1. Continuous time Gauss-Markov Systems: Continuous Time Kalman Filter, Stationarity, Power Spectral Density and the Wiener Filter
   1. The continuous Time Kalman Filter

* Model

Where

%%% Kim’s comment on the model

1. In the mean square sense
2. The measurement in the mean square sense

* The innovation process

where **the estimator**, the estimator error as

* **The conditional estimator**

where is the solution to the **Riccati** equation as (6.12)

%%% Kim’s comment

(6.10) will be

* In continuous time filter, there is no separation as Prediction and estimation.
* The problem is to find the **optimal estimator(optimal filter , optimal observer)** such that

The optimal solution is , which is given in (6.10)

* In discrete system, it may allowed since the time interval between the sampling is finite,the difference of the Brownian motion is defined.
* The Riccati equation is a non-linear differential equation, and in control engineering, it is important for another optimal problem.
* What is another implication of ? This is the minimum mean variance estimator(or observer) , as

**%%%**

* Steady state solution to the Riccati equation

The S.S. of (6.12) is , so that

* Theorem 6.4: then the real parts of eigenvalues of have negative real parts, i.e.,

is asymptotically stable.

6.3 Stationarity

* Definition 6.7. a R.P. is second-order stationary ot wide-sense stationary if

And its correlation(auto) function

Its cross correlation function as

%%% Kim’s comment

Since two independent variables are into one variable as the difference between two variables, in general we may denote for notational simplicity,

%%%

%% Kim’s comment;

1. Brownian RP, which is not stationary
2. White noise , due to their independency %%%

6.4 Power Spectral Densities

6.4.1 Fourier Transforms

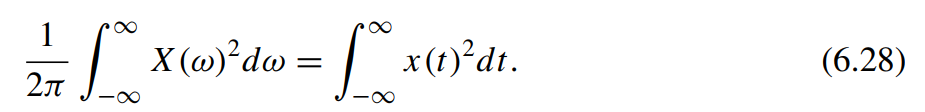
* Fourier transforms

Where

%% Kim’s comment:

There are several definitions on Fourier transform. Now

* Parseval’s theorem



6.4.2 Fourier Analysis Applied to RPs

* Energy and Power

1. Power signal :
2. Energy signal

however

* Energy density in frequency domain

Define the energy density in frequency domain as

Then

%%% Kim’s comment

If the signal has its Fourier Transform , then the energy in ] is

%%%

* Power density

The power density in the frequency domain is

%% Kim’s comment : The energy of power in the frequency domain are called as

The energy spectral density and the power spectral density.

%%%

**Skip the reminder section…**