Passive system

$y=h(t,u)$ is passive if $u^T y\geq 0$. Moreover

- $u^T y=0$: lossless

- $u^T y\geq u^T\phi(u) > 0$, $\forall u\neq 0$: input strictly passive

- $u^T y\geq y^T\rho(u)>0, \forall y\neq 0$: output strictrly passive

1) Memory : Here , $y=h(t,u)$ is a **memoryless system**. such as $y=\mathrm{au}, y=\sin u , y=\mathrm{t}\sin u$.

However

$$y=\int_0^t u(\tau)d\tau\,,\; y=\frac{\mathrm{du}}{\mathrm{dt}}\;\text{ or }\quad \dot{x}=-x+u, y=x\,,\text{or }\dot{x}=f(x,u)\,,y=x$$

or

$$y=h(u)\,,\; h(u)=\begin{cases}0 & -1\leq u\leq 1\\ 1 & \text{otherwise}\end{cases}$$

are memory systems.

2) Regarding $u^T\phi(u)$, here $\phi(u)$ belongs to the sector $[0,\infty)$

3) **Some examples for passivity**

$-y=\mathrm{ku}\,,\,k>0$: passive and input strictly passive and output strictly passive

$-y=k\,u^2$, it is **not passive**

In memory system (state space), Passivity

In the above definition, the passivity is defined over input and output. Now it may extended to include **memory systems .**

- Passivity in the state space

$$\dot{x}=f(x,u), y=h(x,u) \qquad (5.6)$$

The system (5.6) is **passive** if , $V(x)$ continuously differentiable **positive semidefinite function** (here called as the **storage function)** s.t.

$$\mathrm{u}^{\mathrm{T}}y\geq\;\dot{V}=\frac{\partial V}{\partial x}f(x,u),\quad\forall\left(x,u\right)$$

Moreover

-Loseless if $\mathrm{u}^{\mathrm{T}}y\;=\;\dot{V}$

-Input strictly passive if $u^T y \geq \dot{V} + u^T \varphi(u) > 0,\ and\ u^T \varphi(u) > 0,\ \forall u \neq 0$

-Output strictly passive if $u^T y \geq \dot{V} + y^T \rho(y) > 0$, $y^T \rho(y) > 0$

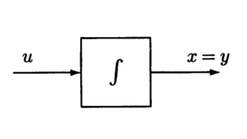
-**Strictly passive** if $u^T y \geq \dot{V} + \varphi(x)$, $\varphi(x) > 0$, PDF -End -

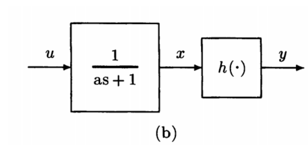
%%% Kim's comment

1) first of all $V(x)$ is a semi- definite positive function

2) in the output strictly passive, $\exists\, \rho(y), \rho > 0$ --> **not all states x** $\rho(x) > 0$ %%%

3) some examples for passivity with memory components





1) Integrator

$\dot{x} = u$, $y = x$

Select $V = \frac{1}{2}x^2$ --> $\dot{V} = x\dot{x} = \mathrm{yu} \dashrightarrow u^T y = \dot{V}$ --> passive and lossless

2) stable linear followed by memoryless

$\mathrm{a}\dot{x} = -x + u$, $y = h(x)$, $\mathrm{x}h(x) > 0 \quad x \neq 0$

Select $V = a\int_0^x h(\sigma)\, d\sigma$, ,

$\dot{V} = \mathrm{a}h(x)\left(\frac{1}{a}\right)(-x + u) = -x\,h(x) + \mathrm{u}h(x) = -\mathrm{x}h(x) + \mathrm{uy}$ --> $\mathrm{uy} = \dot{V} + \mathrm{x}h(x) \geq \dot{V}$

Hence It is strictly passive.

3) integrator with feedforward memoryless

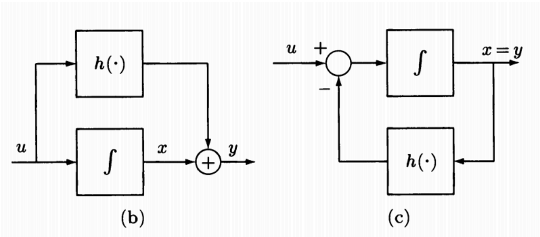


Figure 5.7: Example 5.2

Then the state model is

$\dot{x} = u, y = x + h(u)$

Select $V = \frac{1}{2}x^2$, $\dot{V} = \mathrm{xu} \; - \rightarrow \mathrm{uy} = u(x + h(u)) = \dot{V} + \mathrm{uh}(u)$

