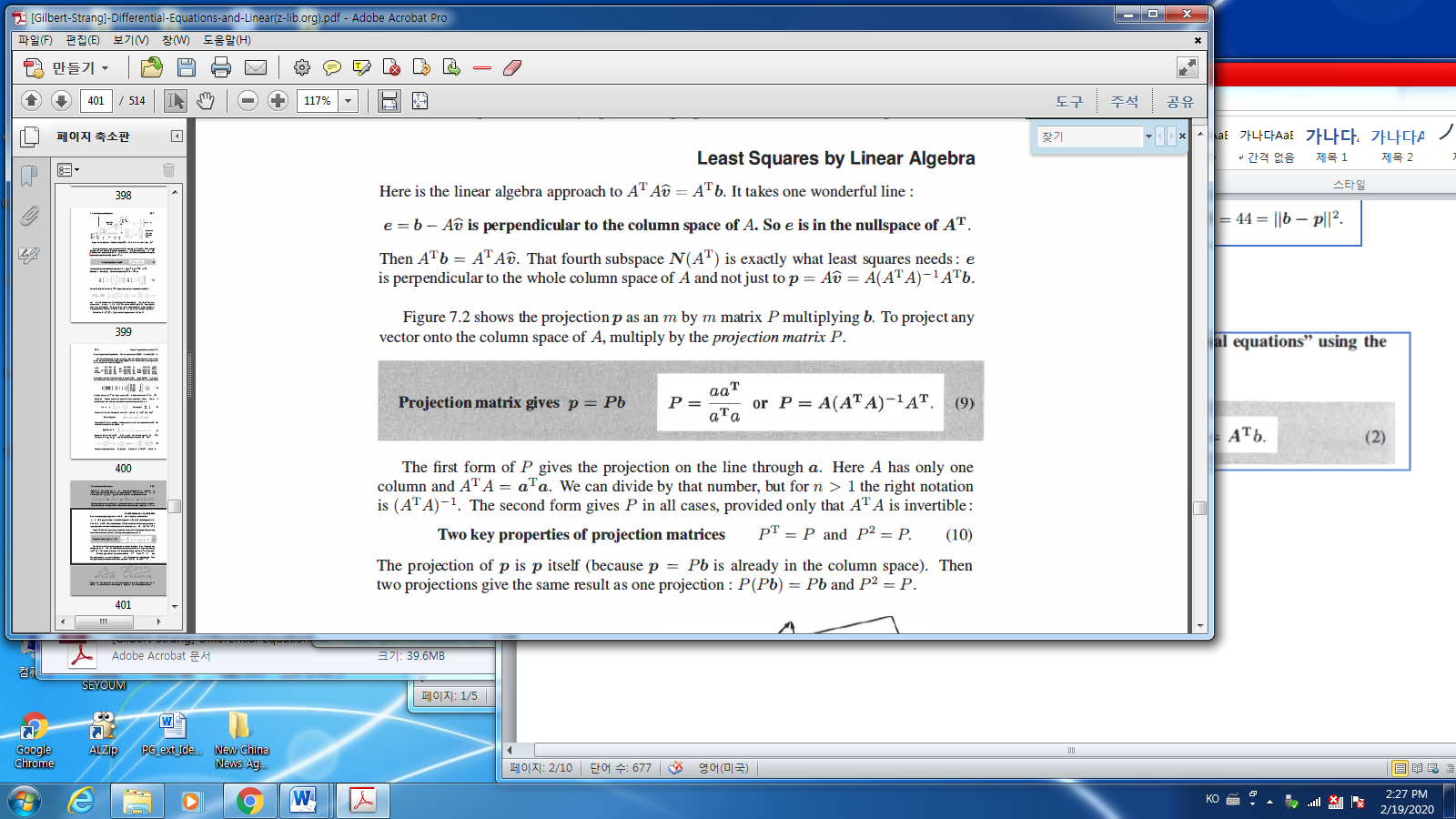
\*Solution: He already got the solution as



* In general the least Square Estimator:

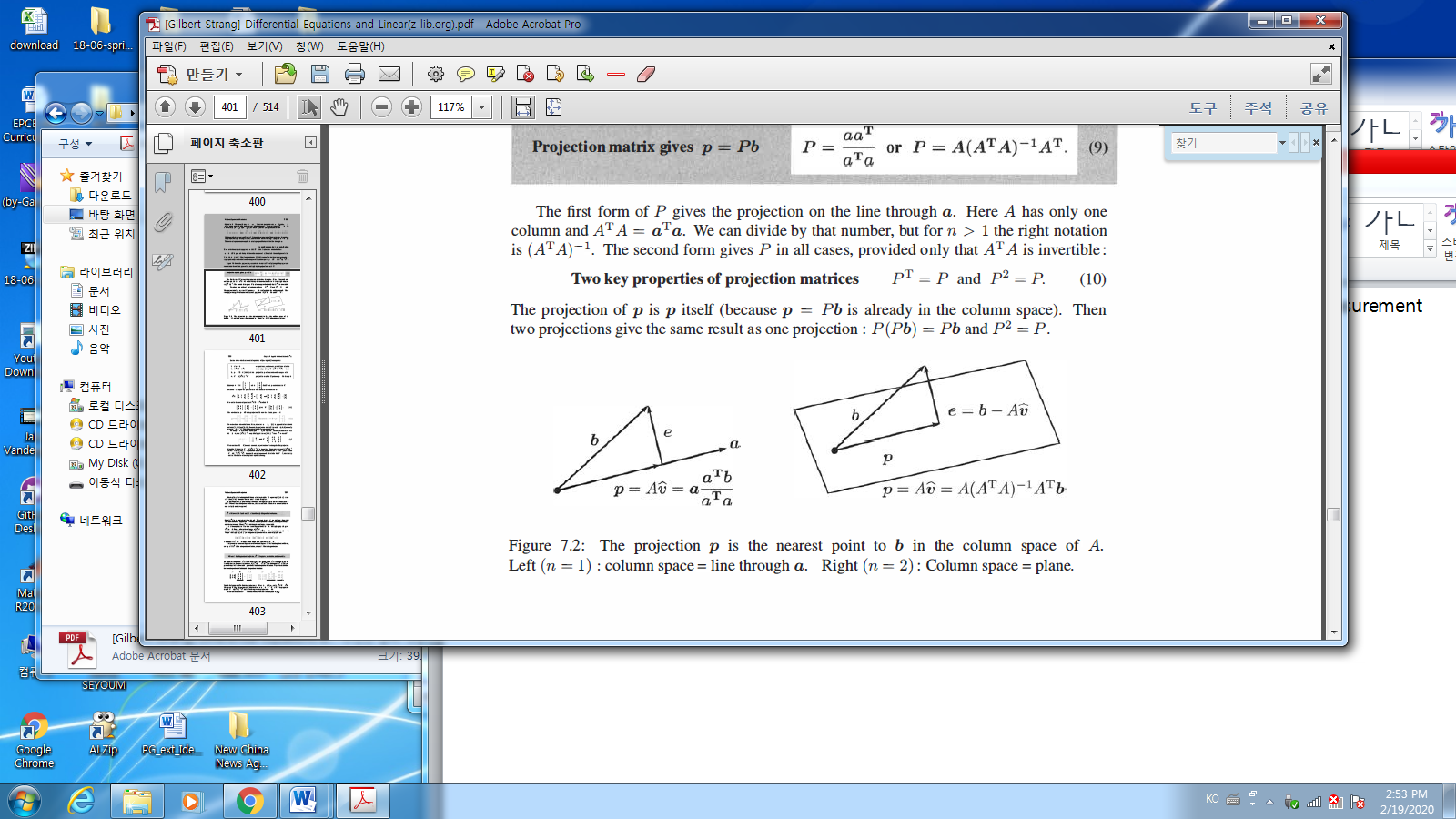


* Fact:



* Projection: vector projects on vector

To project any vector onto the column space , multiply by the projection matrix



The projection vector the vector projects on the vector

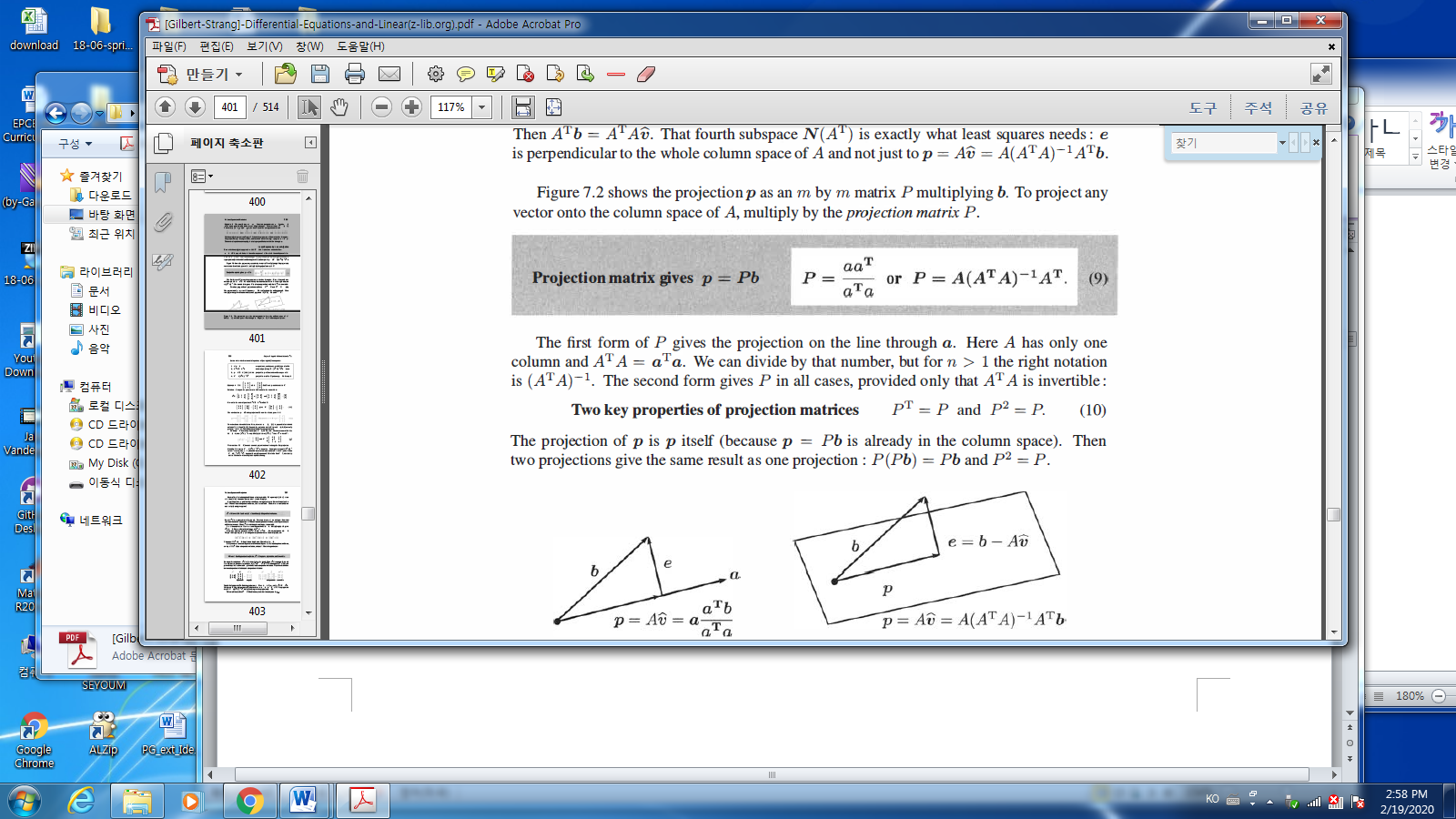
* Kim

Solution: from the picture, which is on the vector .

