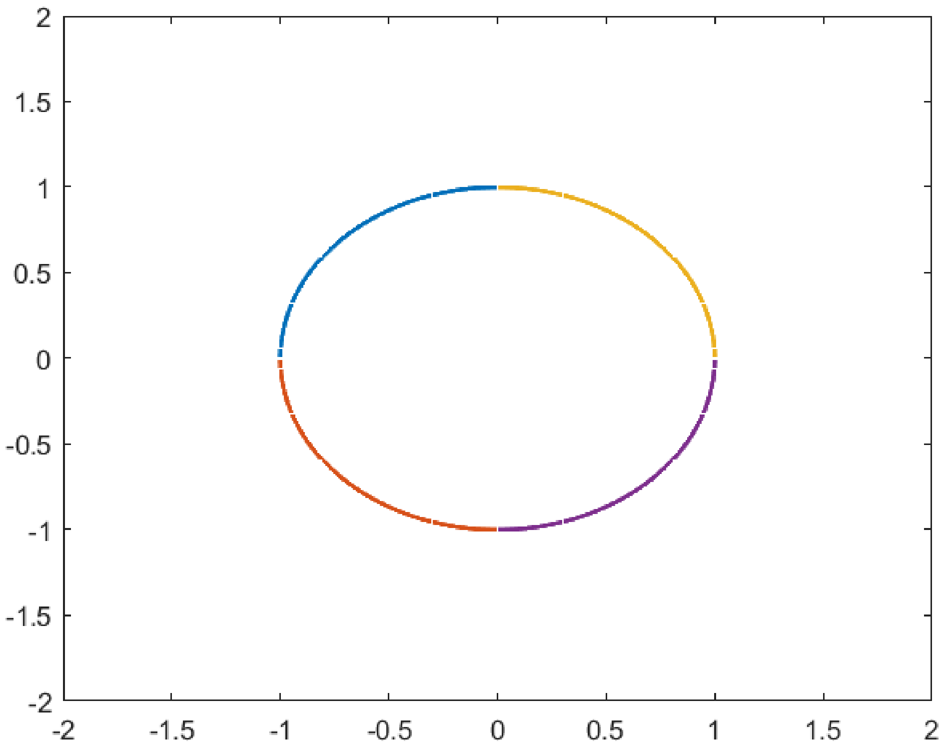
level curve

```
clear all;
f = chebfun2(@(x,y)  x^2 + y^2,'vectorize', 'on');
r = roots(f-1);
plot(r,'linewidth',2)
axis([-2 2 -2 2]);
```



Nyquist Criteria

$T = \frac{G}{1+G}$

# of clockwise encircled the (-1,0) = # of poles of $(1+G(s))$ - # of poles of $G(s)$

Absolute Stability

- Theorem 7.9

Consider a SISO system

$\dot{x} = \mathrm{Ax} + \mathrm{Bu}, y = \mathrm{Cx} + \mathrm{Du}, u = -\psi(t, y)$

where $\{A, B, C, D\}$ is a minimal realization of $G(s)$ and $\psi \in [\alpha, \beta]$. Then the system is absolutely stable if

1) If $0 < \alpha < \beta$, the Nyquist plot of $G(s)$ does not enter the disk $D(\alpha, \beta)$ and encircles it p times in the ccw direction, where p is the number of poles of $G(s)$ with positive real parts.

2) If $0 = \alpha < \beta$, $G(s)$ is Hurwitz and the Nyquist plot of $G(s)$ lies to the right of the vertical line $\mathrm{Re}[s] = -\frac{1}{\beta}$

3) If $\alpha < 0 < \beta$, $G(s)$ is Hurwitz and the Nyquist plot of $G(s)$ lies in the interior of the disk $D(\alpha, \beta)$

-The EnD-

- Some comments:

1) $G(s)$ is positive real if it is Hurwitz and $G(s) + G(-s)^T$ is positve.

2) The Absolute problem is applied to this case.

3) The problem regarding Absolute stability is

Given $G(s)$, find the sector of non-linear (time-varying) memoryless component to ensure the absolute stability.

- Ex. 7.12 Consider

$$G(s) = \frac{24}{(s+1)(s+2)(s+3)}$$

Find the sector of $\psi \in [\alpha, \beta]$, i.e., α, β to guarantee the absolute stability of the closed loop.

1) First investigate $G(s)$

- Is it Hurwitz? yes --> In Theorem 7.9, all three cases are applicable,Here case 3 is investigated.

