1. System Response Methods
2. Frequency Response Method
3. Correlation Method
   1. Correlation Functions
      1. Autocorrelation Function

* Time average of Autocorrelation Function
* Properties of auto correlations



Proof: from(4.3)

define

Then

If , it follows

Q.E.D.

1. If u(t) is periodic, then is periodic
   * 1. White Noise Sequence

* auto-correlation of Definition of white noise
  + 1. Cross-correlation Function
* Definition

As time average

In discrete case



%%% Kim’s comment: delay time estimation using cross correlation

1. Radar ranging system

where the transmitted signal, : the received signal. The range

transmission -- reflection -- reception

The distance between the radar and the object is . Then due to the round trip distance by wave,

which implies

So that the estimate of the time delay leads to estimate the distance

Now assume

Since , if , is the maximum of ,

Hence the estimate of is the maximum of with respect to

1. The received signal is not the delayed signal of the transmitted signal , so that

where is the transmission coefficient of the wave, still this is true unless the noise intensity is not significant.

1. Estimation of location of the wireless phone

Using the geometric method, if there are three station of wireless station whose position are already known, it the distance between the each station to the phone, the location may be estimated.

%%%%

%%% Kim’s comment

The cross correlation is similar but different to the convolution. For example given two signal

The convolution :

The cross correlation :

%%%%%%

* 1. Wiener-Hopf Relationship
     1. Wiener-Hopf Equation
* The in/out cross correlation

Hence

* + 1. Impulse Response Identification Using Wiener-Hopf Equation
* Exm. 4.3: **Impulse response identification**

For asymptotically stable systems, it suffices to determine only **the first elements of ,** so that

* Inputs, and output: , and .After removal of the initial conditions effect, , and
* In/out correlation matrix
* Remember the input is arbitrary. So it is similar as an **empirical method**.
* Also which is computational stable to get the inverse.
  + 1. Random Binary Sequences
* Motivation: Diagonal matrix of

If the input is selected as

Then is a diagonal matrix, which is simple to get as

The problem is how to select a signal as (4.2.3.a)

* Method 1: white noise

See (4.6). But not realizable since

- vary large input

- not genuine random noise.

* Method 2: Random Binary Sequence(RBS)
  + 1. Filter properties of Wiener-Hopf Relationship
* Motivation: Why Correlation method rather than pulse response identification?
* If u(t) is a pulse input, we may get .

But due to measurement noise, the output is corrupted by the noise.

Objective: To cancel the noise

* Method
* The correlation method may filter the output measurement. See the following.

Then

If is uncorrelated, , then

* Correlation method(Wiener-Hopf Relation) will filter out the measurement noise.
  1. Frequency Analysis Using Correlation Techniques
     1. Cross-correlation Between Input-output Sine Waves
* Motivation

The previous Method in frequency domain, sampled input of , a fixed the output is (which is nosy free)

If the output is noise,

To get the , hence from . measure and corresponding to , the noise effect should be cancelled out.

1. Let the input / output be

Multiply the output by the and , denote the result as

where sample time, N : # of sampling

1. Sin term

If , the second term is convergent to zero due to periodic nature, and has no component of the input frequency,

1. Cos term
   * 1. Transfer-function Estimate Using Correlation Techniques

From the previous calculations, the estimate of is

* Algorithm 4.2(textbook ch.5, page 42) to identify

1. Generate a specific frequency a sine wave
2. Apply this signal to the system
3. Determine, for N cycles of the output,
4. Calculate the magnitude and phase of
5. Repeat another

* Kim’s Comment on estimation phase using Quadrature Modulation.

The measured output is

The aim is to estimate the phase delay

1. Multiply with and to get
2. Since

The high pass filer may reject the second terms in each signals, so that

Then the phase will be

1. Multiplication of orthogonal signals is called “Quadrature Modulation”

* Kim’s comment on Estimate the flow velocity in the pipe

1. Problem definition

The velocity of flow :

The velocity of wave in air :

The distance between and :

1. Modeling

The transmit signal

The wave velocity in the pipe

The time delay of received signal is

The received signal

The phase delay due to the time delay

Hence if is estimated, then the flow velocity will be determined

1. To estimate the phase , Quadrature modulation will be applied.

Hence calculate

As the previous example, filtering the , it may possible to estimate the phase delay so that the flow velocity may be estimated.

* 1. Spectral Analysis
     1. Power Spectra
* Definition: Power(auto) spectrum / cross -spectrum
* Power(auto) spectrum = Fourier transform of auto correlation
* Properties
* Power spectrum:
* Nomenclature:

Co spectrum = Real part of

Quadrature spectrum = Imaginary part of

* Quadrature
* In signal processing: Let signal Then the quadrature signal of , is
  + 1. Transfer-function Estimate Using Power Spectra
* Transfer Function Relation

Hence

* If the output is corrupted by noise, then is corrupted by the noise spectrum. (Think of in the time domain case). Hence it is needed to cancel out the noise effect.
  + 1. (Skip) Bias-variance tradeoff in transfer-function Estimates