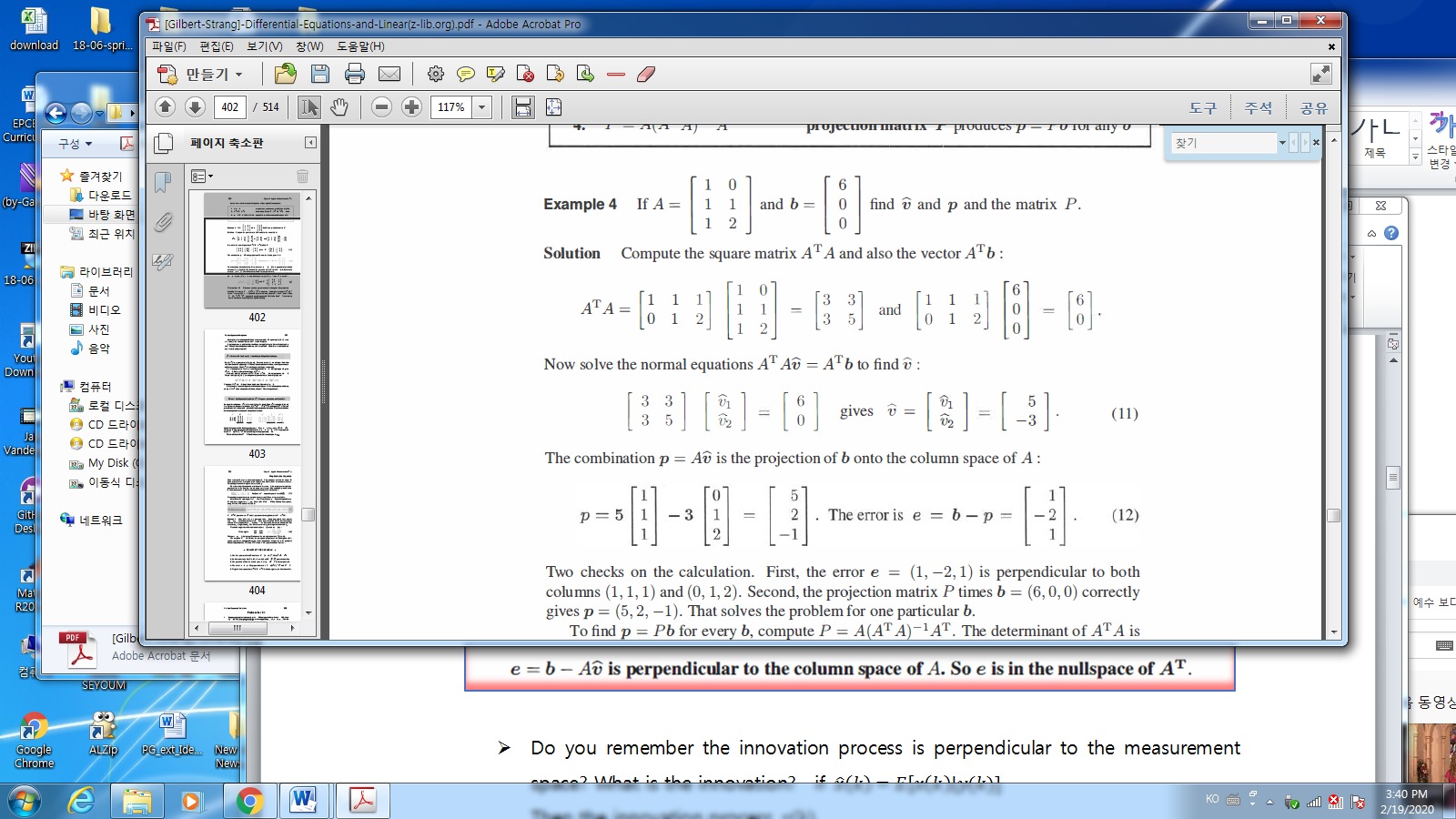
From the definition of dot product

implies

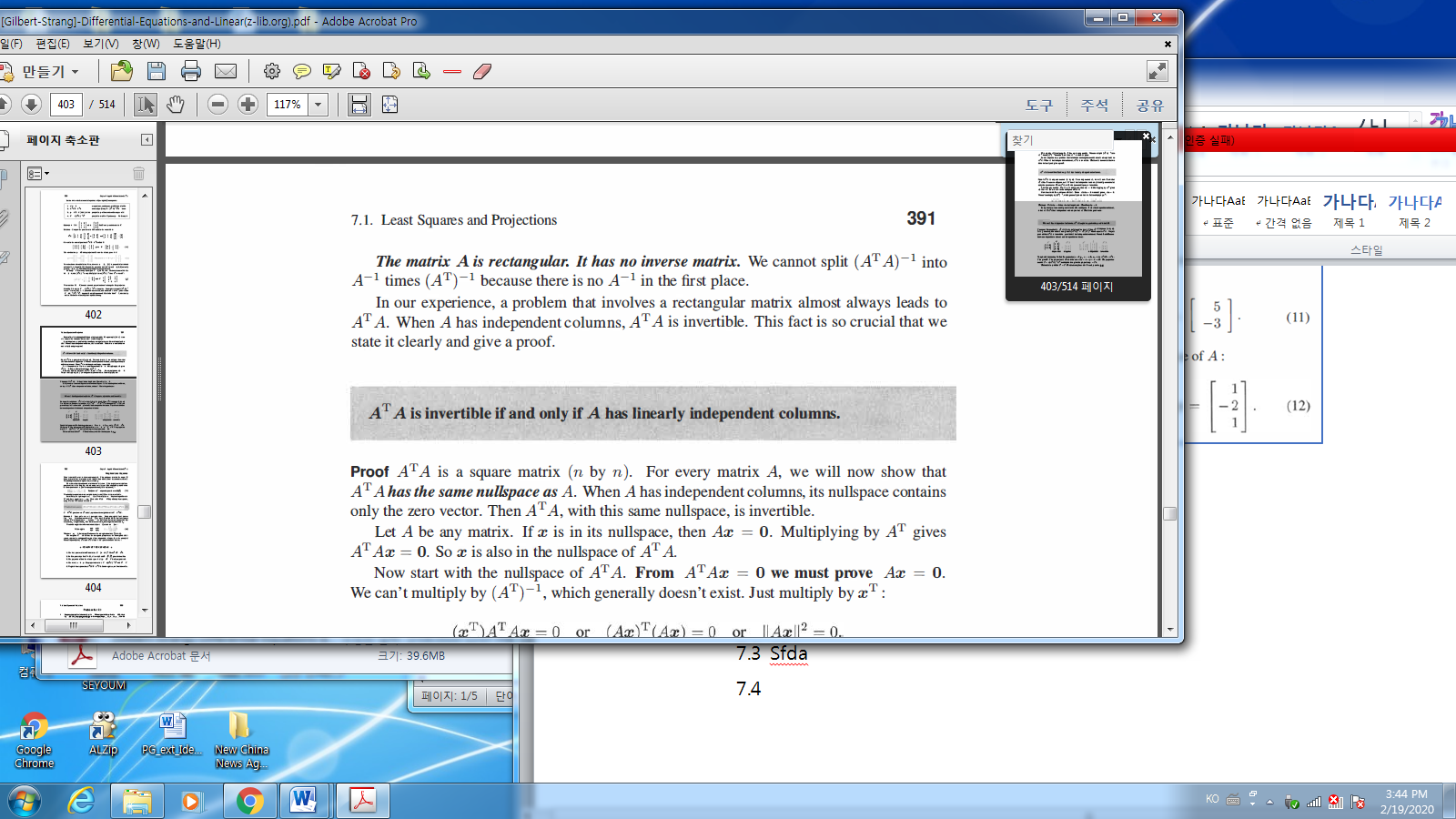
