4. The tracking Problem and Other LQR extensions

In this chapter, the optimal controller to minimize the performance index in CT and DT system are considered

1. The controlled states follow a known trajectory
2. The controlled states satisfy the fixed final value but different from the performance index that of Ch.3
   1. The tracking problem

* Non-linear Systems

1. Problem

* Plant
* Objectives: The plant state (it may be output),tracks the desired reference trajectory over closely.
* One of the performance index is

%%% Kim’s comment

In chapter 3, the final time fixed case, the weighting matrix %%%

1. Hamiltonian Necessary conditions

State equation:

Costate equation:

Stationary condition:

1. Boundary conditions:

given

%%% Kim’s comment :

In Ch.3, (3.2-10), there is no constraint on the final state, i.e., ,

Hence

%%%

1. Optimal controller

* Linear Quadratic Systems – Tracking

1. Problem :

and (it may be output),tracks the desired reference trajectory over closely

1. Necessary conditions:

Hamiltonian is

The Hamiltonian matrix is

1. Optimal affine control

So the optimal trajectory is

1. Minimum cost

where

%%% Kim’s comment

* Derive the optimal controller law using sweep method

From the necessary condition (4.1-11)

And the boundary condition at the final time

Now we may assume

Then from the stationary condition (4.1-12) and the state equation (4.1-10)

Now from the assumption (a), differentiate at both sides

Rearrange it

Since this satisfy for all ,

so that the Riccati Equation with the final value

And

Let us think of the assumption. Since Hamiltonian differential equation is difficult to solve it, Somebody(?) may assume (a).Then he got the solution. !! the method is called as the sweep method to get rid of the various attempts to solve it. “Sweep” the attempts.

%%%

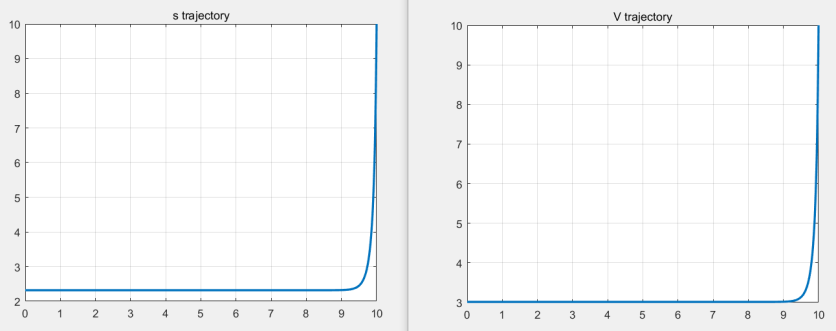
%%% kim’s comment ;

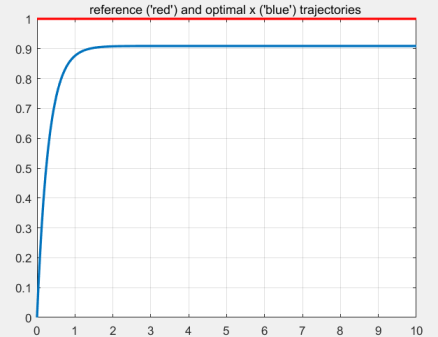
* In the tracking to the reference trajectory , in (4.1-4) the additional control is needed as “affine feedback”
* The Hamiltonian is different from that without the reference tracking case as (3.3-8)
* The closed loop state (4.1-14) has two components. One of them is to stabilize the state, the other is to track the reference trajectory. The affine controller has eigenvalues of the negative that of the closed loop state.
* There are three backward equations as, . Let . The solution is to stabilize the state. The auxiliary trajectory is for the state to track the reference trajectory , the cost auxiliary %%%
* Ex.41.1-1 Scalar

If









1. Suboptimal Tracker

Here, the suboptimal means the final time , the closed loop feedback gain may be constant.

Hence if is reachable(i.e., controllable) and is observable, the Riccati equation has the steady state solution , which leads to constant .

The tracker (the auxiliary )

* 1. Regulator with function on a final state fixed
* Problem and Solutions : Table 4.2-1 : function of the Final-state-fixed LQ Regulator

1. Problem

* System Model
* Performance Index

with additional constraint of fixed final states

1. Necessary conditions

Hamiltonian

1. Optimal control law

%%% Kim’s comment

* Derivation the optimal control using “Sweep Method”
* Here in table 4.2-1, and are all matrixes.
* Here , if , it is same to the free-final state OC( optimal control) so that the optimal controller is due to
* If , it is the same to the fixed final state OC, **however, in this case the state feedback not the open loop in chp.3 !**

1. Hamiltonian :
2. Necessary conditions
3. Boundary conditions : due to the fixed final state,

From the necessary condition at the boundary (3.2-10), where,  **is an additional costate** owing to the final state constraint

Here

So that

3.1) Sweep method

1. Find

Then from the stationary condition

Substitute it in the state equation,

Since the derivative of the costate with the assumption is

With the necessary state equation

Combining it with the necessary costate equation

results in

Rearrange it

In two equations one is the Riccati equation is

with the given . The other auxiliary equation is

If define

with the final value from 3.1)

1. Find

Since

we need to find , which is the additional costate related to the final condition

From the final conditions

We may define at t= T

Differentiate at the both sides

Since

So that we have two equation should be satisfied

And

From (b.1)

implies

Transpose at both sides

Which implies

Hence from (b.2)

And from (b)

Now the optimal controller is

QED %%%

* 1. Second-order variations in the … (Skip)