## Kalman Filter in Discrete Time Linear System

### 1. The scalar case

Let us consider to measure a water level of a tank in whcih the a constant water amount is added. Inside the tank a level senso can measure the water level of the tank.

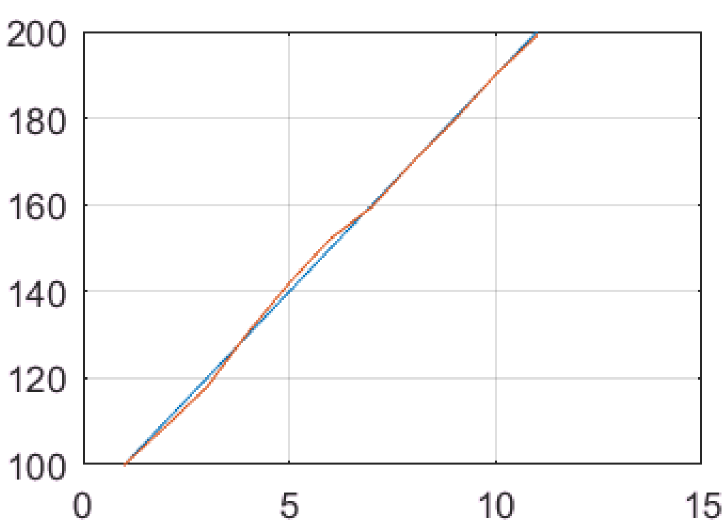
$$w_{k+1} = w_k + a$$
$$z_k = w_k + v$$

Here, $w_k, a, z_k, v$ are the present water level, the rate of inlet water level, the measured water level and the sensor gaussian noise $N(0, \sigma^2)$. One experiment is done such as the measurements are done 10 times as

```
clear all; clc;
% define parameters
a = 10;
sigma = 1;
N = 10;          % the duration if one experiment
figure('Position', [10 10 300 200])
%initialize
w(1)  = 100;     % initial water level
z(1)  = 100;     % assume the first measurement

%process
for k = 1:N
 w(k+1) = w(k)+a;
 z(k+1) = w(k+1)+ sigma*randn;
end

%plot
plot(w); grid on; hold on
plot(z)
hold off
```




Here the problem is to estimate **the inlet rate of the water only the measurement**. One method is first to find the difference of the water level and to average them within the time interval.

```
cSum = 0;   % cumulative sum of the difference between the previous
            % and present water level

for i = 1:N
    d=z(k+1) - z(k);
    cSum = cSum+d;
end
Ave = cSum/N  % one of the estimators
```

```
Ave = 9.0053
```

So are you happy? then it is good. However your sensor does not work properly so that the noise intensity is bigger than this example, you may not be satisfied.

2. the vector case

Now, we may have an additional information of the dynamics which is not used in the previous example properly. Ler us use the plant model to apply a kalman filter. The problem is to estimate of the water inlet $a$, hence we may introduce another state as $a_k$

$$w_{k+1} = w_k + a_k$$
$$a_{k+1} = a_k$$
$$z_k = w_k + v$$

which is in a standard form

$$x_{k+1} = \begin{bmatrix} w_{k+1} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_k \\ a_k \end{bmatrix} = \Phi x_k$$

and the corrupted measurement is

$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} w_k \\ a_k \end{bmatrix} + v_k$$

Problem

1. in the scalar case , do 10 times experiements( the previous example is one time experiment), and estimate the inlet water

2. In the scale case, if the intensity of the sensor noise is increasing or decreasing, analyze your estimator performance

3. In the vector case, we may assume

$$x_{init} = [10; 0]^T$$
$$M_{init} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

What is the best estimator of the inlet water (N=10)?