Hence the can be derived as



* Caution:
* Example



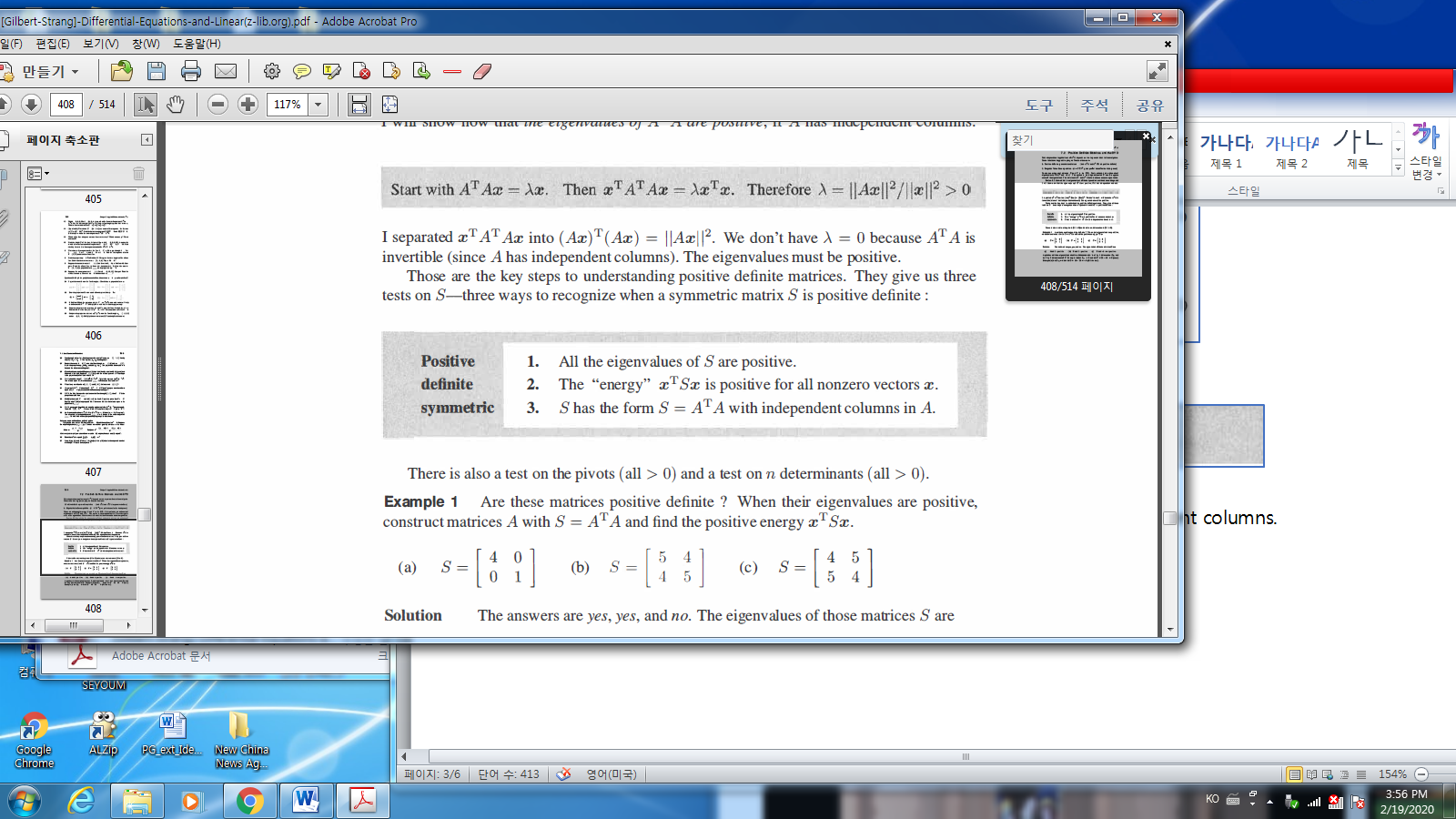
* Fact:



-

🡪 You may remember is invertible iff has linearly independent columns.

* 1. Positive Definite Matrices and SVD
* Definition of a positive definite matrix



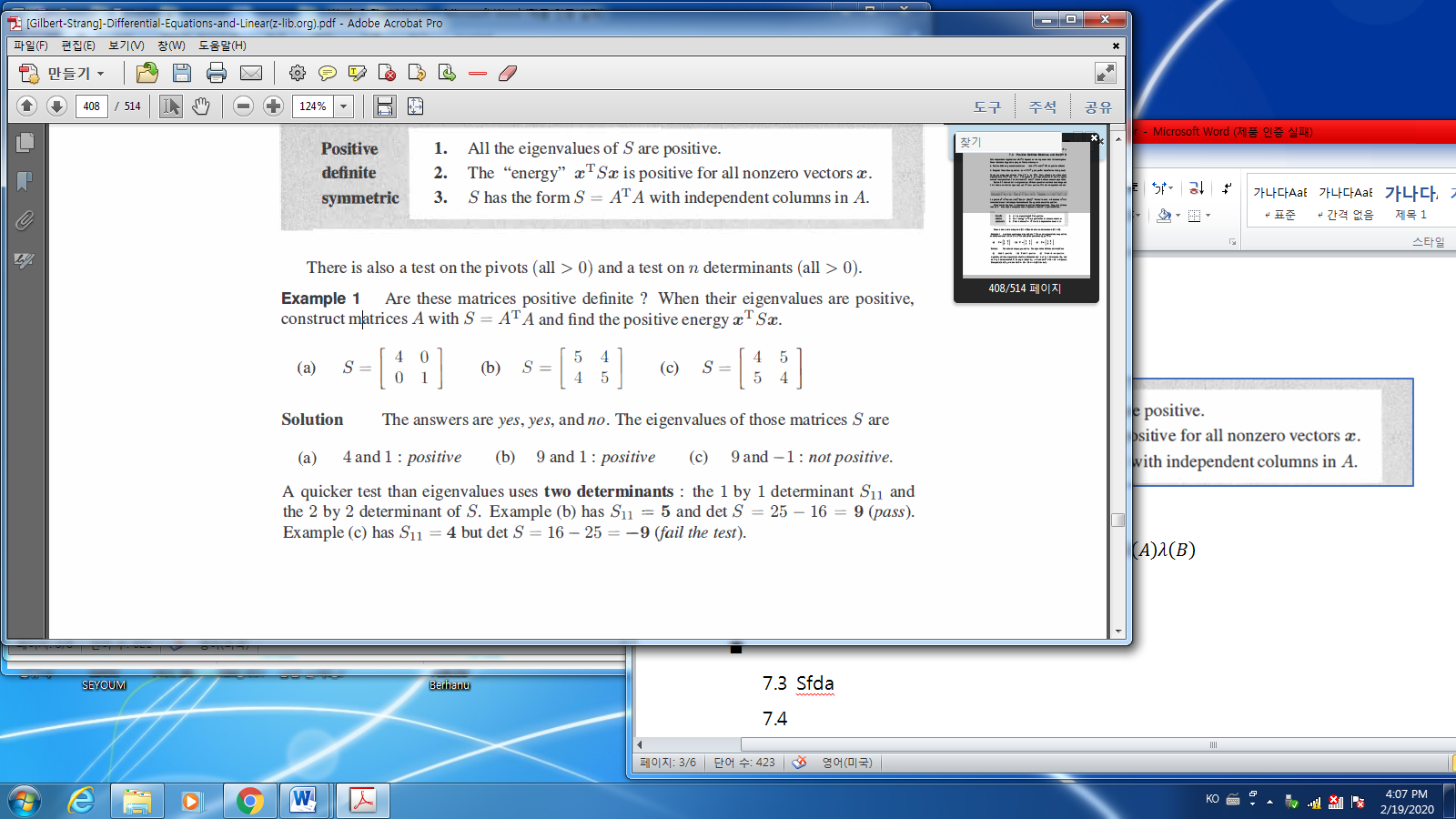
* Remember

1) but .