--> if $\mathrm{uh}(u) > 0$ --> input strictly passive

4) Integrator with memoryless negative feedback

The S.S. model is

$\dot{x} = -h(x) + u, y = x$

Select $V = \frac{1}{2}x^2$, $\dot{V} = x(-h(x) + u) = -yh(y) + \mathrm{uy} \; - \rightarrow \mathrm{uy} = \dot{V} + \mathrm{yh}(y) \; - \rightarrow \text{output strictly positive}$

HW_5.1 (similar to ex.5.4) Consider

$\dot{x}_1 = x_2, \dot{x}_2 = -x_1 + x_1^2 - 2x_2 + u, y = x_2 + u \;\; , t \in [0 \;\; 10]$

1) plot a simulink for this system

2) when $u = 1$, assume all initial points are zero, draw the output *y*

HW_5.2 (simular to Ex.5.5)

$\dot{x}_1 = x_2, \dot{x}_2 = -\sin(x_1) - \mathrm{bx}_2 + u, y = x_2 \;\; , t \in [0 \;\; 10]$

1) plot a simulink for this system

2) when $u = 1$, assume all initial points are zero,draw the output *y*

Passive and positive real transfer function.

https://www.mathworks.com/help/control/ug/parallel-interconnection-of-passive-systems.html

The system can be passive in time domain as $u^T y \geq 0$. How about in frequency domain? Before to investigate to this, we need a new definition : positive real transfer function

- Def.5.4 : An mxm proper rational tensfer function matrix $G(s)$ is (strictly) positive real if

1) all poles of $G(s)$ are in Re[s] <= 0 (< 0)

2) $G(j\omega) + G^T(-\omega)$ is positive semidefinite (definite) real. -EnD-

Here example

$G(s) = \frac{1}{(s+5)}$

```
clear all;
nyquist(tf([1],[1 5]))
```

How about the inverse?

```
nyquist(tf([1 5],[1]))
```

Lemma 5.4 Kalman-Yakubovich- Popov lemma

The linear time-invaraint minimal realization

$\dot{x} = \mathrm{Ax} + \mathrm{Bu}$, $y = \mathrm{Cx} + \mathrm{Du}$ with $G(s) = C(\mathrm{sI} - A)^{-1}B + D$

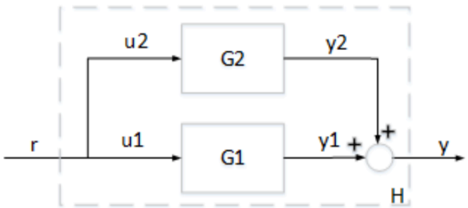
is

- passive if $G(s)$ is positive real
- strictly passive if $G(s)$ is strictly positive real -The EnD-

The Inverse of a passive system is passive. Moreover

- pararrel interconnection of passive systems is passive
- series interconnection of passive systems is passive
- The feedback interconnection of passive systems is passive

1. Parallel interconnection

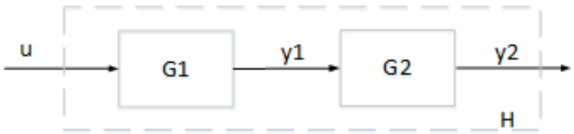


ex:

$$G_1(s) = \frac{(0.1s + 1)}{(s + 2)},\ G_2(s) = \frac{(s^2 + 2s + 1)}{(s^2 + 3s + 1)}$$

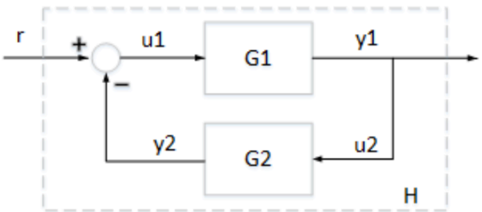
```
clear all;
G1 =tf([0.1,1], [1,2]);
isPassive(G1)
G2=tf([1,2,2],[1,3,10]);
isPassive(G2)
H1 = parallel(G1,G2);
isPassive(H1)
```

2. Series connection




```
H2 = G2*G1;
isPassive(H2)
```

3. Feedback connection




```
H3 =feedback(G1, G2);
isPassive(H3)
```

Which one is not passive? - non-minimum phase

By definition,

1) $G(s)$ is not Hurwitz : understandable

2) evenif $G(s)$ is Hurwitz, it may not be positive real. - non-minimum phase.