2) Draw the Nyquist

```
clear variables;
LW ='linewidth';
num = [24];
den=poly([-1 -2 -3]);
sys = tf(num,den);
nyquist(sys); hold on
```

3) Find an appropriate conditions to the specific problem.

3.1) Let us assume to find a sector with α< 0

Now the largest sector difference $\frac{1}{\beta}-\frac{1}{\alpha}$, i.e., the smallest circle to encicle the Nyquist plot. One candidate is the circle at the center of the origin with the radius of the maximum magnitude of the Nyquist plot. Here in Chap.6 ,

$$\|x\|_{L_\infty} = \max\|x\|$$

so that in matlab

```
[ninf, fpeak] = norm(sys,Inf)
```

```
ninf = 4
fpeak = 0
```

gives the maximum magnitude of the Nyquist to 4.

```
s = chebfun(@(s) s,[0 2*pi]);
f = 4*exp(1i*s);
plot(f,LW,2, '-b')
axis equal
```

Hence the sector for the absolute stability is

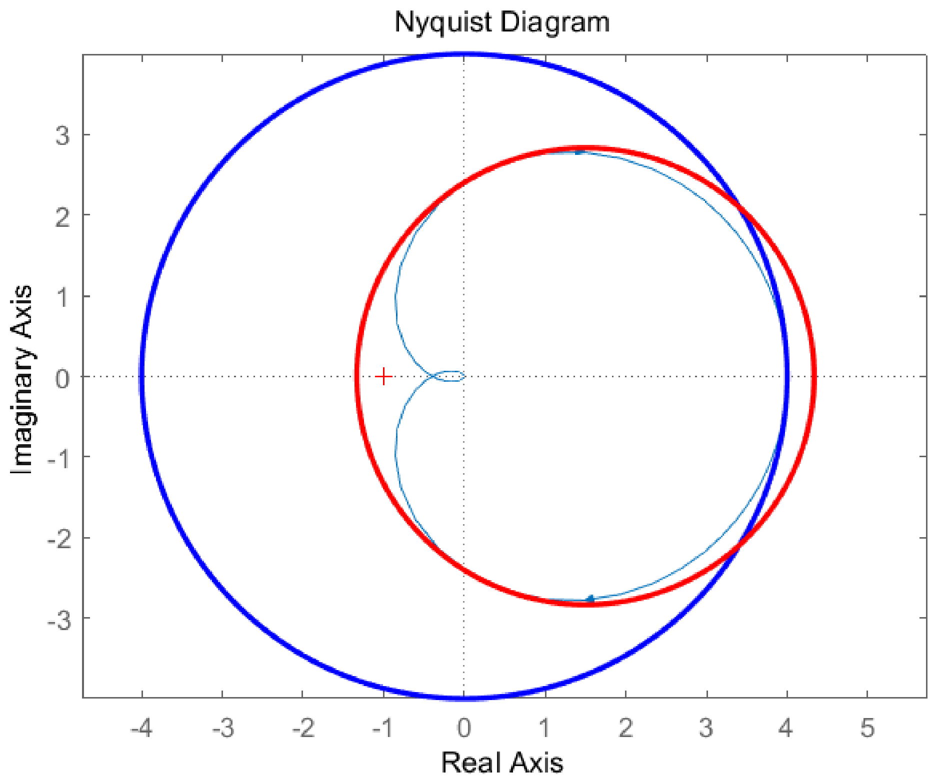
$$\psi \in \left[-\frac{1}{4},\frac{1}{4}\right]$$

Or we may choose a circle at the center of 1.5+j0 with radius 2.834,

```
[ninf, fpeak] = norm(sys,Inf)
```

```
ninf = 4
fpeak = 0
```

```
s = chebfun(@(s) s,[0 2*pi]);
g = 1.5+2.834*exp(1i*s);
plot(g,LW,2, 'r')
axis equal
```



In this case the sector is

$$\psi \in \left[\frac{1}{(-2.834-1.5)}, \frac{1}{(2.834-1.5)}\right] \approx \left[-\frac{1}{4.4}, \frac{1}{1.4}\right]$$

HW._7.1

Let us assume to find a sector with , $0 < \alpha < \beta$,

Find α, β such the the closed loop to be absolute stable

HW._7.2

Let us assume to find a sector with , $0 = \alpha < \beta$,

Find β such the the closed loop to be absolute stable.