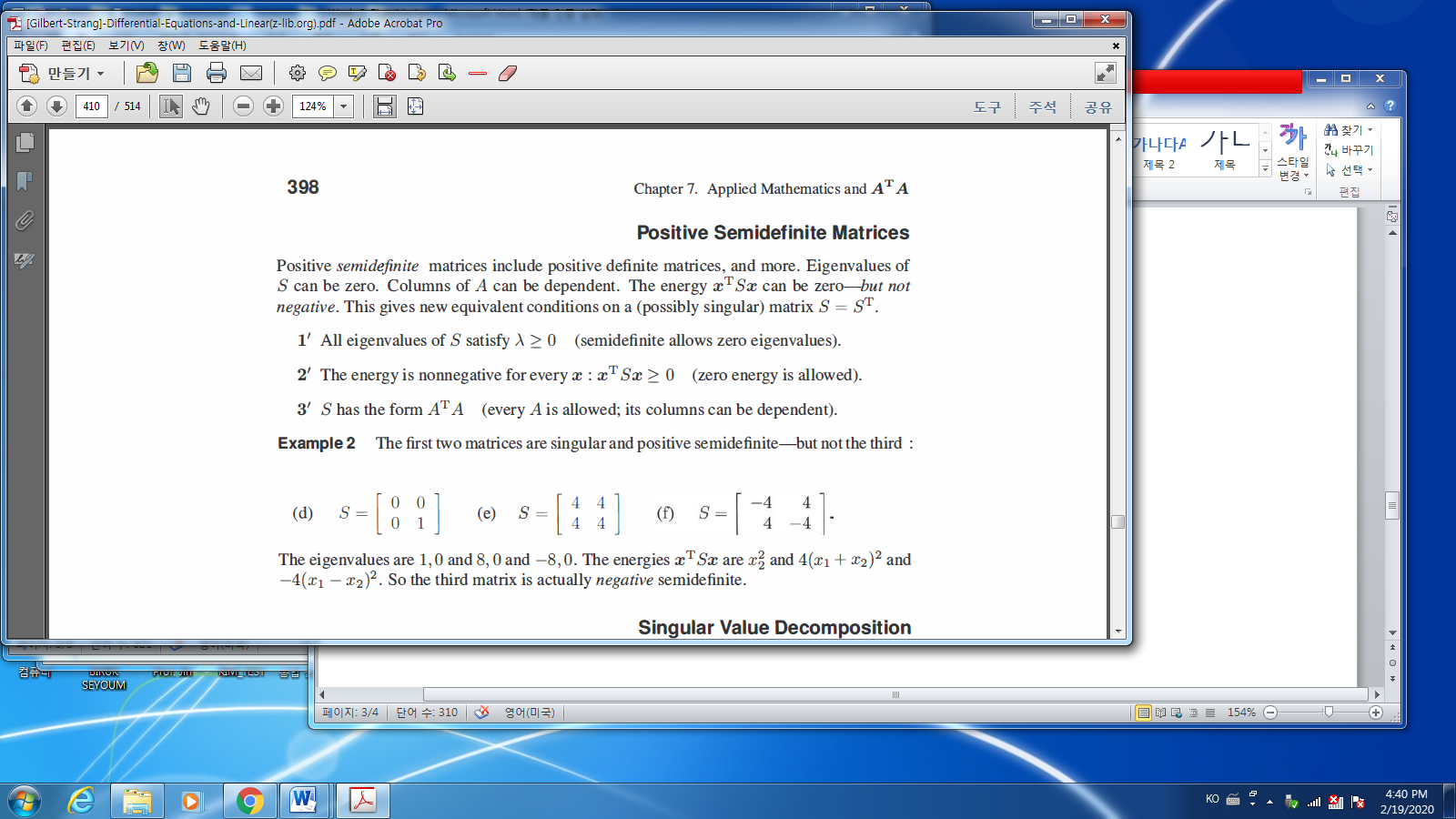
2) Let , Then

Let , Then .

* Example



* Definition: Positive Semidefinite Matrices



%%%%%%%%%%% Let us think one more e-values…

As you know, if is a **square matrix**, then it may be decomposed.

Let . Since e-vectors, which may be a Basis,

Hence

Now find a solution to a linear differential equation

Therefore

And

Together (a) and (b), is decomposed into

%%%%%%% At least square matrices, it is possible to be decomposed. How about not

square matrix? We may have many situations

🡪 since it is solvable

1. ,

since , it is solvable

1. 🡪 not solvable since

So, 3) is not solvable…What can we do best? One of the methods.

**For any matrix** , square or rectangular, e-vectors, can be decomposed as

Singular value decomposition.

* Definition: Singular Value Decomposition



* Let ,

where

Then (2) may be

* If

A

U

* Example
* Orthonomal vector is
* Orthogonality :
* Normalized:
* The sigma value is different from the eigenvalues as

* Even if is a square matrix, e-vectors may not be orthogonal.
* If eigenvalues are different from each other, then the eigenvector are

