1. Conditional Expectations and Discrete Kalman Filter
   1. Minimum Variance Estimation
   2. Conditional Estimate of a Gaussian random vector with Additive Gaussian Noise

* Review : batch model

Model:

%% I will notify means is independent.

Find :

Solution:

Here

Remark.1:

: conditional covariance given by . It is derived by

Do not confuse with : the covariance of

Remark. 2

* The **MV estimator** in (3.14) is .

Also due to the “iterated expectation rule”, it is **unbiased,** i.e.,

* The conditional covariance does **not depend on the measurement .**
* (3.14) is called **“Kalman filter”.** The weighting on the difference between the measurement and the average of the to be estimated is called **“Kalman Gain”**
* **Review – recursive model**

1. **Before measurement**
2. After Measurement :
3. Before measurement
4. After Measurement :
5. Before measurement; ….. continue..
   1. Maximum likelihood estimator

Consider measurement z which is the sum of two R.V.s,

find estimator such that

(Skip-But Later will review this) Maximum likelihood estimator

* 1. **The Discrete-Time Kalman Filter: Conditional Mean Estimator**

**Comment: this is the original Kalman problem, and the most popular estimation problem. Between the sampling time interval , the state is moving by its own dynamics, such as drift.**

* Problem: A dynamic system given as

1. We measure
2. The conditional estimator (the MV estimator) is
3. The prediction the state before measurement

* *general text book notations are as follows*
* The Estimator
* The Predictor 🡪 the upper accents are different ~~

1. Propagating the conditional mean = a priori = prediction

* Remember , not

1. Propagating the variance of the a prior
2. Updating the Conditional Mean: a posterior

* Remember not

1. Updating the covariance: a posteriori

* Define: **Residual 🡪 innovation sequence**

From (3.44)

* Define Kalman Gain: the residual gain :

Hence (3.44.1) is

* Orthogonality Properties of the Conditional Mean Estimator

1. The Innovation Sequence is independent for each

Since, the mean of ,

And,

with these facts, therefore the covariance is

which implies the gaussian are independent.

1. Error is orthogonal to ,