EPCE 6205 1st PG Final Name

1. Consider the measurement is the sum of two gaussian random variables as

$$z = x + y\ , \qquad E[x] = E[y] = 1,\ \Sigma = E[(x\ y)^T(x\ y)] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

To design the conditional estimator of *x* given the $z = 1$,answer the following questions

1.1 Find the transform matrix *P* and $a, b$ such that

$$\begin{bmatrix} x \\ y \end{bmatrix} = P \begin{bmatrix} x' \\ y' \end{bmatrix},\quad P^{-1}\Sigma P = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

1.2 Using the transformed $(x', y'), z$ is expressed into

$$z = \mathrm{c}x' + \mathrm{d}y'$$

which $(x', y')$ are independent. Find the variance of *z*

1.3 Given the measurement $z = 3$, find the conditional estimator of $x'$

1.4 Using the transform matrix *P* , find the conditional estimator of *x*

2. (Non-linear MMSE) : Estimate $(a, b)$ with the governing equation as

$z = \dfrac{a\,b}{(a + \mathrm{b}t)}$, and the measurement data $(1, 1.5), (2, 1.2), (3, 1)$. Using Newton-Gauss method,

assume the initial estimator as $(\widehat{a_0}, \widehat{b_0}) = (1, 1)$. Answer the followings

2.1 Find the Jacobian at $(\widehat{a_0}, \widehat{b_0})$

2.2 Find the next estimator of $(\widehat{a_1}, \widehat{b_1})$

2.3 Find the Jacobian at $(\widehat{a_1}, \widehat{b_1})$

2.4 Find the next estimator of $(\widehat{a_2}, \widehat{b_2})$

3.(Brownian Motion)

Let us define $\beta_t$ be a Brownian, $\beta_t \sim N(0, \sigma^2 t)$. Answer the followings

3.1 Find $E[(\beta_{t_1} - \beta_{t_2})^2]$ if $t_1 < t_2$

3.2 Find $E[(\beta_{t_1} - \beta_{t_2})(\beta_{t_3} - \beta_{t_4})]$ if $t_1 < t_2 < t_3 < t_4$

4.(EKF). Design the EKF

Consider

$\dot{x} = x - x^3 + w,\ \ E[x(0)] = 1,\ E[x(0)^2] = \ 0,\ E[w^2] = \ 1\ \delta(t)$
$z = x^2 + v, E[v^2] = 1\delta\ (t)$

Construct a EKF at $x = 1$

4.1 Linearize the system at $x = 1$

4.2 Construct the Riccati equation $P(t)$ and define the initial point

4.3 Construct the EKF with the solution $P(t)$ to the Riccati equation

4.4 If the steady state solution exists, find the optimal constant observer gain.

If the steady state does not exit, explain why?

5. Non-linear transformed R.V.

Consider a R.V. $x \sim N(1,4)$. A non-linear transformed function

$y = f(x)\ = -x + x^2$

Answer the questions to find the mean and variance of *y*

5.1 Linearize *y* at $x = 0$, find the mean and the variance of *y*

5.2 Linearize *y* at $x = 1$, find the mean and variance *y*

5.3 Introduce a R.V. $\delta x \sim (0,4)$ so that $x = E[x] + \delta x = 1 + \delta x$. Find the mean and variance *y*

5.4 Using Unscented trasnform, find the mean and variance *y*(assume the tunning parameter $\lambda = 2$)

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