**Chapter 9.4 The Observer and Kalman Filgter**

**The model may be written in engineering textbook,** as

Here the White Noise as

Some notations as

The estimator for , and its error between state and the estimator is

Then the error dynamic is

Then the mean of the error should be

And the covariance of the error is

From (L9.4-46)

Here the first term is straight forward, but the second term is a little bit complicate. In this “optimal textbook”,

And

%%% Kim’s comment:

In the Ito differential equation,

Then for

Here is the last term is which is equivalent to (9.4-59)

%%%

Therefore using (9.4-59)

* Deviation of Kalman filter

1. Problem

The plant dynamics

Where

Define the estimator as

and the corresponding error ,

Then the error covariance in the steady state is the solution to

1. Optimal estimator

Define the cost as

subject to (L9.4-63).

1. Hamiltonian

Define Hamiltonian as with a matric Lagrange Multiplier

* The Necessary conditions

From the last equation the optimal observer gain

Plug it into the first equation is

Or

Which is the Riccati equation equivalent to that from the ITO stochastic differential for