1. **HW Week\_10\_1**

Consider a non-linear differential equation

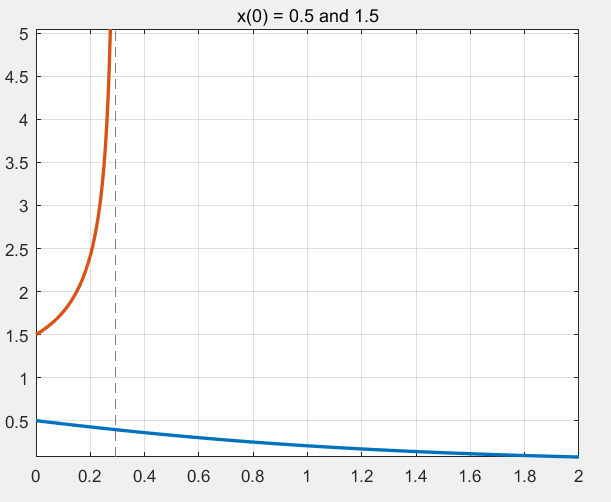
1. Find stationary points?

Sol: the stationary points are

which gives

1. At the initial point draw the trajectory.
2. At the initial point , draw the trajectory. %%%

Sol:



1. **HW\_Week\_10\_2**

Consider a random variable which is a non-linear transform of a gaussian R.V.

1. prove pdf of as (1) and find the mean and the variance if

%% Hint : to get the mean and variance, using symbolic math in matlab

**Sol** : the normal pdf is

Since

Since (<https://en.wikipedia.org/wiki/Leibniz_integral_rule> )

The probability density function of is

The mean and variance are using matlab,

Sol:

clear all

syms y

f = 1/(sqrt(2\*pi)) \* (1/sqrt(y)) \*exp(-y/2);

p = int(f, 0, inf); % probability should be "1" for all range

double(p);

fMean = int(y\*f,0, inf);

double(fMean)

fSq= int(y^2\*f,0,inf);

fVar = fSq - fMean^2;

double(fVar)

Hence the mean and the variance of y is 1 and 2.

1. To get the mean and the variance of y, use Monte Carlo with different number of simulation to see the result, i.e., n = 1000,5000, 50000.

**Sol:**

clear all; clc

N = [1000 5000 10000];

fMean=[ ];

fVar =[ ];

for i = 1:3

pts =randn(N(i),1);

f = pts.^2;

fMean=[fMean;mean(f)]

fVar = [fVar;var(f)]

end

Mean =[ 0.9722 1.0045 0.9926]

Variance = [1.7769 2.1518 1.8843]

1. Prove , Here and we may call this is ‘kurtosis” in the case of

**Sol: The problem is**

For notational simplicity let

Then

Now to integrate we may use several methods

**Method\_1( by some students, even if some notation are incorrect, wonderful !!)**

Define

Now we may differentiate w.r.t “a”

Again differentiate w.r.t

Hence

Since

The integral is

Since , substituting it gives

In conclusion

**Method\_2**

For notation simplicity, then

Now define

Then

Hence this form is in general

which can calculated by partial integral method.

**Method\_3**

Or I will use “symbolic math in matlab”

The code to calculate value of as

clear all; clc

syms x s

f = x^(4)\*exp(-(x^2)/(2\*s^2));

F =int(f,x) % improper integral

Then is

)

Since ,

So that

In conclusion

1. **HW\_Week\_10\_3**

Let , consider a non-linear transform

Find the mean and the variance of using

1. Linearized method

**Sol:** Since , linearize at . Then

Hence

1. Monte Carlo method(N = 100, 1000)

clear all; clc

N = [100 1000];

fMean =[ ];

fVar = [ ];

for i = 1:2

pts = randn(N(i),1);

f = cos(pts);

fMean = [fMean;mean(f)];

fVar = [fVar;var(f)];

end

fMean’

fVar’

Hence mean = [0.5433 0.6137], variance =[0.2563 0.1942]

1. Unscented transform method

**Sol:** the number of sigma points is 3, and

Hence using unscented transform,

Since is a Gaussian, ,

For the covariance the unscented covariance general formula is

Here

Hence

In conclusion

|  |  |  |  |
| --- | --- | --- | --- |
|  | Linearized | Monte Carlo | Unscented |
| mean | 1 | 0.6137 | 0.6131 |
| variance | 0 | 0.1942 | 0.1996 |

------The End -------