* Ex. 3.2-3 : temperature control in a Room

1. Problem formulation

The temperature in the room and the ambient

the rate of heat supply to the room

Define

Then

And the index

Here design the optimal controller to minimize the index such the at the fixed time,the room temperature will be a fixed temperature or free.

1. Hamiltonian
2. The necessary conditions

3.1) first

3.2) second

3.3) together 3.1) and 3.2) solve

Using Laplace transform,

so that

%%%

%%%

1. The final boundary conditions

Still is unknown.

4.1) Fixed final state

First, this is the case of the final state fixed, so that the weighting factor for the final state in the index is zero, so that the index will be

Now from (14)

solving it

Hence the optimal costate

and the optimal control(the optimal heat rate supply to the room)

and the optimal state is

4.2) Free Final state

Remember the index is

The necessary condition at the final time

with (17), solving for ,

From (11)

so that

the optimal trajectory is from (3),

* HW\_Week\_3.3

From (3), (27) prove (29)

4.3) Discussion :

from (29) is not however if

* Ex. 3.3-1 (Zero input) : Propagation of Cost for uncontrolled Scalar system

1. Problem
2. Lyapunov Eq.(Here no Hamiltonian!)

The solution:

so that

So the cost will be

%%% Kim’s comment:

To solve (3), which is backward, define so the changed parameter, (3) will be forward. %%

1. The steady state cost

If from (4) ,

* Ex.3.3-3 (fixed final point) Open-loop Control of Motion Obeying Newton’s law

1. Problem
2. From Hamiltonian and necessary conditions

Then

and

1. Boundary conditions

Since is given,

here the reachability (controllability) Grammian, is the solution of Lyapunov equation

Hence

which yields

Hence

The sate transition matrix is

So the optimal controller is

or

1. Discussion

The optimal control is not dependent of and open loop control. **More control is required to move the system more quickly from one state to another.**

* **Ex. 3.3-5 (free final state) Optimal feedback control of a damped harmonic oscillator**

1. Problem

And

1. Hamiltonian and necessary conditions
2. Boundary conditions

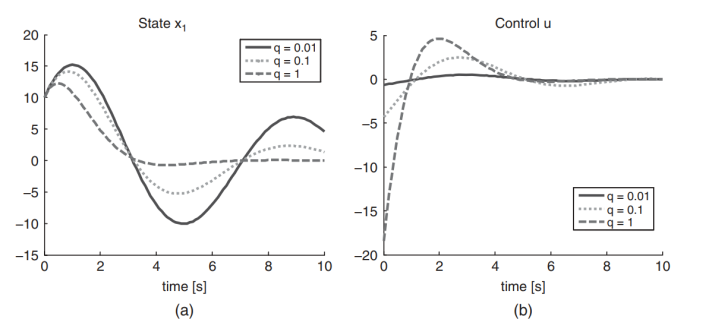
Since , assume satisfies the Riccati equation

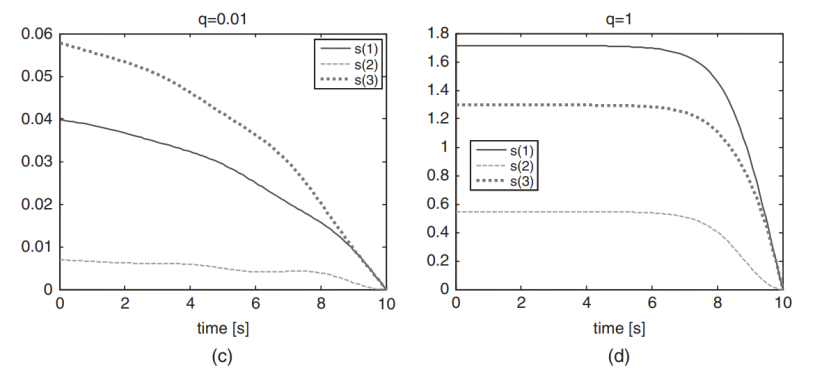
Define

Simplifying the Riccati equation is

The optimal feedback gain is

See the Riccati equation, It is a nonlinear differential equation. How to get the solution? Maybe numerically. See p151, matlab code.





Here an example in the textbook.

If increasing , is almost constant in except near the boundary.

* HW\_Week\_3.4

Run the program to get the graph in the above

%%% The END %%