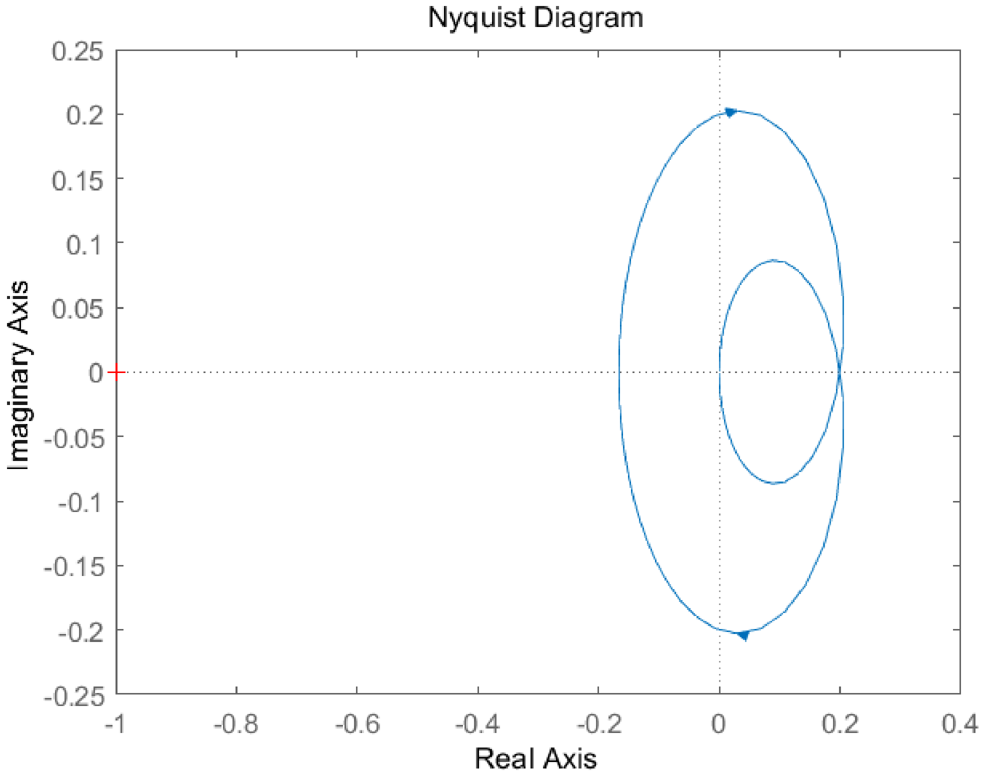
Since $G(s)$ is positive real, then its inverse is also positive real

--> If one of the zeros of the $G(s)$ is not in LHP, it's inverse is not Hurwitz, hence it is not positive real

This needs ato investigate a little more. Which one is not positive real? One of the system is not non-minimum phase, i.e., the zeros of the transfer function are located in RHP. Consider

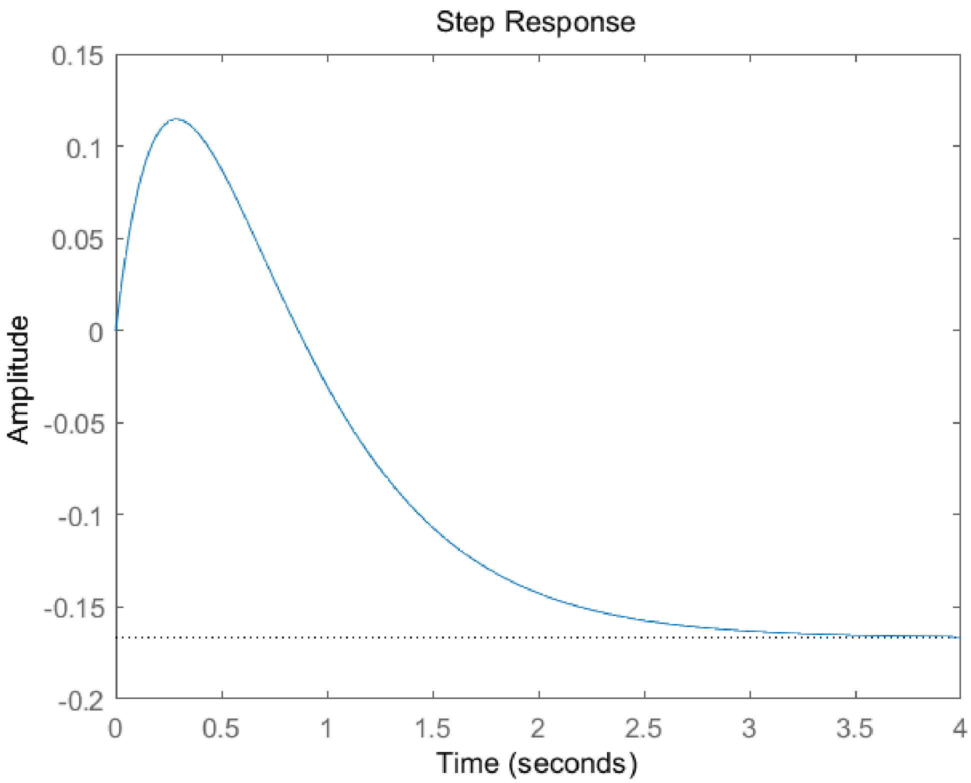
$$G(s) = \frac{(s-1)}{(s^2 + 5s + 6)}$$

```
clear all
G= tf([1 -1], [1 5 6]);
nyquist(G)
```



