1. Monte Carlo

For non-linear filtering, Extended KF is to linearize the no-linear transform to get a linear form so that Linear Kalman Filter algorithm will be adopted. Unscented KF is no need to linearization ( i.e., the first derivatives of the non-linear transform is need), however UKF is need to define additional sigma points to get the estimation thru the non-linear transform.

Now Particle filter defines the sigma points as many as possible compared to UKF( whose number of sigma points is . So in particle filter, as named suggested, the number of sigma points is far large i.e., 100, 1000,… as you want.

* 1. Basic concept
* Monte Carlo Integration

1. Calculate the integral

Consider an integral

If is factorized in such a way that interpreted as a probability density satisfying . Then

where , are independent R.V. here is an unbiased estimator to .

* Example.

Here we may use several methods in matlab.

*clear all;clc*

*LW ='LineWidth';*

*h = @(x) exp(sin(x))*

*x = linspace(-2,5,100);*

*plot(h(x),LW,2); grid on*

*title('plot using anonymous function')*

*integral(h, 0,1)*

In matlab the integral value is 1.6319. We may calculate the integral totally different way.. Monte Carlo method. Here

*N = 2000;*

*pts = rand(N,1);*

*plot(pts,'+')*

*f = exp(sin(pts));*

*sum = 0;*

*for i = 1:N*

*sum = sum + f(i);*

*end*

*sum/N*

Here the value is 1.6304. Of course the value is not equivalent to each other, however, The Monte is simple. Do you believe it? Try a simple one as

* Estimation

Here to estimate the value of . Define a circle with a radius of 1, and bound it in a square. The side of the square has length 2, so the area is 4. We generate a set of uniformly distributed random points within the box, and count how many fall inside the circle. The area of the circle is computed as the area of the box times the ratio of the points inside the circle vs. the total number of points. Finally we know that , so compute

*clear all; clc*

*N = 2000;*

*pts = -1+ 2\*rand(N,2);*

*scatter(pts(:,1), pts(:,2))*

*cts =0;*

*for i = 1:N*

*if pts(i,1).^2 + pts(i,2).^2 <=1*

*cts = cts+1;*

*end*

*end*

*% 4:(pi)R^2 = N : cts*

*cts\*4/N*

1. particle filter

* Concept

Here the non-linear random process

At some fixed time, we may generate random particles to represent a state.

`

X(t)

See a realization of the state. At a fixed time, each particle which is randomly generated, is governed by the process model, i.e.,

Hence the simulated trajectories are projected to next step,

We may not know the mean of the projected at the next step, however, if we measure at the next time step, as which is realized at a specific value, we may guess which projected particles are more likely. i.e.,

at time , we may

* The particle filter

Problem: The system and the measurement equations are

here, are independent with known pdf’s. Find

1. Procedure
2. Assume that the pdf of the initial state is known, randomly generate initial particles on the basis of the

Denote as

1. For
2. Perform the time propagating step to obtain a priori particles
3. Compute the relative likelihood of each particle conditioned on a specific measurement . For example, if

Then the likelihood

And normalize as a probability

1. Generate a set of a “posteriori “ particles on the basis of the relative likelihoods :It is called as **Resampling step.**

For “Resampling” a simple method is introduced as **Sequential Importance Sampling.** With this , we may get sample mean and variance and so on

1. The next step as a prediction

At generate uniform sample points, particles

1. Repeat
2. Some terminology

2.1 Various estimator

* Least Square estimator

Cost function

Solution:

Here the LSE does not depend on the probabilistic characteristic about

* Maximum Likelihood Estimator

Cost function:

Here the MLE does not depend on the probabilistic characteristic about And

MLE is an extension of the LSE considering of the noise characteristics

* The minimum mean variance estimator

Given measurements as

The minimum mean variance estimator is

Here MMVE is dependent of the noise characteristics of

1. Sequential Importance Sampling

Here Sequential Importance Sampling is to find a new particles weighting different with probability, for new particles for ,

