Non-linear least square estimator

Since the formula is given as  
where looks a little bit complicate, so I am not sure it works correctly.

So I test a simple example without noisy measurement.

Consider the following known polynomial

Now, given some measurement, my model is

so that I want to find an estimator for using (1). Due to the non-linearity with respect to ,

we can not use the linear least square estimation procedure. Let us use (1)

1. Measurement data
2. Let us substitute these data into (3),

With the first measurement,

These are the solution to (3), it is not appropriate to my purpose for (1). Let me apply the (1)

1. Calculate the sensitivity matrix(or vector)

Since

yields to

So the sensitivity matrix is

1. Recursive up to
   1. Assume an initial point as

Find the next . First from(4)

And

Hence

* 1. With

Since , continue the next as

And

Hence

* 1. With

Since , continue the next as

And

Hence

* 1. With

Since , continue the next as

And

Hence

* 1. With

Since , continue the next and continue. But the error ~ 0!! Bingo!!

1. Wrap up Non-linear LSE

* Of course my model is noise free model, even if it is noisy, it may works.
* It is difficult to hand write to solve it. However Gauss solved the equation with more unknown parameters by hand. For example The stochastic text book Chapter 4, problem
* In matlab, lsqnonlin (least square non linear) function is provided to solve non linear LSE problem.
* But in non-linear, if the initial point is not good, or the number of data is not sufficient, it may not give the solution.