Hence, the Nyquist plot is located in not only positive but negative region. What are characteristics of input output relation? Assume input is a step, the output is

```
step(G)
```



The steadt state is

$$\lim_{t\to\infty} y(t) \; = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left(\frac{1}{s}\right)\frac{(s-1)}{(s^2+5s+6)} = -\frac{1}{6}$$

It is verified the numerical simulation. But the output in some time interval is moved to the opposite direction. It is different delay time but the direction is the opposite direction to the input.

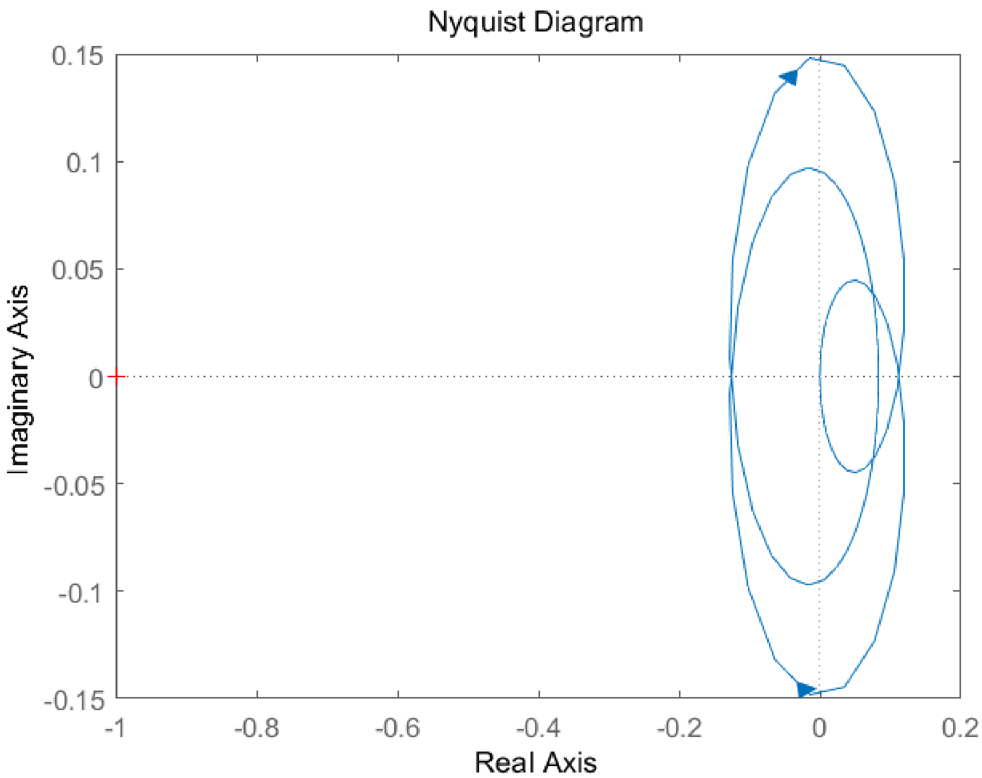
Anorther example as

$$G(s) = \frac{(s-1)(s-2)}{(s^2+5s+6)(s+4)} = \frac{(s^2-3s+2)}{(s^3+9s^2+26s+24)}$$

