Symbolic math - Matlab

- Concept

Perform symbolic comutations from Matlab command line.

1. plot:

Consider to plot a function as

$$f(x) = x^2 + 3x + 3$$

To plot $f(x), x \in [-1, 1]$,

1) numerically

```
clear all
x = linspace(-1,1,100);
figure(1)
plot(x.^2+ 3*x + 3)
```

Here we may define the variable *x* as sampled valued from -1 to 1 with 100 points.

2) symbolic math

Now we may define symbolic object *x* as

```
syms x
figure(2)
fplot(x^2+3*x+3, [-1 1], 'r')
```

in the symbolix math, the variable as "continuos variable" comared to "sampled point" in 1) case. Plot cmd is "fplot", function plot, rather than "plot".

2. As a "variable"

Consider

$$y = 2\frac{(x+3)^2}{x^2 + 6x + 9}$$

then clearly $y = 2$,

```
clear all;
syms x
y = 2*(x+3)^2 / (x^2+ 6*x+9);
simplify(y)
```

The answer is the same as analytically.

3. Solve algebraic equation

Consider to find *x* to satisfy the equation

$$x + 3 = 0$$

or

$$x^2 + 3x + 2 = 0$$

```
clear all
syms x
eqn1 = x+3 == 0;
eqn2 = x^2 + 3*x +2 ==0 ;
S1 = solve(eqn1)
S2 = solve(eqn2)
```

Is it wonderful? more in advance find the solution to the 2-nd order equation.

$$x^2 + \mathrm{ax} + b = 0$$

Here, not only *x* but a, b are symbolic objects.

```
clear all
syms x a b
f = x^2 + a*x+ b ==0;
S1 =solve(f,x)
S2 = solve(f,a)
```

- **Ex.1: More a complicated example be considered.** Let $f(V, V_0, R, C, T) = V - V_0 e^{\left(-\frac{T}{\mathrm{RC}}\right)}$.

Find *R* such that $f = 0$

- Derivatives

Consider $f(x) = \sin(x^2)$. Find $\frac{\mathrm{df}}{\mathrm{dx}}$

```
clear all;
syms x
f = sin(x^2)
Df = diff(f,x)
D=[-3 3];
fplot(f,D); hold on; grid on
fplot(Df,D,'r')
% for the specific value
double(subs(Df,2))
```

Or symbolic function be defined

```
clear all
syms f(x)
% f(x) = sin(x^2);
f(x) = sin(cos(x));
Df = diff(f,x)
% Df = Df(2)
% double(Df)
whos
```

- Linear differential equations (homogeneous)

Find the solution to

$$\frac{\mathrm{dy}}{\mathrm{dt}} = -2y$$

```
clear all
syms t y(t)
D =[0 3];
ode = diff(y(t)) + 2*y(t)== 0;
ySol = dsolve(ode)
```

With a specified initial condition, $y(0) = 2$

```
cond = y(0)==2;
ySol(t) = dsolve(ode,cond)
fplot(ySol(t), D); grid on
```

With an external input as a constant

$$\frac{\mathrm{dy}}{\mathrm{dt}} = -2y + 1$$

```
clear all;
syms t y(t)
ode = diff(y(t),t) + 2*y ==1;
D =[0 3];
cond = y(0)==2;
ySol(t) = dsolve(ode,cond)
fplot(ySol(t),D); grid on
axis([0 3 0 3])
```

Or with an external input $\sin(t)$

$$\frac{\mathrm{dy}}{\mathrm{dt}} = -2y + \sin(t)$$

```
ode = diff(y(t),t) + 2*y == sin(t)
cond = y(0)==2;
```

```
fplot(sin(t),D); hold on; grid on
ySol(t) = dsolve(ode,cond)
fplot(ySol(t),D); grid on
axis([0 3 0 3])
```

- With an unknown parameter.

$$\frac{\mathrm{dy}}{\mathrm{dt}} = \mathrm{ay}$$

where "a" is unknown.

```
clear all
syms y(t) a
ode = diff(y(t),t) + a*y== sin(t)
ySol(t) = dsolve(ode)
```

- **Ex.2 Plot the solution for the different value** $a = 1, 2, 3$

- Here a nonlinear autonomous ODE

$$\frac{\mathrm{dx}}{\mathrm{dt}} = -x + x^2$$

```
clear variables
syms x(t) t
D=[0 5];
ode = diff(x(t),t) + x(t) - x(t)^2 ==0;
xSol(t) = dsolve(ode)
```

Here there are three solutions to the equation,

$$x_1(t) = 0, x_2(t) = 1, x_3(t) = -\frac{1}{\left(e^{(C_1+t)} - 1\right)}$$

The two are trivial if you substitute them into the ODE, however the third is not convinced. Here the third contains a undetermined variable C_1 which is due to undefined initial point. To confirm it is a solution or not, check it

```
clear all;
syms  C t
f = -1/(exp(C+t)-1);
g=diff(f)
h=diff(f)+f-f^2
```