Independent to each other, but may not be orthogonal

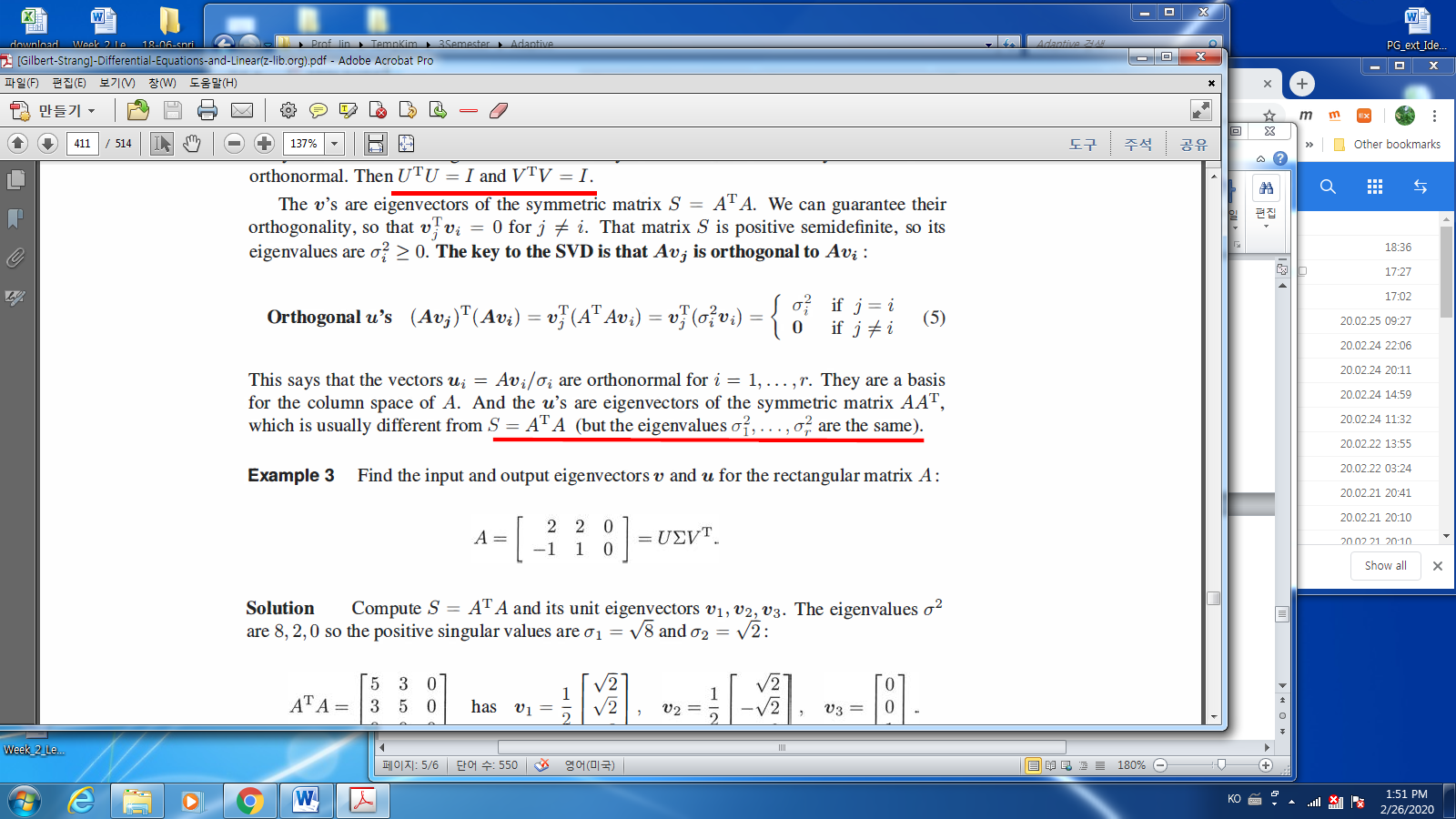
* **If is a symmetric matrix, then e-values are real and orthogonal(Ch.4)**

Hence if , then .

* **Our problem is for any matrix.**
* , there are many orthonormal bases. How about in .How can we find?

?

* Find



Since , are orthogonal,

By the assumption of are orthonormal,

By the assumption of are orthonormal, and normalized

Let the eigenvalues of , be , and assume the corresponding

eigenvector are

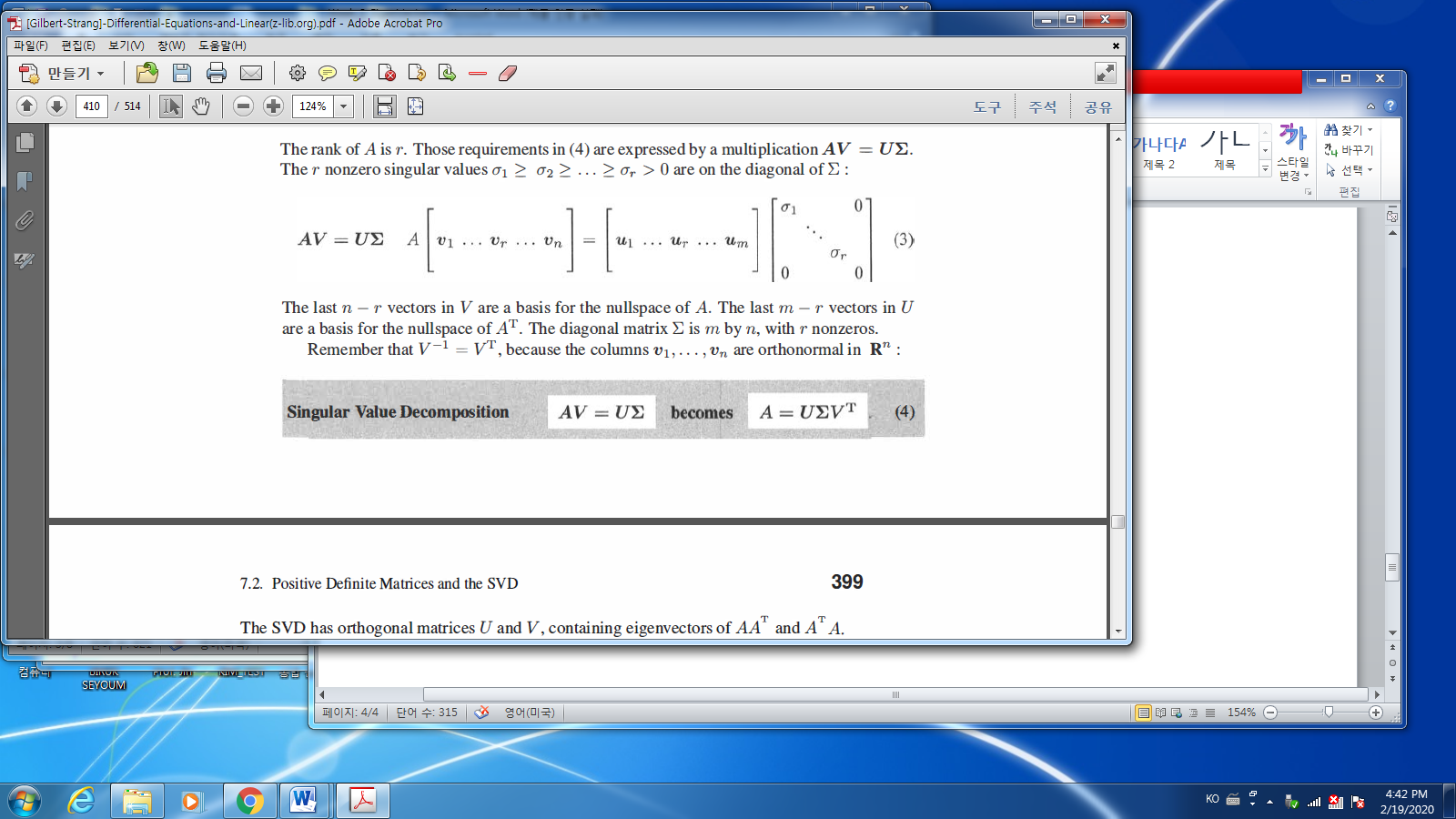
Then from (1)

