ASTU. EPCE . PG Mid\_term PCE6302 . Date Apr.28.2022

1. Given the cost function as

Find the to minimize using

* 1. Gradient method (assume the tuning parameter , the initial point =(5,5)

**Sol**: The minimum value is,

The gradient of is

Hence

Here is matlab

clear all; clc

alpha = 0.5;

x=5;y=5;

N = [x;y];

for i = 1:10

x= N(1); y = N(2);

G =[4\*(x-20)^3 ; 4\*(y-10)^3]

N =N-alpha\*G

end

This method is not stabilized

* 1. Newton’s method (assume the initial point = (5,5)

**Sol:** The Hessian of F is

Hence

Matlab code is

clear all; clc

x=5;y=5;

N = [x;y];

for i = 1:10

x= N(1); y = N(2);

G =[4\*(x-20)^3 ; 4\*(y-10)^3];

H =[12\*(x-20)^2 0;0 12\*(y-10)^2];

N =[x;y]-inv(H)\*G

end

This method gives the optimal solution

1. Given a linear dynamic system (Ex.2.1-2. Page 25)

Consider two cases of whether the final state is fixed or free.

For the case of fixed final state be . And the cost as

Then, the minimum cost with the optimal controller is given as

For the case of free final state

Then, the minimum cost with the optimal controller is given as

2.1) Prove that the minimum cost (b.2) is less than the cost (a)

**Sol:**

Since

And

2.2) In case of the free final state, on which condition, the final state will be convergent to the final reference value , i.e., .

**Sol:**

The optimal trajectory of the fixed final state is

And the optimal of the free final state is

Hence if , i.e., , (b) = (a),

1. (LQ: Continuous system) Consider the following system and the cost function
   1. Find the optimal controller

**Sol:**

The Riccati equation is

gives

Dividing by

The solution is with the optimality condition

Hence the optimal controller is

* 1. If , what is the closed loop pole?

**Sol:** ,the closed loop pole is

* 1. If , what is the closed loop pole?

**Sol:**

* 1. If , what is the closed loop pole?

Sol:

1. (Bang-Bang Control)

Let the dynamic system as

And the input is bounded as

The control objective is to bring the states from any initial point ) to the origin in the minimum time , i.e.,

* 1. Define Hamiltonian

**Sol**: The index for the minimum time may be

Hence the Hamiltonian is

Using the Pontryagin’s Principle, the optimal controller is

* 1. Assume M=1. Let the initial point as , Draw the optimal trajectory in the phase plane

**Sol**: Since , the optimal trajectories in the phase plane are

Eliminating the time variable, then

The switching curve is with

So that

Substituting the initial point value,

which is a parabola trajectory. Here, the intersection at the axis, at

And at the ,

Hence

2 ,0)

)

* 1. If , i.e., if there is no constraint on the magnitude of the input, Draw the asymptotic optimal trajectory in the phase plane

**Sol:**

When ,the intersection points at the axis are asymptotically as

And at the axis, the asymptotical points are

Hence as , the minimum time trajectory is

)

%% Strange? I think so… This case is a bang-bang type of not only controller but also the trajectories. Mathematically, it is understandable, since to travel from another point in minimum time, the maximum control effort is needed. And to force the speed to be zero is done in advance to position to be zero.

1. Following system and the cost function

Consider Table 8.1-1(page 303). If the output matrix is the identity matrix, , verify the optimal controller, , is equivalent to the steady state optimal controller at page 155

**Sol:**

In table 8.1-1,and

Substitute then

The above equation is the same to the steady state ARE of so that .

-QED-