1. Generate a random number that is uniformly distributed on [0, 1]
2. To find a corresponding “j”, Accumulate the likelihood’s into a sum, one at a time, until the accumulated sum is greater than r. That is

If “j” is found, then

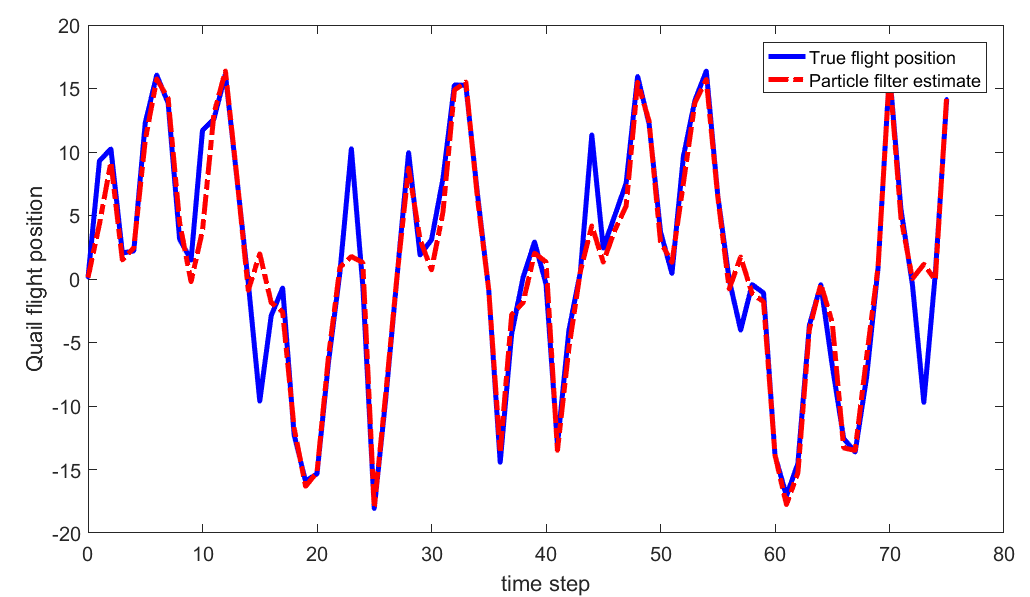
Therefore, if priori is more probable, the new particles is assigned to that more frequently, and vice versa.

Example

Given a scalar system

If we guess the initial point as , and the number of particles is 100,

then using particle filter,



**Discretization of continuous time system**

1. Why discretization
2. Physics was modeled in continuous modeling, however, in present computer period, everything in computer is in discrete. However, as you see, almost all formular related to physics are continuous modeled. What is velocity? What is capacitance in capacitor? And so on.
3. The controller in the continuous system is generally implemented in by OP-amp, which is corrupted by the noise inherently. Nowadays the controllers are realized by computer which is discrete. The discrete system is more flexible, less noise corrupted and so on.
4. You may see data, which is saved, are all discrete. It is impossible to save the continuous data. If you get data, to fertilize it, the corresponding system should be discretized.
5. Discretization for linear systems
   1. Problems

Given a linear dynamic system as

Find the solution in

* 1. Solution

multiplying by at both sides yields to

Rearranging it

Since the above is

Integrating w.r.t.

Hence

Which is

If the input is constant between sampling instants, i.e., , then the above equation is

In conclusion

, to the discrete system

where

* 1. Example

The equivalent D.S. is

And

Where

which leads to

In conclusion

1. Stochastic Linear System
   1. Model

Here is a white noise(white stochastic process)., such that

%% kim’s comment: The expectation of time delayed two random process is called as a auto-correlation function, denoted as . %%%

%%% Kim’s comment

The delta function is defined as

And its time shift formula is

%%%

* 1. Discretization

The solution is

* 1. Stochastic Integral -🡪 more formally we will see

Define a random variable as

so that

3.3.2 mean of

3.3.3 Covariance

Since

So that

In conclusion, given a continuous system

A discretization is

Where

Where

* 1. Example

where

Then

With

%% Reference

[1]”Optimal State Estimation”, example 15.1