* 1. Conditional Estimate of a Gaussian Random Vector with Additive Gaussian Noise
* Modelling
* %% I will notify means is independent.
* Problem: find

To get the solution, we need

1. Since are gaussians, is a gaussian
2. , and are correlated.
3. How to find given ?

* Solution

**where**

* The minimum variance estimator of

= the conditional expectation given

~ mean of x + (scale) (measurement – mean of x)

* Uncertainty: if the variance of a RV is large, the uncertainty is large.

%%% Kim’s comment : Scaler case

In the scalar case, everything is a scalar not a vector or a matrix. So it may help to understand the basic concept of the (3.14), which is called as a Kalman filter.

First of all, the conditional covariance of

* How to find ?

-first find : He use a trick. Define a new RV as

-second find : This is simple. As you know

* Tip: (3.5) Matrix inverse

Verify the inverse in (3.5) 🡺

And 🡨 You may try

* Tip.
  + 1. (Skip) Simplification of the Argument of the Exponential
    2. (Skip) Simplification of the Coefficient of the Exponential

%%% Kim’s comment:

Given a measurement R.V.’s which is the sum of two independent Gaussian R.V.’

1. The is a Gaussian as
2. The conditional mean of given is a Gaussian as

where

And the conditional variance of given

%%% Kim’s comment:

The filter (3.14) is called a Kalman filter in case of batch process. What is the batch process? Well

First of all we may understand the (3.14), then after that we may think about the batch process.

* Scalar case (3.14)

1. If the noise is very small, i.e.,

* The MV is only dependent of the measurement

1. If the noise is very large, i.e.,

* The MV is only dependent of the state regardless of the measurement %%%%
  + 1. Processing Measurements Sequentially

%%% Kim’s comment : Batch process / real time process (sequential measurement)

* Difference between batch/ sequential process

Example: a random process as

Find the average of

1. Batch Process
2. Recursive process

Define

Then

1. Merits for recursive way

* The memory size is lower than the batch type
* The result can be acquired at every step, which is more informative

1. The static Kalman (3.14) is a batch process. In dynamic system, it is more efficient to calculate the Kalman gain at the sample time.

* Extension of (3.2): Sequentially measuring of
* **Goal:** find sequential the conditional mean of given the sequential measurements

Where in (3.32) means corresponding to in (3.13) indexed n.

* Solution

With the sequential measurements, construct it as a batch model

Since

Hence

Substituting (3.21) and (3.20) into (3.14) gives

Multiplying (3.22) through by gives

where a new variable **defined** as

And **define** additional terms as

We get

For iteration calculation, define new variables at measurement

And



* Iteration with at time step

Multiplying by then



Or

* Important:

1. With the last measurement , the conditional mean of can be calculated by (3.14) or (3.32)



1. To get the , in the batch process you need more memory compared to the real time process
2. In general, .Hence we need recursively using (3.28)
3. Kalman gain:
   * 1. Statistical Independence(orthogonal) of the Error and the Estimate

From (3.32) at every measurement, there is a correction between the new measurement and the previous estimator .

Let us define the error

%%% Kim’s comment

By definition two R.V’s are independent if

Hence if they are independent, then

Now by definition two R.V.’s are orthogonal in the mean sense is

Hence in the text book,

does not imply the independence.

However, in linear algebra, two vectors are orthogonal in Linear algebra

If their dot product is zero, i.e.,

And if two vectors are orthogonal, then they are independent.

1. Ex. : orthogonal and independent
2. are independent but not orthogonal.



* 1. Maximum likelihood estimator

(Skip-But Later will review this) Maximum likelihood estimator

* 1. **The Discrete-Time Kalman Filter: Conditional Mean Estimator**
* Problem: A dynamic system given as

1. We measure
2. The conditional estimator (the MV estimator) is
3. The propagated estimator is

* *The propagated estimator:*

= a Priori Estimator / a Predicted estimator

= measurements are previous compared to a posteriori estimator







* *In general text book notations are as follows*
* The Estimator
* The Predictor 🡪 the upper accents are different ~~

1. Propagating the conditional mean = a priori = prediction

* Remember , not

1. Propagating the variance of the a priori
2. Updating the Conditional Mean: a posteriori

* Remember not

1. Updating the covariance: a posteriori

* Define: **Residual 🡪 innovation sequence**

From (3.44)

* Define Kalman Gain: the residual gain :

Hence (3.44.1) is

* Orthogonality Properties of the Conditional Mean Estimator

1. The Innovation Sequence is independent for each

Since, the mean of ,

And,

with these facts, therefore the covariance is

which implies the gaussian are independent.

1. Error is orthogonal to ,

* Example 3.9 (estimating the speed of a car)
* Problem: estimate traveled distance by car.

1. The speed of car = 55 mi/hr, for 1 hour, implies the distance is 55 mi
2. The trip meter shown 55.3 mi

What is the best estimated distance with two information?

* Method

1. The system is modelled as
2. We may guess
3. The variance by the measurement: the quantization of the trip meter is = 0.1(mi) 🡪 The variance of the trip meter ,
4. The variance by the process: the velocity varies +/- 1 (mi/hr), the variance of the process
5. The best estimator of the distance in mean square sense,
6. Comments:  
   - the variance of a uniform RV in (a,b) is

-. Compare