1. Conditional Expectations and Discrete-Time Kalman Filtering

* Introduction
* Model: the measurement is modeled as the sum of two 2 RVs.
* Known fact
* Unknown fact (You may have to calculate )

* Given a measure of , What is an estimator of For example
* Kim: Example

If you measure your temperature, there are two R.V. Let two random variables,: temperature, measurement sensor noise.



You measured yours as .Which is the best estimator of your temperature?

1. The simple one may be the estimator is . So how to get ?

If you assume they are Gaussians, . Or you may statistical average.

Is this the best?

1. The answer is no. The best one is

How to get this? Later we will show if they are Gaussian, then

**Where this is the BASIC, FUNDAMENTAL LAW…**OK… Then the problem is how to find if they are not Gaussian. It is Example 3.8. It is not simple…



1. The **minimum variance estimator**
   1. Some textbooks, it is called as the **minimum mean square error**

<https://en.wikipedia.org/wiki/Minimum_mean_square_error#:~:targetText=In%20statistics%20and%20signal%20processing,values%20of%20a%20dependent%20variable.>

* 1. Identification : the **least (mean) square error**

1. The maximum a posteriori estimator

Bayesian probability

* : = the posteriori PDF
* : = the priori PDF
* : the likelihood

%% Kim

The posterior : after measurement, estimate the source

The priori : before measurement

%%%%

* 1. Minimum Variance Estimation
* Problem statement – static parameter estimation

where .

* The minimum variance estimator
* Remark 3.1.argmin
* (Skip): Define convex distance function

Is a non-negative and convex distance function.

-. Convex function: a function is convex if for any points, and some scalar such that

-. Example of a convex function

* (Skip): Define a loss function , such that

-.

-.

(Skip) Theorem 3.2(Sherman’s Theorem). Let x be a random vector with mean, , and density,. Let be a loss function as defined above. If . Is symmetric about and unimodal(i.e., has only one peak), then minimizes .

* Without observation, the minimum variance estimator.

Sol:

* Remark Sherman’s theorem is generalized of the above estimator.

%%%% Kim’s comment

Minimum variance estimator:

Minimum variance (conditional) estimator:

* There are two MVE. But is the MVE also, the expectation.
* Usually MVE is the conditional minimum variance estimator

And in Linear algebra

Least Square solution = least square error estimator:

* Solution : not to be confused with MVE

%%%

* Theorem 3.6. Given the equation(3.1). **if the estimate is a function of , then the minimum variance estimate is the conditional mean.**

(Skip) Proof: let be an estimator of ( that means given is a constant).

Then for

Since the cross term is

.

Then

Since are independent of and is positive semidefinite,

1. is equivalent to

* Remarks:

1. It means
2. It is **unbiased**

%% Kim, comment

It is important that

For example,

* The solution is

: which is a R.V. dependent of the initial point R.V.

What is MVE? Yap

Now what is

%%%%

**The following example by M.Idan should be carefully understood. The main goal is to verify**

%%% kim’s comment

Let tow random variables which are independent with pdf as Define

Find pdf of Z

Solution:

Now

dy

=

* This integral is called the convolution integral.

Examp.

Define Find

f

1. For 0, i.e., since (a.1), f ,
2. For , i.e., from (a.1)
3. For , from (a.1)
4. For from (a.1)
5. In conclusion

X

Y

z = x+y

Or you may plot then calculated.

f

1. Plot each pdf
2. Change the graph one of them

g(-x+z)

g(-x)

g(x)

-1+z z

1

-1

1. Increase from negative infinity to positive infinity to integrate the convolution integral

-1+z z

g(-x+z)

f(x)

1. If , there is no overlapped portion, so that f
2. If

There is overlapped portion so that

f

1. If , there is a overlapped portion so that

f

1. If If , there is no overlapped portion so that

f

-1+z z

f(x)

g(-x+z)

-1+z z

g(-x+z)

f(x)

%%%%%%

* **Ex.3.8: conditional mean and variance of the sum of two RV’s**

**In tutorial We well consider it.**