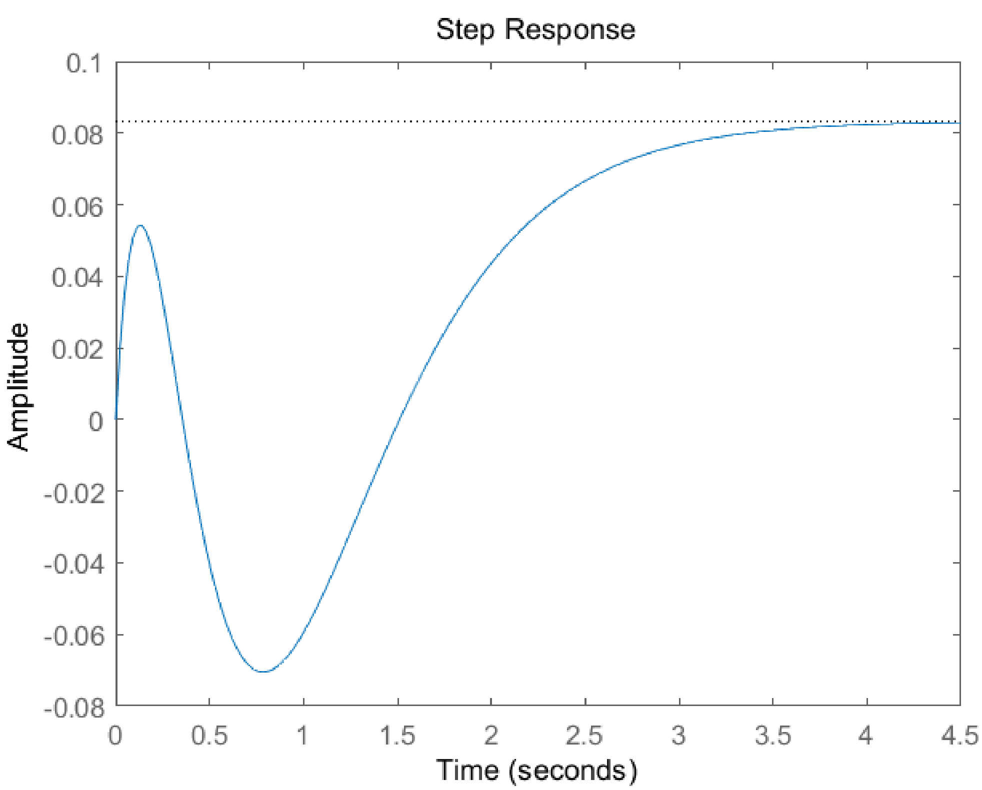
```
clear all
z = poly([1 2]);
p = poly([-2 -3 -4])
```

```
p = 1×4
     1     9    26    24
```

```
G= tf([z], [p]);
nyquist(G)
```


Nyquist Diagram
Imaginary Axis
Real Axis
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-1
-0.8
-0.6
-0.4
-0.2
0
0.2

```
step(G)
```


Step Response
Amplitude
Time (seconds)
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

You may see the output in some time interval is going to negative direction different to the steady state value. Hence if you measure the output before the steady state, you may be incorrect steady state estimation.

SInce for any linear state feedback, even if you may change poles to any points, however the zeros can not be changes, so that non-minimum system can not be minimum by any linear state feedback.

One of the physical systems whos transfer function is non-minimum is an inverted pendulum on a moving cart.

HW_5.3 Why an inverted pendulum on a moving cart(p. 357) is non-minimum phase?

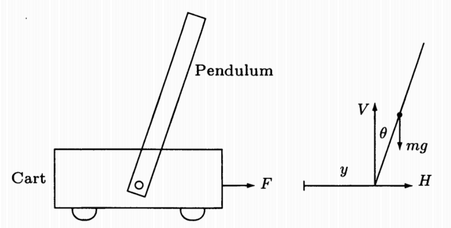


Figure A.12: Inverted pendulum on a cart.

1) The pendulum is initially located upward as in the figure. Without input force *F* to which direction of the cart will be moved (left or right)?

2) In order to stabilize the pendulum (at $\theta_e = 0$) to which direction of the force is applied? -- The End -

5. Related to the stability especially feedback system.

In these example, if the system of a positive real transfer function with a feedback(negative) positive real function, the system is passive. How about stability?

To check the stabilty of the closed loop system, Nyquist criteria may be used, i.e.,

The Nyquist plot of $1 + G_1 G_2$ should be in RHP

--> If $G_1 G_2$ is not encircled (-1, 0), then the closed loop system is stable.

--> the phase of $G_1 G_2$ is not greater than -180 degree

Now if G_1, G_2 are positive real, then

$\angle G_1 \le 90, \angle G_2 \le 90 \; -\to \; \angle G_1 G_2 \le 180$

which implies it is not encircled (-1,0), so that **the closed loop is stable! Remember the closed system is positive real.**

One additional moments is the G_1, G_2 should be postive real.

6. Now how about non-linear system?

See the Theorem 7.1. Wonderful~~