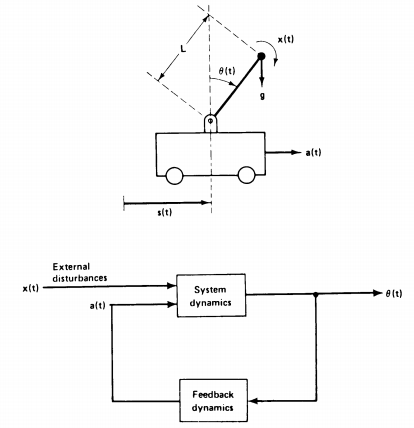
Engineering Model

1. Inverted Pendulum

* In the literature, there are several kinds of modeling of an inverted pendulum, which depends on its application.
* The pendulum is mounted on the moving cart, their dynamic models ae different.
* Hence to stabilize the unstable pendulum , the controllers are different (totally different)

1. The first example
   1. The physical model [1] -



* 1. Mathematical model

The torque by the gravity, , is

The torque by the acceleration is

The torque by the external disturbance x, is

The total torque equation is

which is linearized as

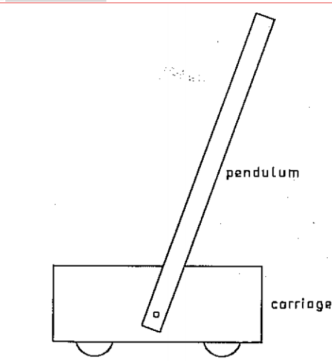
The transfer function from the disturbance to the angle

i.e.,

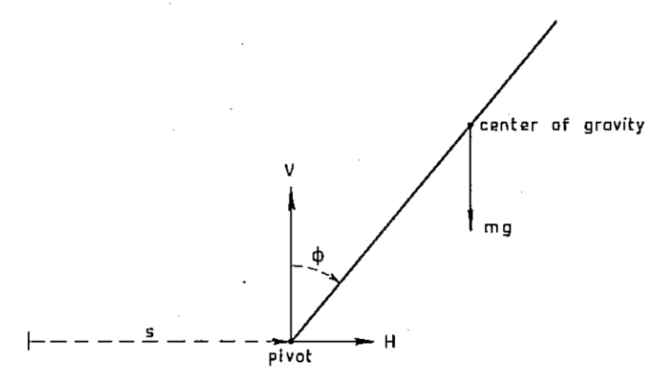
* 1. The stabilizing controller is suggested as PD
  2. Realization



1. On the cart Inverted pendulum on the cart.
   1. Mathematical Model



* 1. mathematical model



1. Terminology

The horizontal force

The vertical force

The external force

the friction coefficient of the cart in the horizon direction

the friction coefficient of the pendulum in the rotational direction

1. Mathematical model
   1. For the horizontal force

And the dynamics is

Plugging it into gives

Arranging it

%% Here are two activate forces, One is due to the slanted pendulum , the other is the external force .. make sense? %%%

* 1. For the vertical force

so that

Arranging it with respect to the vertical force

And its dynamics

Together with the vertical and horizontal forces,

Arranging it gives

1. Combining two equations

Substitute (2) into (1)

Arrange in terms with

So that

And with the same way

1. Comparison with others
   1. Sivan, Kwaterrnaak -

Here the assumption is the cart movement does not depend on the pendulum’s movement , i.e.,

Is it reasonable?

* 1. Brunton “ Data-Driven..”

)/

)

Here the inertia of the pendulum is smaller than i.e..

But the rod inertia is . (and the denominator term ‘s should be

1. Controller design - Linearization
   1. Controller design

The inverted pendulum is the common problem to design a stabilizing controller at the unstable stationary point, i.e., to keep the pendulum at the up position, which is unstable stationary point. In non-linear system, it is difficult to design stabilizing controller whose system is unstable.

1. Lyapupov method: there are several papers to design a non-linear controller based on Lyapunov method.
2. Time optimal / energy minimization optimal controller (?)
3. Linearization
   1. Linearization
4. Plant
5. Linearization at the up-position

Here , then,

And

Then the approximated system may be, the translational motion,

The rotational motion may be

1. The controller design

Two eq’s (1)’ and (2)’ are linear w.r.t. . So there are plenty of linear controllers to stabilize an unstable system.Good~~

There are another problem… as you know, how many sensors to stabilize it? In our problem, there is 4th ODE, so 4 sensors are enough. Too many isn’t it?