1. Frequency Response Method
   1. Empirical Transfer-function Identification
      1. Sine wave testing

* Motivation: Using sampled input of , and the output

, find . So In order to get “Bode-plot”, we need infinite

* + 1. Discrete Fourier Transform of Signals
* **The Discrete Fourier Transform(DFT)**

-. sampled at

where

%%% kim’s comment

Nomenclature: in (3.1), , which means is discrete! In general

Continuous and discrete is discriminate as

continuous angular frequency

discrete angular frequency

But in this textbook, is used in both case. Remember.

%%%%%%

-. For

Hence for a is the period of

* Question: what is the maximum frequency of (3.1)?
* Ex.3.1 Sine-wave signal

-. Given

where and

-. Find when

Hence

For simplification, using the following relationship:

so that

%%% Kim’s comment

Prove (3.2)

%%%%%%%%%%

* Periodogram: the plot of values of
  + 1. Empirical Transfer Function Estimate
* Empirical Transfer Function Estimate(ETFE) for Samples systems
* For LTI

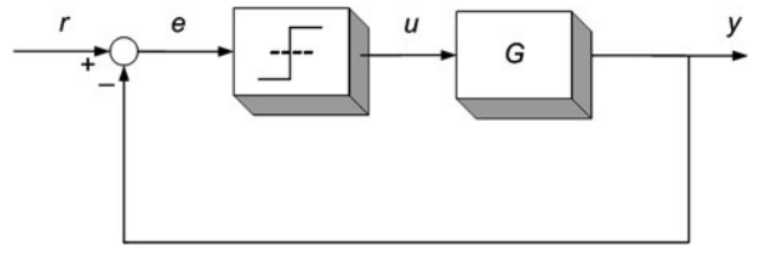
Now for a sampled system, , the estimate of is

* The empirical transfer function estimator
* Ex. 3.2 : Empirical Transfer Function Estimator : matlab test.

%%% Kim’s comment

If you have to estimate your plant, and if you do not want to apply special inputs(such as impulse, step, sine wave), **you may save your input and output (perhaps in a long time)**, then apply discrete fourier transform to estimate your plant. %%%%

* + 1. Critical point identification(Describing function method)
* In a system with a relay, which is not a linear component, hence the system is a non-linear system.
* Sometimes stable limit cycle oscillations
* What is the limit cycle? 🡪 we should learn in non-linear (control) system.



* Problem:

Assume a plant be a time delay linear or non-linear system. And assume without the input (the output may be self oscillate with a negative feedback. Now estimate of critical point of , in Nyquist plot

1. Method: Relay Identification
2. In the backward loop of , insert a relay with the amplitude
3. Increase the magnitude of h so that the error will be start to be oscillate with a fixed frequency
4. Measure the oscillation frequency the amplitude
5. Evaluate
6. Estimate the critical point
7. Analysis
8. The closed loop system is unstable at the critical point. Because the closed loop system characteristic equation is
9. So the system may be unstable with gain
10. In a real system, without the relay, in order to be stable, the proportional gain should be limited less than

%%% Kim’s Comment

1. You may use a proportional gain instead of the relay to calculate the critical point.
2. In real systems, time delay of the system is popular. Time delay systems are not stable

in strict sense. For example the denominator of the transfer function of a closed loop feedback with the time delay

Then for a fixed there are many poles. Is this stable? You may draw the Nyquist plot for this transfer function, and find the critical point due to the delay.

* 1. Discrete-time Transfer Function
     1. z-transform
* Discrete Fourier transfer function of a sampled system
* z-transform
* as a Laplace transform goes to 0 to ,

%%% Kim

Find the output of

* In frequency domain

Define

Hence

The z-transfer function is

Now the Taylor series of is

Using the formula

Then

In time domain

* In time domain analysis

So they are equivalent

%%%%%%%

* transfer function in domain

Then the z-transform of

%% Kim’s comment

Prove (3.14)

%%%%

* + 1. Impulse response identification using Input-output data

For t=0



In matrix form

Since , the matrix , for , is difficult to get it. But in general is converge to zero fast, so it may be possible to reduce the size of N

* Shift operator
  + 1. Discrete-time Delta operator - skip