Hence the third is also a solution. Let us define the initial point $x(0) = 1$

```
clear variables;
syms x(t) t
D=[0 5]
ode = diff(x(t),t) + x(t) - x(t)^2 == 0;
cond = x(0)==1;
xSol(t) = dsolve(ode, cond)
```

As you may expect from the solution,

$x_1(t=0) = 0$, which is the satisfy $x(0) = 1$, hence it is not a solution

$x_2(t=0) = 1$, which satisfies $x(0) = 1$, hence it is a solution

$x_3(t=0) = -\frac{1}{\left(e^{(C_1+t)} - 1\right)}\Big|_{t=0} = -\frac{1}{e^{C_1} - 1} = 1$, which satisfies if $e^{C_1} = 0$

which means $C_1 = -\infty$ which is not a solution. The only solution to this general ODE is $x(t) = 1$. This is one of the difference between L-ODE.

- **Ex.3 Consider**

$$\frac{\mathrm{dx}}{\mathrm{dt}} = -x + x^2$$

1) linearize at $x = 0$ and check the stability near x=0. Let us define $x(0) = 0.0001$. draw the trajectory.

2) linearize at $x = 1$ and check the stability near x=1. Let us define $x(1) = 1.0001$. draw the trajectory.

- If a non-homogeneous case, let us define the external force as constant =1, 1

$$\frac{\mathrm{dx}}{\mathrm{dt}} = -x + x^2 + 1$$

Find the solutions

```
clear variables
syms x(t) t
D=[0 5];
ode = diff(x(t),t) + x(t) - x(t)^2 ==1;
xSol(t) = dsolve(ode)
```

Here the solutions are quite different to the homogeneous case!!

- Another non-L ODE

Consider

$\frac{\mathrm{dx}}{\mathrm{dt}} = -x + x^3$

```
clear variables
syms x(t) t
D=[0 5];
ode = diff(x(t),t) + x(t) - x(t)^3 == 0;
xSol(t) = dsolve(ode)
```

Let us draw a trajectory of the solution with the initial point $x_0 = 1.0001$

```
clear variables
syms x(t) t
D=[0 5];
ode = diff(x(t),t) + x(t) - x(t)^3 == 0;
cond = x(0) == 0.99999
```

cond =

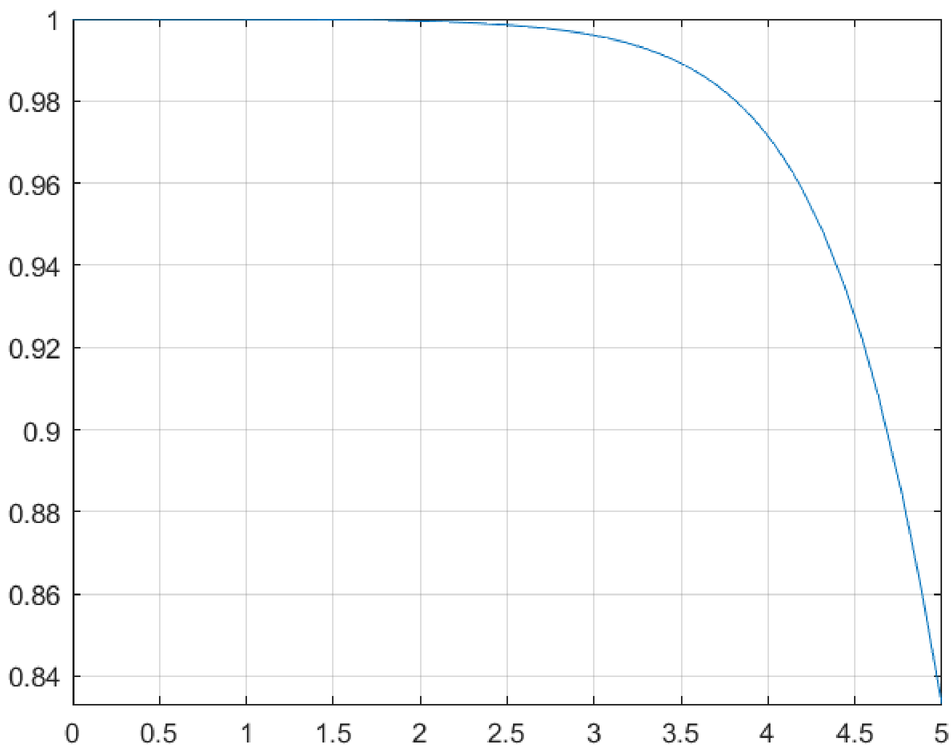
$$x(0) = \frac{99999}{100000}$$

```
xSol(t) = dsolve(ode, cond)
```

xSol(t) =

$$\sqrt{\frac{1}{e^{2\,t+\log\left(\frac{199999}{9999800001}\right)}+1}}$$

```
fplot(xSol(t),D); grid on
```



Hence this with $x_0 = 1.0001$ will be unbounded as time increase.

Ex.4 In the following cases, regarding stability, anayize with different initial conditions as

1) $x_0 = 0.9999$

2) $x_0 = 0.0001$

3) $x_0 = 0.5$