Hence

Hence

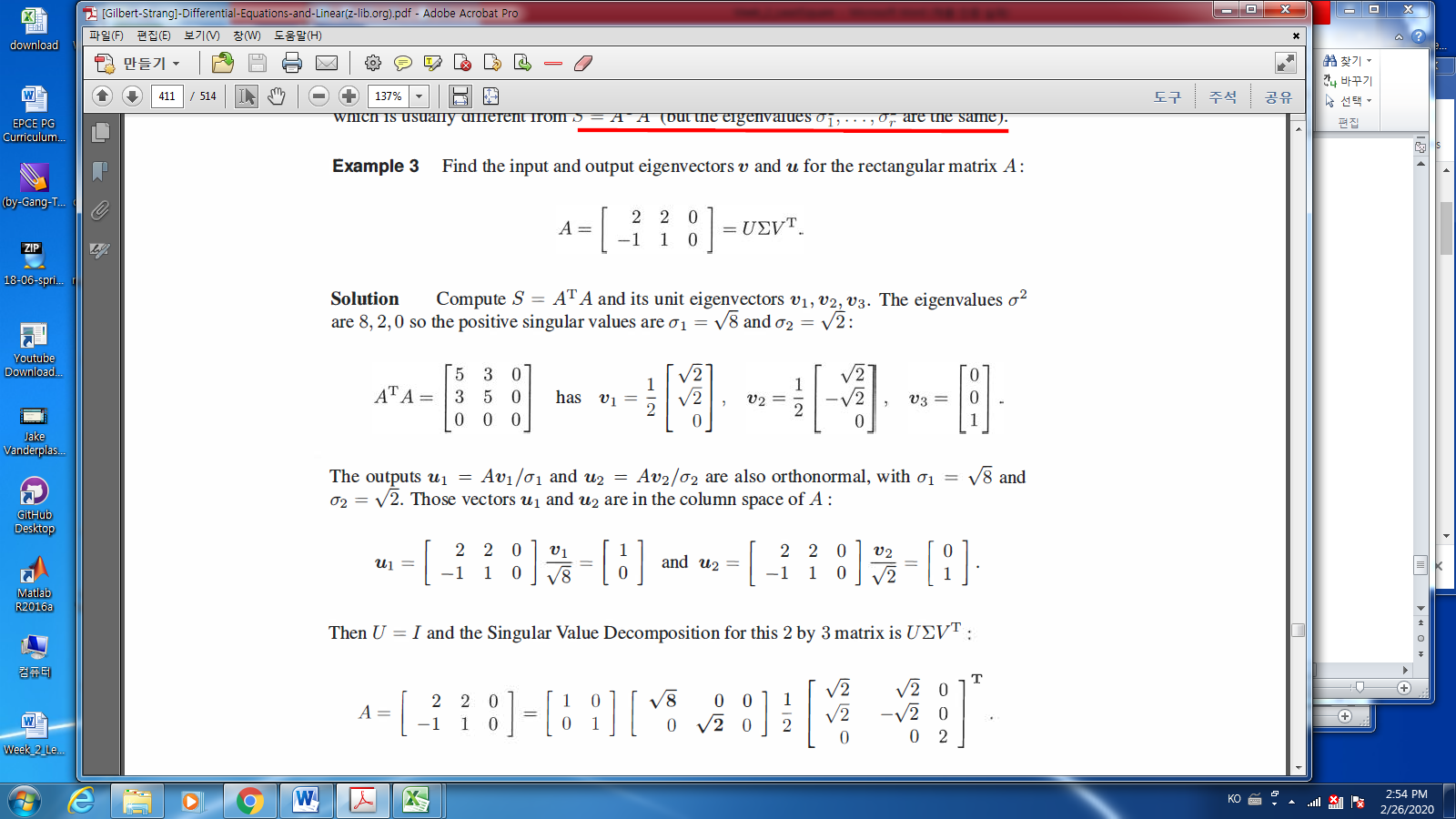
About , from the , if , then , and if then you may

find the others using “Gramm-Schmidt” until the number of is “n”.



* Fact:

, ,



* Kim’s Gram – Schmidt orthogonalization

<https://en.wikipedia.org/wiki/Gram%E2%80%93Schmidt_process>

Given a linearly independent set of vectors in

Find an orthogonal vectors such that

such that they are orthogonal, i.e.,

1. Find an orthogonal vectors

* Pick up any vector , define
* Calculate
* Calculate
* Repeat until

1. Normalize such as
   1. Boundary conditions replace initial conditions -skip
   2. Laplace’s Equation and -skip
   3. Networks and the Graph Laplacian - skip

%% The first exam. Candidate problems.

Ch.7.1 10/11 / 21 /22

Ch.7.2 3 / 4 / 12 / 27

Who is interested in Ch.7.2 problem 32~~