1. Linear estimator

The following signal is a vector in the space of square integrable, periodic signals on the interval [-1,1] as

Let’s define a subspace spanned by the vectors with

* 1. Find an orthonormal basis of

**Sol:** By Gramm-Schmidt procedure

1. Pick up and normalize to get
2. Calculate next orthogonal vector selecting

Normalizing it to give

1. Calculate next orthogonal vector selecting

For normalizing, the norm of is

which leads to , is

* 1. Find the least square estimator   in S of

**Sol:**

The coefficients to the orthonormal basis are

Hence the best estimator   (linear least square estimator) in subspace S is

%% Is it strange since not -1. However the error is define as

%%%

1. Regression – Linear and nonlinear

With the measurements

2.1) If the governing equation is following, find the least square estimator

**Sol:** Substituting the date to get a standard Linear least square form is

Hence

2.2) If the governing equation is following, find the least square estimator using Newton-Gauss method

**Sol**:

1. First to get the Jacobian;
2. For initial guess the measurement error is

Hence .

You are very luck since due to the best selection for the initial estimator !!

1. Consider a non-linear transform of a Gaussian random variable

2.1) Find the mean of

**Sol:**

2.2) Find the variance of i.e.,

Since

And

Calculating it each term is,

Summarizing it leads to

1. Kalman Filter

Consider the following random process

If   and , find

**Sol(2021\_Week\_6\_Chapter3\_control):** First the prediction step,

Hence for before measurement,

After measurement,

Hence at

Then the estimator at  

1. Kalman- non-linear (notations are same to that of materials)

Consider the following nonlinear stochastic process,

If   and , using EKF, find the estimator

**Sol(2021\_Week\_9\_EKF\_UT**):The nonlinear transform is only in the process model as

First the prediction step, the jacobian is

Here

Hence for before measurement,

After measurement,

Hence at

Then the estimator at  

-The END -